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STUDY OF NONLINEAR VIBRATION OF LAMINATED COMPOSITE PLATES USING VARIATIONAL APPROACH

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ABSTRACT

In this paper free vibration characteristics of laminated composite plates is considered. A model is developed for a composite layer based on the consideration of non-linear terms in Von-Karman's non-linear deformation theory. The governing partial equation of motion is reduced to an ordinary non-linear equation and then solved using He's variational approach method. The variation of frequency ratio of the Isotropic and composite plates is brought out considering parameters such as aspect ratio, fiber arrangements (orientation), number of layers and modal ratios.

Keywords: laminated composite plate, nonlinear analytical analysis, nonlinear vibration, classical plate theory, he's variational approach method.

1. INTRODUCTION

Laminated composite plates due to their high specific strength and stiffness have been increasingly used in a wide range of civil, aerospace and mechanical applications. By tailoring the sequence of the stacks and the thickness of the layers, composite laminates' characteristics can be matched to the structural requirements with no difficulty. To use composite laminates efficiently, an accurate knowledge of vibration characteristics is essential. Vibration not only creates excessive noise and wastes energy but also may result in catastrophic failures. These phenomena when the system operates around its natural frequencies would be even more disastrous.

Many publications have dealt with the linear vibrations of laminated composites. In these cases the equation of motion is obtained easily and then by a reduction into a generalized eigenvalue problem, frequencies and mode shapes are determined. However in many working conditions, plates are subjected to large amplitudes and a nonlinear frequency analysis is required as a result. For the case of nonlinear free vibration of composite plates, obtaining an exact solution because of the complexity of the equation of motion has been found to be particularly difficult. The first approximate solution developed by Chu and Herman [1] in 1956 was the start point for so many other numerical methods introduced in the following years such as the finite element method (FEM) [2], the discrete singular convolution method (DSC), the strip element method, and the Ritz methods [3].

Singh *et al*. [4] used direct numerical integration to study non-linear vibration of rectangular laminated composite plates in 1990. Using Kirchhoff hypothesizes and Von-Karman strain-stress relations, they derived governing equations. They also employed harmonic oscillating assumptions and investigated large amplitude vibrations for various arrangements.

Although numerical approaches are applicable to a wide range of practical cases, approximate analytical methods provide highly accurate solutions and a deep physical insight. One of the main approximate analytical approaches on nonlinear vibration analysis is Perturbation Method. This method is effective just in solving weakly nonlinear differential equations. Because of the limited application of the perturbation methods, newer approaches have been developed during recent years which are more powerful. For example; the vibrational behavior of quintic nonlinear in extensional beam on two-parameter elastic substrate based on the three mode assumptions is investigated by Sedighi [5]. He employed parameter expansion method to obtain the approximate expressions of nonlinear frequency-amplitude relationship for the first, second, and third modes of vibrations. An efficient iterative method is applied to the analytic Treatment of Nonlinear Fifth-order Equations by Saravi and Nikkar [6]. Ghaffarzadeh and Nikkar [7] applied a new analytical method called the variational iteration method-II (VIM-II) for the differential equation of the large deformation of a cantilever beam under point load at the free tip. Askari *et al*. [8] applied He's energy balance method and He's variational approach to frequency analysis of nonlinear oscillators with rational restoring force. Sedighi and Daneshmand [9] studied nonlinear transversely vibrating beams by the homotopy perturbation method with an auxiliary term. Barari *et al*. [10] studied non-linear vibration behavior of geometrically non-linear Euler-Bernoulli beams using variational iteration method and parameter perturbation method. He's Variational Approach (VA) is used to obtain an analytical solution for the Bratu's equation by Saravi *et al.* [11]. Bagheri *et al*. [12] studied the nonlinear responses of clamped-clamped buckled beam. They used two efficient mathematical techniques called He's variational approach and Laplace iteration method in order to obtain the responses of the beam vibrations. Salehi *et al*. [13] applied two efficient methods to consider large deformation of cantilever beams under point load. Younesian *et al*. [14] studied free oscillations of beams on nonlinear elastic foundations by VIM. Askari *et al*. [15] applied higher order Hamilton

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approach to nonlinear vibrating systems, and many other problems solved by these methods [16-19].

