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# MODELING OF EXTREMITY DISTAL FIELD HEMODYNAMIC

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**ABSTRACT**

In this paper the construction of a model of the distal vessels is considered on the basis of O. Frank's and A. P. Favorskii's works. An algorithm for numerical solution of the equation of hemodynamics is proposed. A. P. Favorskii's model is added refined equation of state, the explicit scheme is applied to the intermediate spatial filtering of high-frequency oscillations of the velocity profiles, pressure and cross-section.

**Keywords:** signal, hemodynamic, modulation, section, vessel.

## 1. INTRODUCTION

Nowadays many devices are manufacturing, realizing the removal of photoplethysmogram (PPG) signal of limbs and its analysis. At the same time a process of its generating and influence on PPG of various factors both biological, and external character is investigated not completely. The problem of creation of adequate model on which it is possible to investigate these dependences with acceptable accuracy, is actual.

Historically the first attempt to replicate the cardiovascular system performance has been made in O. Frank's work [1, 2]. In Frank's model the following allowances are made:

- All large vessels are being modelled by one elastic reservoir which volume is proportional to pressure.
- The vessels represent the rigid tubes.
- Elastance and resistance for each vessel's group are constant in time and in space.
- The transient processes in blood streams are not being concerned.

A. P. Favorskii [3] has offered an algorithm of the numerical solution of hemodynamic equations on the vessels graph including the main organs of vascular system. The laws of contiguity and impulse conservation are put in basis of the mathematical description of blood movement in cardiovascular system. The transversal size of the vessels is considered small in comparison with their length that allows to pass to quasi-one-dimensional approach.

In this work A. P. Favorskii's model is supplemented by improved equation of condition describing in more details the distal field of the vessel. Besides, the obvious scheme is applied to digital differentiation of equations system with mediate spatial filtering of high-frequency fluctuations of profiles of speed, pressure and section [4].

## 2. PROBLEM STATEMENT AND PRELIMINARIES

### 2.1. Equations system of hemodynamic

For the purposes of this work it is possible to consider the hemodynamic problem as one-dimensional movement of incompressible fluid. In this case the blood movement can be described by the following system of nonlinear partial differential equations [5, 6]:

$$\begin{cases} \frac{\partial S}{\partial t} + \frac{\partial uS}{\partial x} = \varphi(t,x,S,u) \\ \frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2} + \frac{p}{\rho}\right) = \psi(t,x) \end{cases} \quad (1)$$

where ***t*** - the time, ***x*** - the longitudinal coordinate, ***u(t, x)*** - the rate of blood flow along the vessel, ***p(t, x)*** - an average blood pressure inside the vessel, ***s(t, x)*** - the area of cross-section of the vessel, $\rho$ - the blood density, $\varphi(t,x,S,u)$ - the blood inflow or outflow, $\psi(t,x)$ - the sum of external and friction forces.

The first equation (1) represents the continuity equation, and the second - the movement equation.

For the case concerned in this work $\varphi(t,x,S,u) = 0$ as in the vessel the invariance of blood volume is assumed.

$\psi(t,x)$ in (1) it is possible to provide as:

$$\psi(t,x) = f_e(t,x) + f_R(t,x),$$

where $f_e(t,x)$ - an external force, $f_R(t,x)$ - the friction force caused by the blood viscosity.

Let's consider further a case of fluid laminar stream for which the friction force can be defined as follows: $f_R(t,x) = \frac{8\pi\nu u}{s}$ where the kinematic viscosity $\nu$ = 0.033 cm$^2$/sec [7].

For drawing up the closed system it is necessary to add the system (1) of condition equation describing an interaction of flow and the elastic vascular vessel's wall. Such equation describes an empirical dependence of the area of cross-section on pressure in the stream: s= s (p), in the assumption that the vein represents an elastic passive tube. Let's choose a piecewise and linear approximation of the dependence s = by s (p) [4]:

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$$s(p)=\begin{cases} s_{\min}+\dfrac{s_{\max}-s_{\min}}{p_{\max}-p_{\min}}(p-p_{\min}), & p_{\min}\le p \\ s_{\min}, & p<p_{\min} \\ s_{\max}, & p>p_{\max} \end{cases} \quad (2),$$

Where

$s_{\min}$ = the minimal section of the vessel,

$s_{\max}$ = the maximal cut of the vessel,

$p_{\min}$ = the minimal pressure in vessel section

$p_{\max}$ = the maximal pressure in vessel section.

