



GALLOPING OF SMALL ASPECT RATIO SQUARE CYLINDER

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ABSTRACT

A mathematical model of galloping is considered in a quasi-steady approximation. Normal force aerodynamic coefficients C_y of square cylinder are measured in the subsonic wind tunnel at different angles of attack α . The cylinder aspect ratio is 10. The same cylinder is tested with end plates as well. A new function for the approximation of the aerodynamic coefficients dependence on the angle of attack is used. Krylov-Bogoliubov method is applied. Mathematical model allows predicting the critical air velocity at which oscillations occur. The amplitude of the oscillations can be calculated as a function of flow velocity. It appeared that the end plates significantly change the aerodynamic coefficients at low angles of attack. The critical air velocity reduces. The inflection point appears on the dependence $C_y(\alpha)$. Results of mathematical simulation are verified in the wind tunnel experiments. Square cylinder is suspended on two springs across the flow generated in the wind tunnel working section. The tension of the spring is measured with semiconductor tensoconverter. The output signal from tensoconverter is recorded in a computer file for further processing. In a separate experiment, the elastic damping system is measured. The calibration of instruments allows determine the amplitude of steady oscillations of the cylinder. The results of calculation and experiment are in good agreement for all the stable modes of oscillation.

Keywords: galloping, aspect ratio, wind tunnel, mathematical model.

INTRODUCTION

Galloping is self-excited oscillations of bluff bodies in the wind flow caused by a specific dependence of the aerodynamic force on the angle of attack. Galloping can result in destruction of structures. The quasi-steady mathematical model of bluff body galloping was developed in the middle of 20th century. One of the first studies of modeling was published in the paper of Parkinson and Brooks [1]. A more complete model of the galloping of elastically fixed bluff body in the air flow was proposed by Parkinson and Smith [2]. In these papers, the galloping of prisms with a square cross-section was considered. Later Novak [3] studied the cylinder with a rectangular cross-section. In the last decade, a number of studies revealed the aeroelastic instability of many bodies having different cross section [4 - 8]. Barrero-Gil *et al.* [9] considered the galloping of square cylinders in cross flow at low Reynolds numbers using finite element method. Hémon and Santi [10] performed the experiments with a flexible cylinder clamped at both ends. In all the papers mentioned above bodies of infinite aspect ratio were considered. However, the bodies of small aspect ratio also can oscillate under the action of a fluid flow. The example of such body is the rope way cabin [11]. In the present paper we study the transverse galloping of elastically fixed square cylinder of small aspect ratio.

MATHEMATICAL MODEL

Let axis z be normal to the base of the cylinder. The cylinder can move only along the axis y . The incoming flow has a constant velocity v and is directed along the axis x . The cylinder has a mass m , a length L (along z), a width H and a height H (along the axes of x and y respectively). The displacement from the equilibrium position of the cylinder is y . The cylinder is

held with an elastic holder. The stiffness of a holder is k , and viscous damping force of the holder is equal $r dy/dt$. This force is always directed against the velocity of the cylinder dy/dt . Quasi-steady assumption is adopted, according to which the aerodynamic coefficients depend only on the angle of attack. The normal component of the aerodynamic force acting along the axis y , is

$$C_y = s\rho_0 v_r^2 / 2.$$

Here ρ_0 – density of the medium, $s = HL$ – face area of the cylinder, C_y - the normal force coefficient, v_r – relative velocity of the cylinder. This velocity is the sum of the own cylinder velocity ($0, dy/dt$) and the free stream velocity with opposite sign ($-v, 0$). Thus, the relative velocity and the tangent of the angle of attack are calculated as follows:

$$v_r = v \sqrt{1 + \left(\frac{1}{v} \frac{dy}{dt}\right)^2}, \quad \tan \alpha = -\frac{1}{v} \frac{dy}{dt}.$$