In the present paper VA method is used to obtain approximate analytical solutions for nonlinear vibrations of a thin laminated composite plate. VA method is used to achieve nonlinear natural frequency and its excellent accuracy for the wide range of amplitude values is satisfied. A comparison with results in other articles has been done to validate the answers.

2. MATHEMATICAL MODELLING

Consider a thin laminated composite plate of length *a*, width *b*, and thickness *h* as shown in Figure-1. A positive set of coordinate system demonstrated in a way that reference surface is taken as the mid plane and its origin is considered at the corner of the plate. The plate is supposed to be simply supported along all its edges. In addition, no slip condition between the layers is assumed.

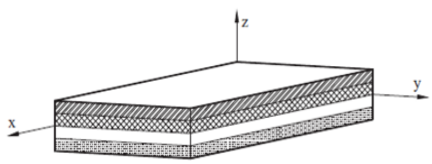

Figure-1. Schematic of the laminated composite plate[20].

Assume *u*, *v* and *w* are the displacements of an arbitrary point of the plane in *x, y* and *z* axis, u_0 and v_0 to be the corresponding displacements of that point in the mid-plane and ε^0 κ to be the mid-plain strain and mid-plane curvature respectively, mechanical shear relation can be demonstrated as [20]:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{1}$$

Following Von-Karman's strain-displacement assumptions the in plane strain, shear strain and plane curvatures can be expressed as:

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2 \\ \dfrac{\partial v_0}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2 \\ \left(\dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} + \dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y}\right) \end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial w_0}{\partial y} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \tag{3}$$

According to Classical Theory of Elasticity, the strain-stress relations for each layer can be derived as:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}_k = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix}_k \tag{4}$$

Where the numbers 1, 2, 6 referred to principal axis of each layer. $(Q_{ij})_k$ $(i = 1,2,6)$ are the coefficients of the reduced stiffness matrix at the *k*th layer and are defined as:

$$\begin{aligned} Q_{11} &= \frac{E_{11}}{1-\nu_{12}\nu_{21}} \\ Q_{22} &= \frac{E_{22}}{1-\nu_{12}\nu_{21}} \\ Q_{12} &= \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}} \\ Q_{66} &= G_{12} \end{aligned} \tag{5}$$

Below stress-strain relations are obtained from the axis transformation of the each layer stress-strain equations referred to global axes:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \left(\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right)_k \tag{6}$$

And $(\bar{Q}_{ij})_k$ $(i = 1,2,6)$ are plane stress-reduced stiffness coefficients.

Constitutive equations which relate force and moment resultants to the strains through an appropriate integration along the thickness can be developed as:

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$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{pmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} + \begin{pmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{pmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{7}$$

$$\begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{pmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} + \begin{pmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{pmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{8}$$

where,

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k - h_{k-1}\right) \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k^2 - h_{k-1}^2\right) \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(h_k^3 - h_{k-1}^3\right) \end{aligned} \tag{9}$$

A_{ij}, B_{ij}, and D_{ij} are called extensional stiffnesses, bending-extensional coupling stiffnesses and bending stiffness respectively. The equations of motion are derived from Hamilton's principle as follows [20]:

$$0 = \delta\int_{t_1}^{t_2} (U - T)d\tilde{t} \tag{10}$$

Where U, the potential energy of the plate and T, the kinematic energy of the plate are given by:

$$U = \frac{1}{2}\int_V \left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + 2\sigma_{xy}\varepsilon_{xy}\right)dz\,dx\,dy \tag{11}$$

$$T = \frac{1}{2}\int_0^a\int_0^b \left(\sum \rho_i h_i\right)\dot{w}^2 dx\,dy \tag{12}$$

Substituting Eqs.(11) and (12) into the Hamilton's equation of motion and with an appropriate integration along the thickness and then by equating $\delta u_0, \delta v_0$, and δw_0 coefficients to zero, one obtains the below relations:

$$\begin{aligned} &\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \\ &\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial}{\partial x}\left(N_x \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial x}\left(N_{xy}\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial y}\left(N_{xy}\frac{\partial w}{\partial x}\right) \\ &\qquad + \frac{\partial}{\partial y}\left(N_y \frac{\partial w}{\partial y}\right) = \rho h \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{13}$$