In this work it is conducted the modification of equation of vessels condition (2) so that to take into to consideration the dependence ***s = s (p)*** on coordinate of section along the vessel i.e. the dependence turns to a kind ***s = s (p, x)***. The distal vessel on the extremity has the dilation with more resilient walls which can be taking into account indicting the dependence $s_{\max}=s_{\max}(x)$. In elementary case it is possible to apply the linear dependence:

$$s_{\max}=\begin{cases} s_{\max 0}, & x<m \\ s_{\max 0}+\dfrac{s_{\max 1}-s_{\max 0}}{L-m}(x-m), & x\ge m \end{cases} \quad (3),$$

Where

$s_{\min}$ = the minimal section of the vessel,

$s_{\max 0}$ = the maximal section of the vessel,

$s_{\max 1}$ = the maximal section of the vessel,

$p_{\min}$ = the minimal pressure in the vessel section,

$p_{\max}$ = the maximal pressure in the vessel section,
$m$ = the initial point of terminating field of the vessel,
$N$ = the vessel length.
The required dependence *s(p, x)* is gained by substitution of (3) into (2).
Let's converse the system (1) to the kind which is more convenient for solving:

$$\begin{cases} \dfrac{\partial S}{\partial t}+S\dfrac{\partial u}{\partial x}+u\dfrac{\partial S}{\partial x}=0 \\ \dfrac{\partial u}{\partial t}+\dfrac{\partial u}{\partial x}+\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}=f_e(t,x)+f_R(t,x) \end{cases} \quad (4).$$

The expressions (4), (2) and (3) compound the closed system solving which allows to determine the functions *u(t, x)*, *s(t, x)* and *p(t, x)*.

For solution it is required also an initial and boundary conditions for these functions.

In order to build-up the incremental scheme we use the following approximations for private derivatives:

$$\begin{aligned} \frac{\partial S}{\partial t}\Big|_i^{New} &= \frac{S_i^{new}-S_i}{\tau} \\ \frac{\partial u}{\partial t}\Big|_i^{New} &= \frac{u_i^{new}-u_i}{\tau} \\ \frac{\partial S}{\partial x}\Big|_i &= \frac{S_{i+1}-S_{i-1}}{2h} \\ \frac{\partial u}{\partial x}\Big|_i &= \frac{u_{i+1}-u_{i-1}}{2h} \\ \frac{\partial p}{\partial x}\Big|_i &= \frac{p_{i+1}-p_{i-1}}{2h} \end{aligned} \quad (5)$$

Substituting (5) into (4), we will express $S_i^{New}$ and $u_i^{New}$:

$$\begin{aligned} S_i^{New} &= S_i-\frac{\tau}{2h}\left(S_i(u_{i+1}-u_{i-1})+u_i(S_{i+1}-S_{i-1})\right) \\ u_i^{New} &= u_i-\frac{\tau}{2h}\left(u_i(u_{i+1}-u_{i-1})+\frac{1}{\rho}(p_{i+1}-p_{i-1})\right)+\tau\cdot f_{вi}-\frac{8\tau\pi\nu u_i}{s_i} \end{aligned} \quad (6)$$

$S_i^{New}$ also $u_i^{New}$ represent the values of distributional functions of sectional area and blood flow rate on new temporal layer. After each evaluation the values $S_i^{New}$ and $u_i^{New}$ are being copied into arrays $S_i$ and $u_i$ respectively.

The values of elements of array $p_i$ are calculated by means of the function ***p (s)*** being the inverse function from function (2).

**2.2. Boundary conditions**

For flow rate on the vessel extremities we will use the extrapolation of first order:

$$\begin{aligned} u_1 &= 2\cdot u_2-u_3 \\ u_N &= 2\cdot u_{N-1}-u_{N-2}, \end{aligned} \quad (7)$$

Where

$u_1$ = a flow rate at the beginning of the vessel,

$u_N$ = a flow rate at the end of the vessel.

Let's similarly determine the boundary condition on the vessel extremity for its section:

$$S_N=2\cdot S_{N-1}-S_{N-2} \quad (8)$$

The boundary condition for pressure on the extremity of the vessel is defined as $p_N=p(S_N)$, i.e. through the inverse function of the dependence (2).

