VOL. 9, NO. 12, DECEMBER 2014 ISSN 1819-6608

ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

MATHEMATICAL MODEL OF CYCLIC LOADING OF MATERIAL MR

Alexander Aleksandrovich Troynikov and Konstantin Vladimirovich Boyarov
Samara State Aerospace University (SSAU), Moskovskoye shosse, Samara-443086, Russia

ABSTRACT

Mathematical model in the shape of combination of exponent, power and exponential functions has been presented, which allows to build the library of hysteresis loop at cyclic loading of material MR, and to perform quantitative analysis of elastically damping characteristics. Characteristics of mathematical model are connected with technological parameters of material MR by means of dilations' coefficients [1].

Keywords: mathematical model, material MR, characteristics, vibration isolator, hysteresis loop, calculation.

1. INTRODUCTION

The process of cyclic loading of vibration isolators made of MR material may be described by the library of hysteresis loops in "loading-movement" coordinates. In [2] work physical model of MR material is presented as a system of cores (Figure-1), which are immovably clamped in root section. On the ends of cores "the boots" are located in such way than in no-load condition there would be clearance between them, and in loaded condition the contact would be possible for small platforms. Outer load is applied to end section of the first core. In the process of deformation of core system, some part of load from the first core is transmitted to further cores via contact surfaces of "the boots".

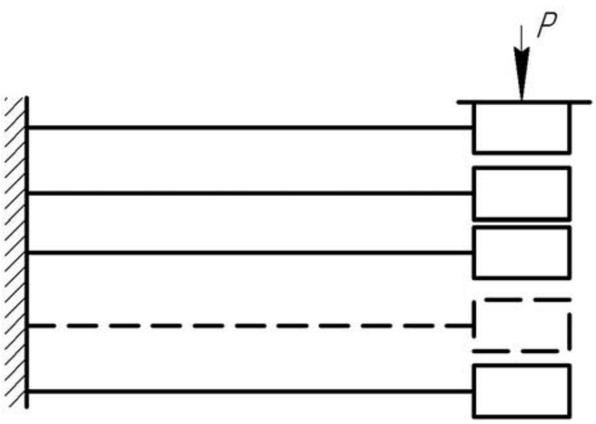

Figure-1. The model of contact interaction between elements in MR.

According to primer approach, MR product is presented as a system of structural damping, which is composed of core systems, sequentially and parallely connected with each other. With the help of physical model it is possible to describe the process of cycling loading of vibration isolator in steps (with zigzag lines), here is the loading process:

$$P_i = \frac{Y_{\text{э}} c_i}{1-\frac{3hi}{4l}f}\sum_{i=1}^{m} i; \quad Y_i = Y_{\text{э}} i;$$

here is unloading process:

$$P_j = \frac{Y_{\text{э}} c_j f}{2+f\sum_{j=1}^{m_1} j}\sum_{j=1}^{m_1} j; \quad Y_j = 2f\,\frac{Y_{\text{э}}\sum_{j=1}^{m_1} j}{2+f\sum_{j=1}^{m_1} j;}. \tag{1}$$

Here $Y_{\text{э}}$ is a clearance between model's cores; Y_i is isolator's deformation at the first stage; c_i is core's stiffness at j-stage of unloading system; f is frictional coefficient of elements' surfaces; h and l stand for cores' geometrical parameters.

2. METHODS

While developing vibration isolation systems, when it is required to calculate several variants of vibration isolators, as well as in the process of their operation at transient states, the usage of correlations (1) is associated with a range of difficulties. First, for building library of hysteresis loop that describe elastically frictional properties of products made of MR, a huge volume of calculation works is required. Second, solving dynamical problems with similar analytical methods becomes difficult, when loading and unloading processes also need to be expressed as analogue functions, and the model describes these processes via polygonal functions.

Thus, for more widespread practical use of physical modeling's results, the process of cycling loading of MR products should be presented as a mathematical model, which would be useful for solving dynamic tasks. There are mathematical models that doffer according to method of hysteresis loops' description, processes of cyclical deformation of constructional damping and according to types of used vibration isolating and damping devices [3-6]. We offer mathematical model, expressed in combination of exponent, power and exponential functions has been presented, which allows building the library of

VOL. 9, NO. 12, DECEMBER 2014 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

hysteresis loop at cyclic loading of material MR in more convenient way for solving dynamic tasks.

Let us write an equation of elastic line of core system (considering (1) f=0) in relative coordinates:

$$\eta = a\Phi_1(\xi)\Phi_2(\xi), \tag{2}$$

where $\eta = P/T_0$; $\xi = Y/Y_0$; P is load; Y is moving; T_0, Y_0 are coefficients of process' dilations by load and movement, accordingly, [1]; a is coordinating invariable.