By assuming simply supported boundary condition, the relations below are considered for displacement equations to satisfy the boundary conditions:

$$\begin{aligned} u &= U(t)\,Sin\,\frac{2m\pi x}{a}\,Sin\,\frac{n\pi y}{b} \\ v &= V(t)\,Sin\,\frac{m\pi x}{a}\,Sin\,\frac{2n\pi y}{b} \\ w &= W(t)\,Sin\,\frac{m\pi x}{a}\,Sin\,\frac{n\pi y}{b} \end{aligned} \tag{14}$$

And $U(t)$, $V(t)$, and $W(t)$ are the maximum displacements of plate center point along principal axes x, y, and z respectively. $U(t)$ and $V(t)$ can be expressed in terms of $W(t)$ using first two equations of Equation (13) and then by employing Galerkin method and substituting $U(t)$ and $V(t)$ in terms of $W(t)$ into Equation (13), the governing equation can be written as:

$$\frac{d^2W(t)}{dt^2} + \alpha_1 W(t) + \alpha_2 V^2(t) + \alpha_3 W^3(t) = 0 \tag{15}$$

Supposing W_{max} as the maximum vibration amplitude of the plate center, the initial conditions of the center of the plate can be expressed as:

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$$W(0) = W_{\max} \quad , \quad \frac{dW(0)}{dt} = 0 \tag{16}$$

3. BASIC CONCEPT OF THE PROPOSED METHOD

The variational approach method to nonlinear oscillators was first proposed by Chinese mathematician, Ji-Huan He [21]. We give a brief introduction of the method. To clarify the basic ideas of proposed method consider the following second order differential equation:

$$u'' + f(u) = 0, \qquad u(0) = A, \qquad u'(0) = 0 \tag{17}$$

where $f(u)$ is a nonlinear function of u, u' and u". For simplicity, if function f depends on u only, its variational functional can be obtained as:

$$J(u) = \int_0^T \left\{ -\frac{u'}{2} + F(u) \right\} dt \tag{18}$$

where F is the potential, $\frac{dF}{du} = f$ and T is the period of the nonlinear oscillator.

We assume that its approximate solution can be expressed as:

$$u = A\cos(\omega t) \tag{19}$$

where A and ω are the amplitude and frequency of the oscillator, respectively. Substituting Equation (4) into Equation (3), and setting $\frac{dJ}{dA} = 0$, we can obtain an inexplicit amplitude-frequency relationship of Equation (1).

4. APPLICATION OF THE PROPOSED METHOD

Rewriting Equation (15) in the standard form of Equation (18) results in the following equation:

$$\frac{d^2\eta(t)}{dt^2} + \omega^2\eta(t) = f\left(\eta(t)\right) \tag{20}$$

Where

$$\begin{aligned} & f\left(\eta(t)\right) = \omega^2\eta(t) - N\left\{\eta(t)\right\}, && \lambda_1 = \alpha_1 + \left(P + F_{0th} - F_{1th}\right)\alpha_P + \alpha_L + \alpha_{Sh} \\ & N\left\{\eta(t)\right\} = \lambda_1\eta(t) + \lambda_2\eta^2(t) + \lambda_3\eta^3(t), && \lambda_2 = \alpha_2, \quad \lambda_3 = \left(\alpha_{NL} + \alpha_3\right) \end{aligned} \tag{21}$$

In this part, we solve Equation (15) via variational approach. We can, now, easily obtain the following variational formulation:

$$J(u) = \int_0^{\frac{T}{4}} \left\{ -\frac{\dot{\eta}(t)}{2} + \frac{\lambda_1}{2}\eta(t)^2 + \frac{\lambda_2}{3}\eta(t)^3 + \frac{\lambda_3}{4}\eta(t)^4 \right\} dt \tag{22}$$

Assume that its approximate solution can be expressed as:

$$\eta_0(t) = A\cos(\omega t) \tag{23}$$

where ω is the frequency to be determined and A is the amplitude of oscillation. Substituting Equation (23) into Equation (22) results is

$$J(u) = -\frac{1}{2}(\omega A\sin\omega t) + \frac{\lambda_1}{2}(A\cos\omega t)^2 + \frac{\lambda_2}{3}(A\cos\omega t)^3 + \frac{\lambda_3}{4}(A\cos\omega t)^4 \tag{24}$$