Boundary conditions for pressure at the beginning of the vessel we will set as a special time sequence (triangular impulse), imitating the blood pressure outburst at systole [8]. Let's accept the systoles periods

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equal to one second, the pulse width of pressure we will accept equal to 100 msec. (Figure-1).

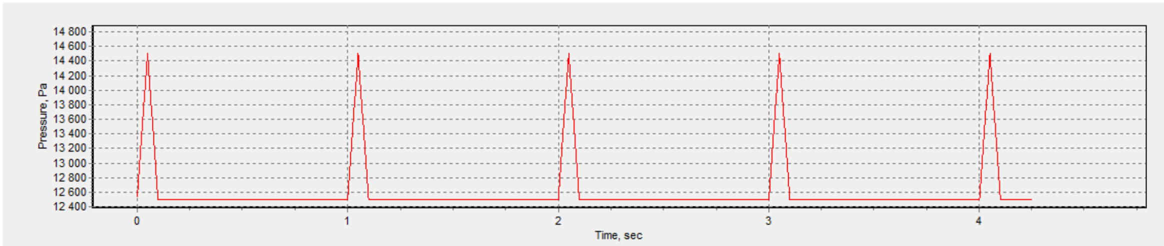


**Figure-1.** The sequence of triangular pulses simulating a surge in blood pressure during contraction of the heart.

$p_1$ for each time stratum we gain from impulse model, and the boundary conditions $S_1$ at the beginning of the vessel we determine by means of the expression (2):

$$S_1 = S(p_1) \tag{9}$$

Solving the problem of system (6) with boundary conditions (7), (8), (9) and with data from impulse model, on each time stratum we gain three arrays $u_i$, $S_i$ and $p_i$, respectively representing the instantaneous distributions of velocity, section and pressure along the vessel.

The PPG signal is proportional to the object optical density which is at first approximation proportional to the summary diameter of distal part of arteries. For estimation of PPG signal level it is possible to summarize the estimations of vessel diameter in its terminal field (the vessel section is proportional to the quadrate of diameter for the round vessel) [9]:

$$Fp^{New} = \sum_{i=m}^{N} \sqrt{S_i} \tag{10}$$

Where

$Fp^{New}$ = the next point of estimation of PPG trend on the current time stratum
m = the initial point of vessel terminal field
N = the vessel length

While using the explicit scheme (6) it is very often the progressing solution oscillations arise leading to the solution disorder. The usual approach to oscillations reduction - is the step diminution on time $\tau$ . However the test made for the scheme (6) have shown that the oscillations are not present at $\tau < 10^{-8}$, that leads to the major time expenditures at modelling operation. For oscillations reduction in [10] the special term is inducted into system (4), so-called the *mathematical viscosity*. In this work the other approach is used. According to the Doppler ultrasonography (US) it is known that the frequency spectrum of the flow rate compounds the unities of kiloherzes, i.e. at sonic speed in the water equal 1500 m/sec. the order of oscillation wave lengths makes 150-1500 mm. In our examinations the digitization step on length *h=0.5 mm* that is much less then the given value. For elimination of oscillations adverse effect it is enough to eliminate the frequencies with waves lengths $\lambda = 2h = 1\text{мм}$, or with frequency of 1.5 MHz.

For similar filtering the following set of non-recursive filters of low frequency was used:

$$un_i = (u_{i-1} + u_i + u_{i+1})/3 \; un_i = (u_{i-1} + 2u_i + u_{i+1})/4, \quad un_i = (u_{i-1} + 4u_i + u_{i+1})/6. \tag{11}$$

All the filters have a symmetrical, concerning the center, set of coefficients with the peak central coefficient. Such filters have a unitary transfer ratio and lack of phase distortions.

## SIMULATION RESULTS

The program has been developed for examination of hemodynamic model in Delphi 7 environment in Pascal language. The modeling has been conducted for the vessel in length of 50 mm with average sectional area 0.3 mm$^2$ . The distal field of the vessel compounded 10 mm, there at the sectional area of this field at maximal pressure can reach 2.8 mm$^2$. The peak and minimal pressure in the vessel is 15000 and 10000 Pa respectively. The model impulse of pressure has an amplitude of 2000 Pa. By means of the program the pressure trends (Figure 2-4), the flow rates (Figure 5-7), the velocity profiles (Figure 10-11), the pressures (Figure 8-9) and the vessel sectional areas (Figure 12-13) and the signal PPG kind presented in Figure 14 are gained.