The aspect ratio λ of the cylinder is defined to be the ratio of the length L of the cylinder to the width H : $\lambda = L/H$. The equation of transverse motion of the cylinder is given by

$$m \frac{d^2 y}{dt^2} + r \frac{dy}{dt} + ky = C_y s \frac{\rho_0 v_r^2}{2}. \quad (1)$$

Coefficient of aerodynamic force can be approximated by polynomial function of the angle of attack or tangent of the angle of attack [2, 8, 9, 12]. However, for the small aspect ratio cylinders, as shown below, this polynomial approximation describes well the



experimentally determined dependence $C_y(\tan \alpha)$ only for small angles of attack. In the paper [13], we propose the following approximation of the dependence $C_y(\tan \alpha)$:

$$\begin{aligned} C_y &= B_{01} + B_1 \tan \alpha, \quad \text{if } \tan \alpha < -w, \\ C_y &= \sum_{i=0,1,3,\dots} A_i (\tan \alpha)^i, \quad \text{if } -w < \tan \alpha < w, \\ C_y &= B_{02} + B_1 \tan \alpha, \quad \text{if } -w < \tan \alpha. \end{aligned} \quad (2)$$

The Equation (1) of the cylinder motion can be transformed to a dimensionless form by choosing $\sqrt{m/k}$ as the unit of time and $2r\sqrt{m}/(\rho_0 s \sqrt{k})$ as the unit of length:

$$\frac{d^2 Y}{d\tau^2} + Y = \mu \left(C_y V^2 - \frac{dY}{d\tau} \right), \quad \mu = \frac{r}{\sqrt{mk}}, \quad (3)$$

where Y , V and τ are dimensionless transverse displacement of the cylinder, the dimensionless flow velocity and dimensionless time:

$$Y = y \frac{2r}{\rho_0 s} \sqrt{\frac{m}{k}}, \quad V = v \frac{2r}{\rho_0 s}, \quad \tau = t \sqrt{mk}.$$

The second order differential equation can be transformed to a system of two first-order equations:

$$\frac{dY}{d\tau} = Z, \quad \frac{dZ}{d\tau} + Y = \mu (C_y V^2 - Z). \quad (4)$$

For the solution of Equation (4) the Krylov-Bogoliubov method [14] was applied. The method is based on the averaging principle when the exact differential equation of the motion is replaced by its averaged version. Let suppose that

$$Y = \rho \cos(\tau + \varphi), \quad Z = -\rho \sin(\tau + \varphi), \quad (5)$$

where ρ and φ are the slowly varying amplitude and phase of oscillation. After substitution the expressions (5) to the system Equation (4) the first equation of the system should be multiplied by $\cos(\tau + \varphi)$, the second one should be multiplied by $-\sin(\tau + \varphi)$. The summation of these two equations after averaging leads to differential equation for slowly varying amplitude ρ of the oscillations. The equation for phase φ one can obtain by the same way, but first equation of Equation (4) should be multiplied by $-\sin(\tau + \varphi)$, the second one should be multiplied by $-\cos(\tau + \varphi)$.

Thus, the first approximation of Krylov-Bogoliubov method [14] leads to a system of differential equations for slowly varying amplitude and phase:

$$\begin{aligned} \frac{d\rho}{d\tau} &= -\mu \frac{A_1}{2} \left[\left(V + \frac{1}{A_1} \right) \rho + \frac{3}{4} \frac{A_3}{A_1 V} \rho^3 + \dots \right], \quad \text{if } \frac{\rho}{V} \leq w, \\ \frac{d\rho}{d\tau} &= \mu \left\{ \frac{\rho}{2} - V^2 \frac{B_{01} - B_{02}}{\pi} S + \right. \\ & B_1 V \left(\frac{\rho}{2} - \frac{\rho}{\pi} R + \frac{wV}{\pi} S \right) + A_1 V \left(\frac{\rho}{\pi} - \frac{wV}{\pi} S \right) + \\ & A_3 \frac{\rho^2}{V} \left[\frac{3\rho}{4\pi} R - \frac{wV}{\pi} S \left(\frac{3}{4} + \frac{3w^2 V^2}{2\rho^2} \right) \right] + \\ & \left. A_5 \frac{\rho^4}{V^3} \left[\frac{5\rho}{8\pi} R - \frac{wV}{\pi} S \left(\frac{5}{8} + \frac{5w^2 V^2}{12\rho^2} + \frac{w^4 V^4}{3\rho^4} \right) \right] + \dots \right\}, \\ & \text{if } \frac{\rho}{V} > w, \end{aligned} \quad (6)$$