Making J stationary with respect to A, according to He's method, we obtain:

$$\frac{\partial J}{\partial A} = -\frac{1}{2}(\omega\sin\omega t) + A\lambda_1(\cos\omega t)^2 + A^2\lambda_2(\cos\omega t)^3 + A^3\lambda_3(\cos\omega t)^4 = 0 \tag{25}$$

Finally collocation at $\omega t = \frac{\pi}{4}$, the frequency can be approximated as:

$$\omega = \sqrt{\lambda_1 + \frac{3}{4}\lambda_3 A^2} \tag{26}$$

and zero-order approximate solution is:

$$u = A\cos\left(\sqrt{\lambda_1 + \frac{3}{4}\lambda_3 A^2}\, t\right) \tag{27}$$

In order to verify the precision of the suggested method, current results were compared with other articles. Table-1 illustrates the frequency ratio of square and rectangular plates. It is observable that our results are in excellent agreement with the results provided by other references. It can be observed that the second displacement coefficient is zero for square plates.

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Table-1. Frequency ratio of isotropic rectangular and square plates using various methods $(\upsilon = 0.3)$.

W/h	$(a/b=1)$			$(a/b=2)$		
	VA	**HAM**	**Ref [4]**	**VA**	**HAM**	**Ref [4]**
0.2	1.0251	1.0252	1.0208	1.0285	1.0285	1.0254
0.4	1.0967	1.0967	1.0809	1.1091	1.1091	1.0982
0.6	1.2056	1.2056	1.1743	1.2307	1.2306	1.2097
0.8	1.3423	1.3422	1.2937	1.3819	1.3818	1.3505
1	1.4991	1.4988	1.4327	1.5541	1.5537	1.5124
1.2	1.6703	1.6698	---	1.741	1.7404	---
1.4	1.852	1.8512	1.7503	1.9384	1.9376	1.877
1.6	2.0412	2.0404	---	2.1435	2.1425	---
1.8	2.2364	2.2353	---	2.3543	2.353	---
2	2.436	2.4347	2.2828	2.5694	2.5679	2.4798

In Table-2 the Variation of frequency ratios versus non-dimensional amplitude ratio for symmetrical and non-symmetrical square plate arrangements are shown. The symmetrical arrangement plates have the same frequency ratios values.

Table-2. Frequency ratio of composite square plate with different arrangements using various methods $(E_1/E_2 = 40, G_{12}/E_2 = 0.5, \nu_{12} = 0.25)$.

W/h	**[0°/90°/0°/90°]**			**[0°/90°/90°/0°]**			**[90°/0°/0°/90°]**		
	VA	**HAM**	**Ref [4]**	**VA**	**HAM**	**Ref [4]**	**VA**	**HAM**	**Ref [4]**
0.25	1.0575	1.0575	1.0634	1.0509	1.0509	1.0535	1.0509	1.0509	1.0535
0.5	1.2121	1.212	1.2388	1.1892	1.1891	1.2038	1.1892	1.1891	1.2038
0.75	1.4308	1.4305	1.4832	1.3874	1.3872	1.4172	1.3874	1.3872	1.4172
1	1.6886	1.6881	1.7679	1.6232	1.6227	1.6691	1.6232	1.6227	1.6691
1.25	1.9703	1.9694	---	1.8827	1.8819	---	1.8827	1.8819	---
1.5	2.2671	2.2659	2.4000	2.1574	2.1563	2.2355	2.1574	2.1563	2.2355
1.75	2.5739	2.5724	---	2.4422	2.4408	---	2.4422	2.4408	---
2	2.8876	2.8857	3.0729	2.7342	2.7325	2.8439	2.7342	2.7325	2.8439

5. CONCLUSIONS

In this study, a composite laminated plate model is established to verify the vibrational behaviors of composite plate. Von-Karman's assumption and efficient approximate method (VAM) are employed to derive the nonlinear governing equation of motion. Analytical expressions are presented for nonlinear natural vibration analysis of composite laminated plate. Comparing with other Results, it is shown that the approximate analytical solutions are in very good agreement with the corresponding solutions.

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