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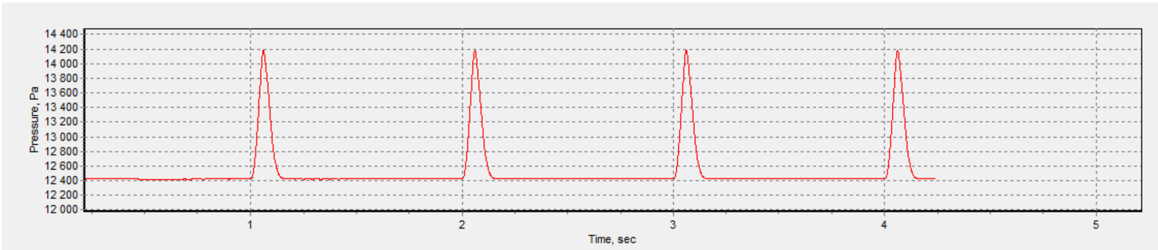


**Figure-2.** The pressure trend (in the point 10th from the vessel beginning).

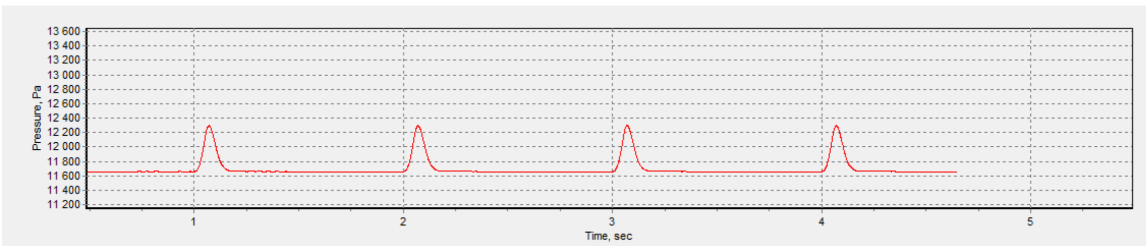


**Figure-3.** The pressure trend (in the point 70th from the vessel beginning).

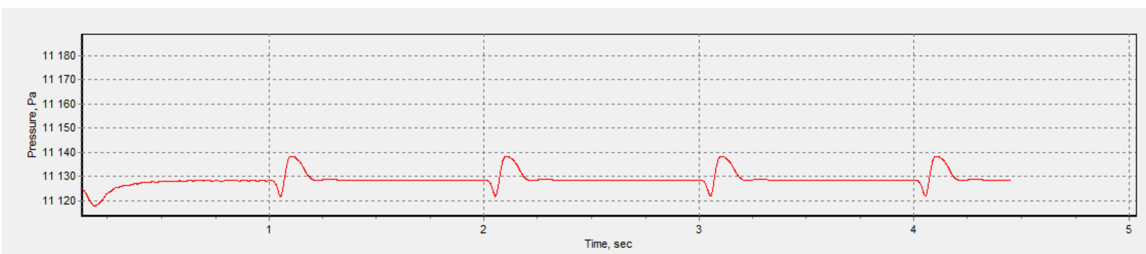


**Figure-4.** The pressure trend on distal field (in the point 90th from the vessel beginning).

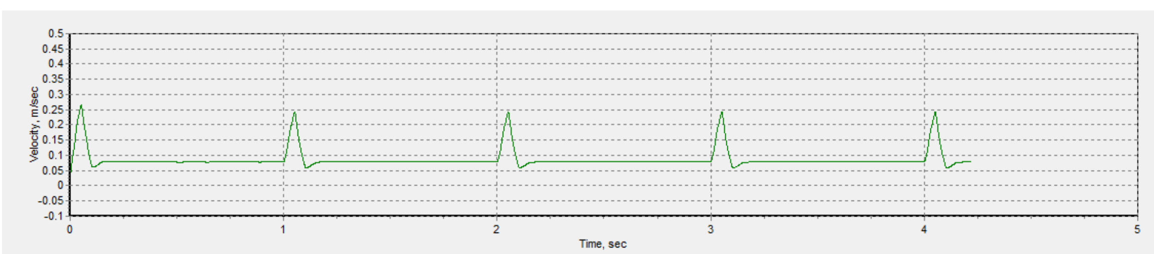


**Figure-5.** The flow rate trend (in the point 10th from the vessel beginning).

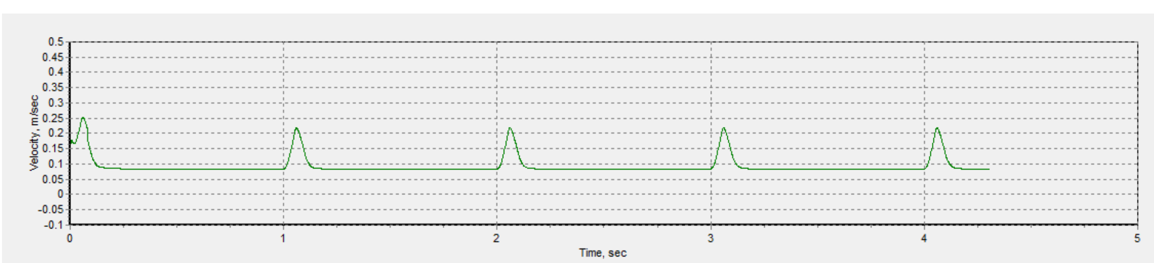


**Figure-6.** The flow rate trend (in the point 70th from the vessel beginning).

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**Figure-7.** The flow rate trend on distal field (in the point 90$^{th}$ from the vessel beginning).