$\Phi_1(\xi) = c_i(\xi_i)$ function defines system's stiffness. In coordinates $\ln c_i;\ \xi$ this function is linear, thus, it is subjected to exponential law:

$$\Phi_1(\xi) = a_i e^{c\xi} \tag{3}$$

$\Phi_2(\xi) = \sum_{i=1}^{m} i$ function expresses the sum of geometric progression's components, with a common difference that equals 1 (since i=1, 2, 3, ..., m);

$$\sum_{i=1}^{m} i = S_i = \frac{i(i+1)}{2},$$

which at i >5 may be expresses as a power function:

$$S_i \approx \frac{l^2}{2} = a_2\xi^b. \tag{4}$$

Let us substitute (3) and (4) in (2), and we will receive equation of elastic curve of core system in generically:

$$\eta = a\xi^b e^{c\xi},$$

where $a = a_1 a_2$.

Considering that the process of MR product's deformation is built by summing processes of deformation of core systems (consequently and parallely connected with each other), let us write equations of loading and unloading processes of vibration isolator (Figure-2):

$$\eta_н = a_н \xi^{b_н} e^{c_н \xi}$$
$$\eta_p = \eta_m - a_p(2\xi_0 - \xi)^{b_p} e^{c_p(2\xi_0 - \xi)} \tag{5}$$

where $\eta_m = a_н(2\xi_0)^{b_н} e^{c_н 2\xi_0}$ is a vertical axis of top of hysteresis loop; $a_н, b_н, c_н, a_p, b_p, c_p$ are coefficients of approximating function of loading and unloading branches of hysteresis loop.

Approximating function of loading and unloading must coincide with hysteresis loop, built according to correlations (1) in points O, C and B, E (Figure-2). Coincidence in points O and C provides loop's closeness, and coincidence in points B and E provides determination accuracy of dilations' coefficients. Besides, for provision of minimal error of loop's form it is necessary to make sure it is best approximated to model loop at the beginning of loading and unloading processes. These points should be butted out at $\xi = 0,25\xi_0$ (for loading processes) and at $\xi = 1,75\xi_0$ (for unloading processes). A point (Figure-2) corresponds to the heel region of loading branch, D point stands for the areas with the biggest non-linearity $(d^3\eta/d\xi^3 = 0)$ of unloading branch.

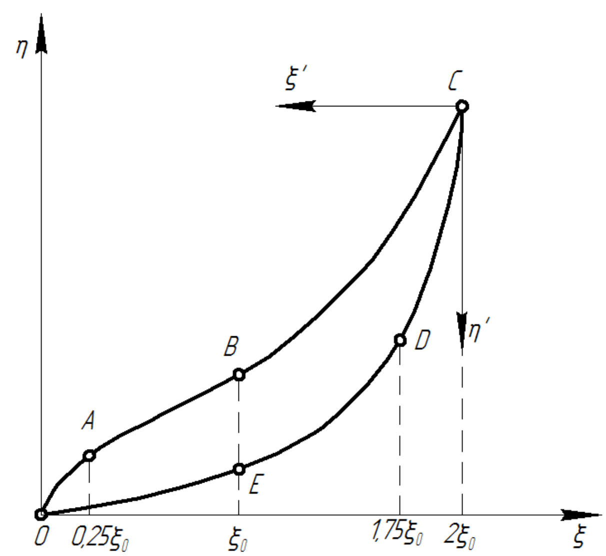

Figure-2. Hysteresis loop of vibration isolation of material MR.

Thus, based in conditions of the best fitting of approximating functions, vibration isolator's loading process must go through points $A(\eta_A; 0,25\xi_0)$, $B(\eta_B; \xi_0)$; while unloading process must go through points $E(\eta_E; 0,25\xi_0)$; $D(\eta_D; \xi_0)$; $C(\eta_C; 2\xi_0)$ of modeling hysteresis loop. Equation (5) coefficients are defined by method of "selected points". After taking logarithms, equation systems are composed for loading processes in coordinates η, ξ:

ARPN Journal of Engineering and Applied Sciences

$$\begin{aligned} \ln \eta_C &= \ln a_н + b_н \ln 2\xi_0 + 2\xi_0 c_н; \\ \ln \eta_B &= \ln a_н + b_н \ln \xi_0 + \xi_0 c_н; \\ \ln \eta_A &= \ln a_н + b_н \ln 0,25\xi_0 + 0,25\xi_0 c_н. \end{aligned} \qquad (6)$$

and for loading processes in coordinates η', ξ':

$$\begin{aligned} \ln \eta_C &= \ln a_p + b_p \ln 2\xi_0 + 2\xi_0 c_p; \\ \ln \eta_E &= \ln a_p + b_p \ln \xi_0 + \xi_0 c_p; \\ \ln \eta_Д &= \ln a_p + b_p \ln 0,25\xi_0 + 0,25\xi_0 c_p. \end{aligned} \qquad (7)$$