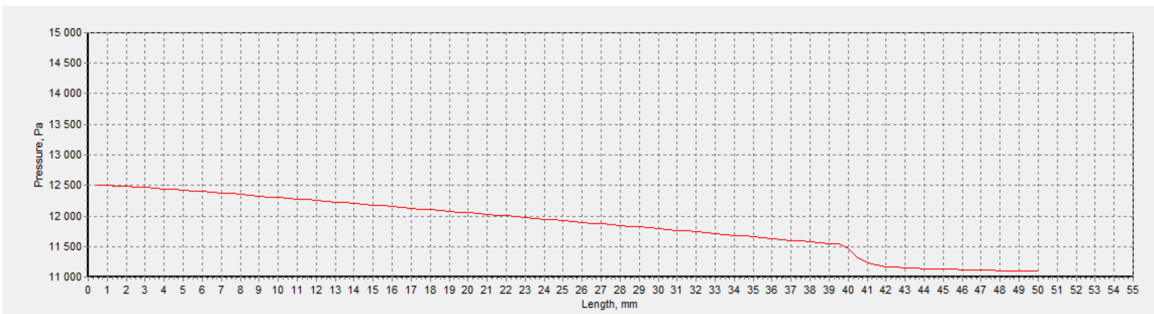


**Figure-8.** The pressure profile in the diastole.

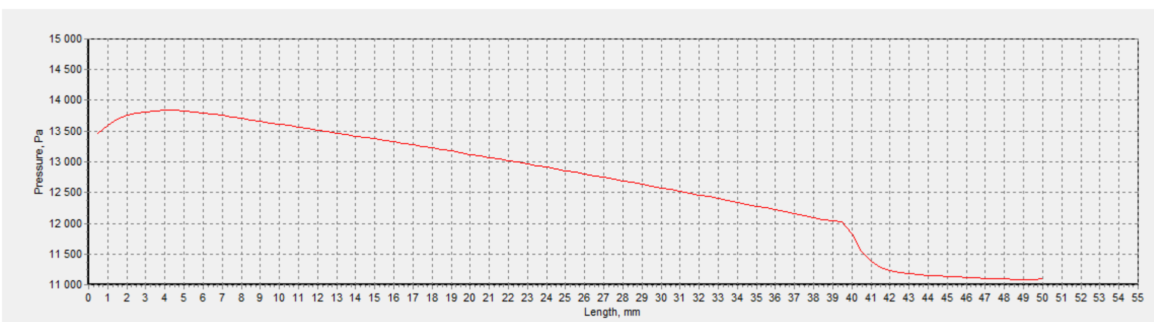


**Figure-9.** The pressure profile in the systole.

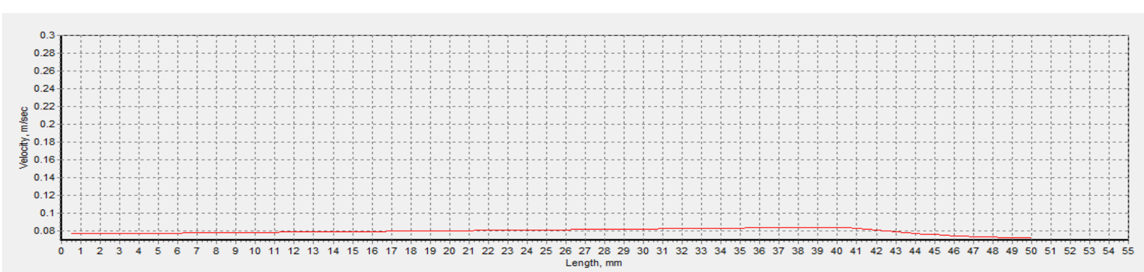


**Figure-10.** The flow rate profile in the diastole.

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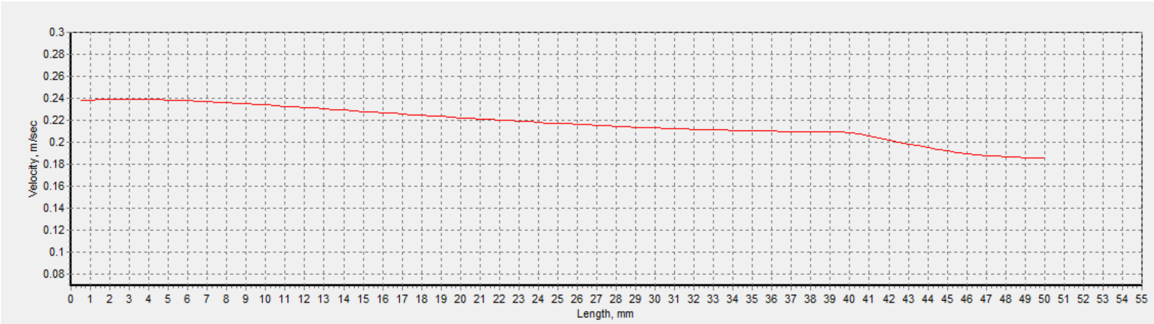


**Figure-11.** The flow rate profile in the systole.

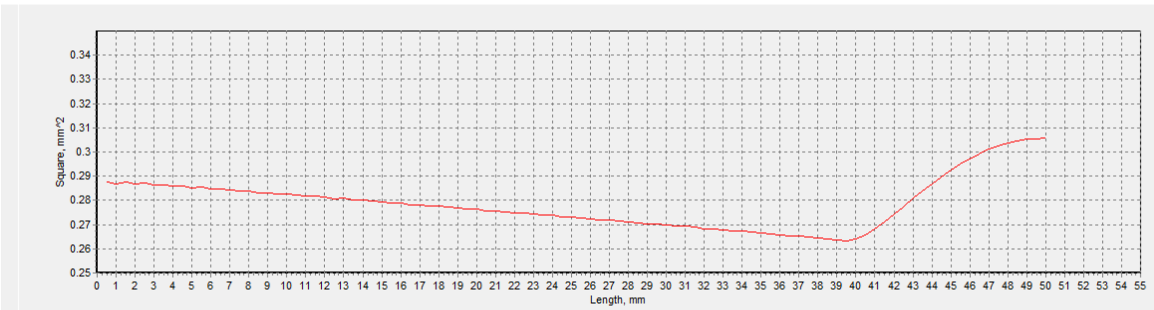


**Figure-12.** The profile of the vessel area in the diastole.

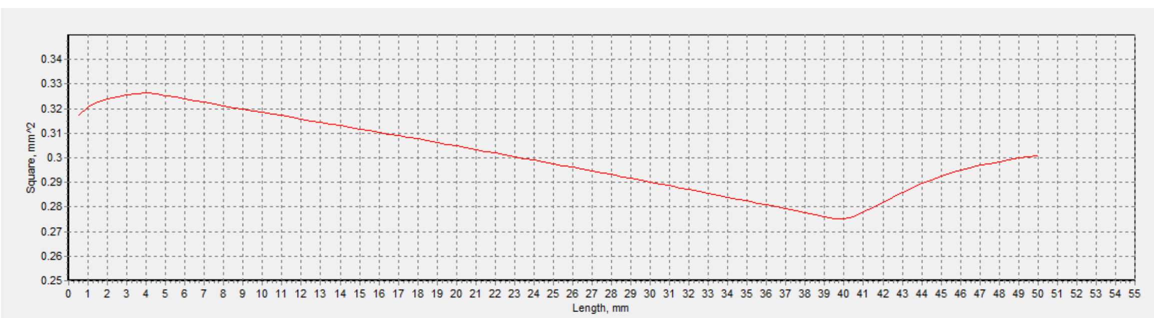


**Figure-13.** The profile of the vessel area in the systole.

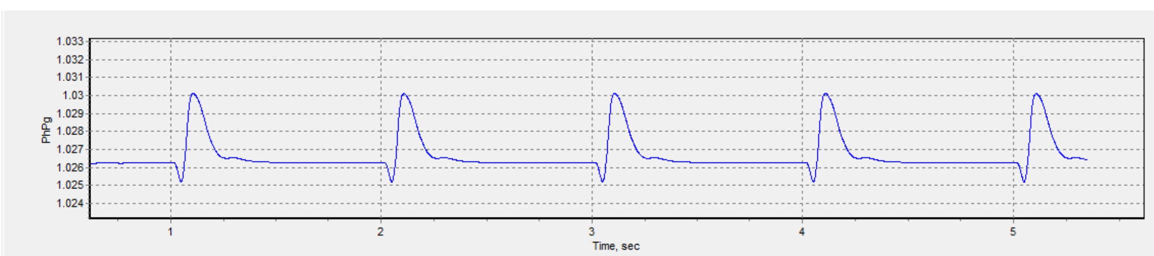


**Figure-14.** The view of PPG signal.

The problem on gravitation impact on process of PPG recording is rather important. The developed model allows to generate the PPG signal taking into account the provision of the limb. For this purpose in system (4) it is enough to lay $f_e(t,x) \neq 0$. The situation when the extremity is lowered was simulated. The results of modelling operation are shown in

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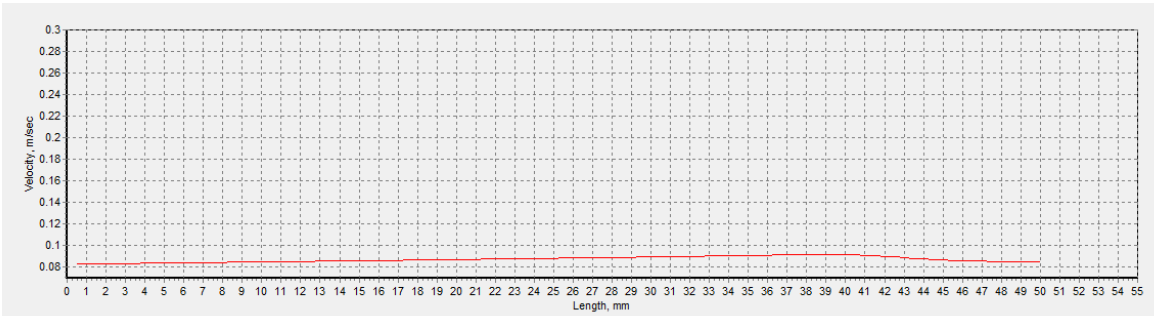