Then systems of equations (6) and (7) are solved with respect to required coefficients:

$$b_H = \frac{\ln\left(\dfrac{\eta_B^2}{\eta_C \eta_A}\sqrt[4]{\dfrac{\eta_C}{\eta_B}}\right)}{1,25\ln 2};$$

$$c_H = \frac{\ln\dfrac{\eta_C}{\eta_B} - b_H \ln 2}{\xi_0};$$

$$a_H = \exp\left(\ln \eta_C - b_H \ln\left(2\xi_0\right) - c_H 2\xi_0\right);$$

$$b_P = \frac{\ln\left(\dfrac{\eta_E^2}{\eta_m \eta_D}\sqrt[4]{\dfrac{\eta_m}{\eta_E}}\right)}{1,25\ln 2};$$

$$c_P = \frac{\ln\dfrac{\eta_m}{\eta_E} - b_P \ln 2}{\xi_0};$$

$$a_P = \exp\left(\ln \eta_m - b_P \ln\left(2\xi_0\right) - c_P 2\xi_0\right);$$

After substitution of numeric values of $\eta_A, \eta_B, \eta_C, \eta_D, \eta_E$, linear connections of approximating functions' coefficients are found with relative amplitude of vibration isolator's deformation:

$$\begin{aligned} k_1 &= 0,84\xi_0^{-0,543} e^{-0,0836\xi_0}; \\ k_2 &= 0,166\,\mathrm{tg}\frac{\pi(3-\xi_0)}{7,25} + 0,4; \\ k_3 &= \frac{0,0098}{(\xi_0+0,1)^3} + \frac{0,098}{(\xi_0+0,1)^2} - \frac{0,0382}{\xi_0+0,1} + 0,362; \end{aligned} \qquad (8)$$

$$\begin{aligned} k_4 &= \exp\left(\ln \eta_m - k_5 \ln 2\xi_0 - k_6 2\xi_0\right); \\ k_5 &= 0,05\,\mathrm{tg}\frac{\pi}{8}(3,8-\xi_0) + 0,37; \\ k_6 &= \frac{0,0022}{(\xi_0+0,05)^3} - \frac{0,0414}{(\xi_0+0,05)^2} - \frac{0,055}{\xi_0+0,05} - 0,025; \end{aligned} \qquad (9)$$

Having substituted correlations form (8) to (5), we will present mathematical model of cyclical loading of vibration isolators made of MR material in extended variables:

$$\eta_H = k_1 \xi^{k_2} e^{k_3 \xi}; \qquad (10)$$

$$\eta_p = \eta_m - k_4\left(2\xi_0 - \xi\right)^{k_5} e^{k_6(2\xi_0 - \xi)}; \qquad (11)$$

3. CONCLUSIONS

For the purposes of checking correspondence of received correlations to requirements, specified for models, along with control of dependencies' dimensions, orders and characters (collation of qualitative conclusions), there has been performed control of extreme situations, mathematical closeness and physical sense of the end result. It has been stated that suggested mathematical model allows uniquely solve task concerning MR products' deformation and to perform quantitative and qualitative analysis of the main characteristics.

REFERENCES

[1] Buzitsky V.N, Troinikov A. A. 1990. Raschët sleeve shock absorbers of the MR material, working on compression. In.: Seismic resistance and nadëzhnostdvigateley and aircraft systems. Kuibyshev: Kuai. pp. 14-15.

[2] Belousov A.I., Troinikov A.A. 1985. Mathematical model of cyclic loading of vidration isolator vade of MR. Soviet Aeronautics (English translation of Izvestiya VUZ, Aviatsionnaya Tekhnika). 28(3): 17-21.

[3] Yan H., Wang L., Jiang H.-Y., Ulannov A.M. 2010. RETRACTED ARTICLE: Analysis of the basic mechanical parameters of metal rubber materials. ICACTE 2010 - 2010 3rd International Conference on Advanced Computer Theory and Engineering, Proceedings, 1, art. No. 5578992, pp. V1396-V1399.

[4] Ponomarev Yu.K., Ermakov, A.I., Simakov, O.B., Mikhalkin I.K. 2013. Metallic counterpart of rubber: A material for vibration and shock protection. Metal Science and Heat Treatment. 55(1-2): 8-13.

[5] Yan H., Jiang H.-Y., Liu W.-J., Hao Z.-D., Ulannov A.M. 2010. Analysis of acceleration response of metal rubber isolator under random vibration. Wuli Xuebao/Acta Physica Sinica. 59(6): 4065-4070.

[6] Yan H., Jiang H.-Y., Liu W.-J., Ulannov A.M. 2009. Identification of parameters for metal rubber isolator with hysteretic nonlinearity characteristics. Wuli Xuebao/Acta Physica Sinica. 58(8): 5238-5243.

[7] Jiang H.-Y., Hao D.-G., Xia Y.-H., Ulanov A.M., Ponomarev Y.K. 2008. Damping characteristics calculation method of metal dry friction isolators. Journal of Beijing Institute of Technology (English Edition). 17(2): 173-177.