**Figure-15.** The flow rate profile in the diastole at gravitation impact.

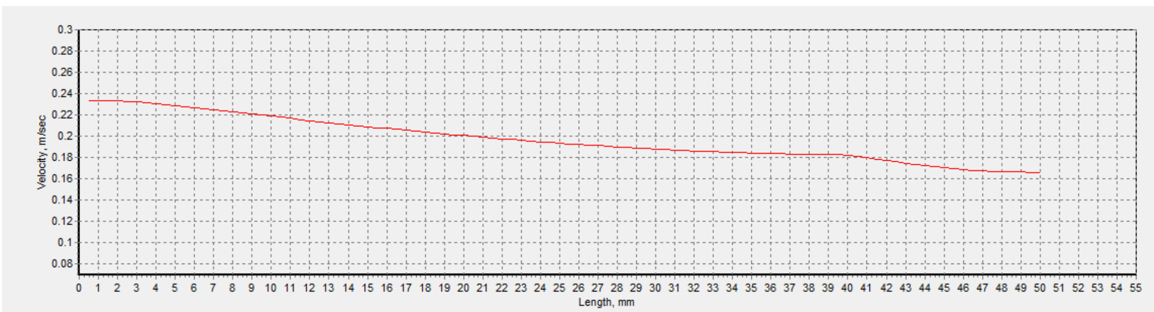


**Figure-16.** The flow rate profile in the systole at gravitation impact.

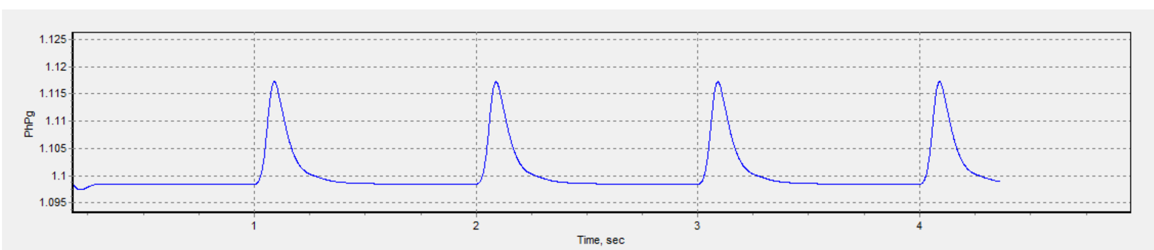


**Figure-17.** The PPG signal view at gravitation impact.

From the drawing it is visible that the PPG amplitude has considerably grown in comparison with fig. 14, and the shape of its ascending branch has varied.

## CONCLUSIONS

In this work the known hemodynamic model of the distal vessel that has allowed to simulate the PPG waveform taking into account the modification of vessel geometry during cardial cycle has been modified. It is shown that a little change of the shape of terminal part of the vessel leads to the significant modification of PPG amplitude and shape. It is necessary for developers of PPG gauges to consider an influence of mechanical actions on the tissue transparent field to exclude the artifacts removing origin.

The gained dependences of PPG amplitude and shape on slope angle of the extremity can help to the doctors with development of new tests for estimation of hemodynamic character.

The particular interest is represented by the offered modification of the explicit scheme of partial differential equations system integration, reducing the arising probability of stray oscillations.

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## REFERENCES

[1] Sagawa K. and Lie R. K. 1990. Schaefer J. Translation of Otto Frank's paper "Die Grundform des Arteriellen Pulses"//J. Mol. Cell. Cardiol. 22: 253-277.

[2] Astrakhantseva E.V., Gidaspov V. Yu. and Reviznikov D. L. 2005. Mathematical modelling operation of large veins hemodynamic. Mathematical modelling operation. 17(8): 61-80.

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[3] Koshelev V. B., Mukhin S. I., Sokolova T.V., Sosnin N. V. and Favorskii A.P. 2007. Mathematical modelling operation of cardiovascular system hemodynamic taking into account the neuroregulation influence. Mathematical modelling operation. 19(3): 15-28.

[4] Allen J. 2007. Photoplethysmography and its application in clinical physiological measurement. Physiological Measurement. 28: 1-39.

[5] S.S. Simakov and A. S. Kholodov. 2009. Computational study of oxygen concentration in human blood under the low frequency disturbances. Mat. Mod. Comp. Sim. 1(283-295).

[6] Abakumov M. V., Ashmetkov I.V., Esikova N. B., Koshelev V. B., Mukhin S. I., Sosin N.V., Tishkin V. F., Favorskii A.P. and Hrulenko A.B. 2000. Procedure of mathematical modelling operation of cardiovascular system. Mathematical modelling operation. 12(2): 106-117.

[7] Sviridova N. V. and Vlasenko V.D. 2014. Modelling operation of hemodynamic processes of cardiovascular system on the basis of data of peripheral arterial pulsing. Mathematical biology and bioinformatics. 9(1): 195-205. URL:<http://www.matbio.org/2014/Sviridova_9_195.pdf>.

[8] Pham T.D., Thang T.C., Oyama-Higa M. and Sugiyama M. 2013. Mental-disorder detection using chaos and nonlinear dynamical analysis of photoplethysmographic signals. Chaos, Solitons and Fractals. 51: 64-74.

[9] Glass L. 2009. Introduction to controversial topics in nonlinear science: is the normal heart rate chaotic. Chaos. 19: 1-4.

[10] Zamir M. 2007. The Physics of Coronary Blood Flow. New York: Springer, 2005. 408 p. 17. Shelhamer M. Nonlinear Dynamics in Physiology. A State-Space Approach. Singapore: World Scientific. p. 345.