$$\frac{d\varphi}{d\tau} \rho = 0,$$

where

$$S = \sqrt{1 - \frac{w^2 V^2}{\rho^2}}, \quad R = \arcsin\left(\frac{wV}{\rho}\right).$$

In this paper, a cylinder with a square cross-section is considered, but one can apply the mathematical model to the oscillations of bluff bodies of different shapes, including asymmetrical.

If the body is not symmetrical, the even expansion coefficients for the normal force are not equal to zero, but after the application of the Krylov-Bogoliubov procedure corresponding terms would be equal to zero. Application of the Krylov-Bogoliubov method is possible, if the aerodynamic force and the resistance force are smaller than the elastic force. In this case, the oscillation of the body is closed to the harmonic one. The characteristic time of change of the oscillation amplitude is much greater than the oscillation period.

The first formula of Equation (6) implies that the solution $\rho = 0$ becomes unstable when the dimensionless flow velocity V exceeds the critical value of $V_* = -1/A_1 > 0$.

Thus, to describe the oscillation of the cylinder it is necessary to measure the dependence of the aerodynamic normal force coefficient on the angle of attack and determine the parameters w , A_1 , B_{01} , B_{02} and B_2 .

EXPERIMENTAL DETERMINATION OF THE AERODYNAMICAL COEFFICIENTS AND MATHEMATICAL SIMULATION OF OSCILLATIONS

The cylinder with square cross section was taken. Cylinder aspect ratio was $\lambda = 10$. The cylinder was tested with end plates as well. The experiment was conducted in the closed return wind tunnel with open working section. Reynolds number Re was equal to $1.0 \cdot 10^5$. Cylinder was rigidly fixed in the aerodynamic balance. Measurements of



aerodynamic forces were conducted at angles of attack from -30° and 30° .

For cylinder without end plates satisfactory normal force approximation could be obtained by putting $A_3 = 0$. Figure-1 shows the dependence of the approximation C_y on $\tan \alpha$ for cylinder without end plates. Equation (7) describes the dependence of the aerodynamic coefficients of the angle of attack.

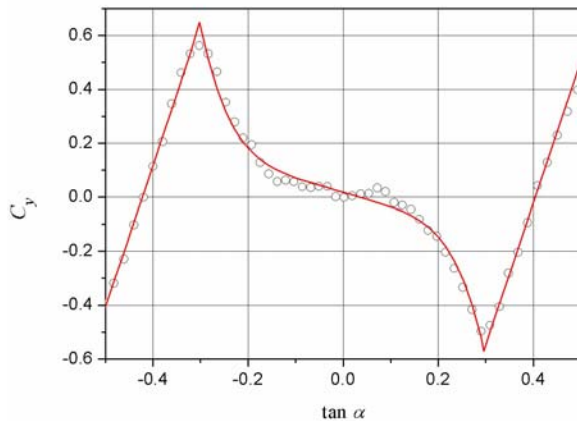


Figure-1. Aerodynamic coefficients C_y for cylinder $\lambda = 10$ without end plates. Experimental data and approximation.

$$\begin{aligned} C_y &= 2.180 + 5.140 \tan \alpha, & \text{if } \tan \alpha < -0.3, \\ C_y &= 0.016 - 0.456 \tan \alpha - 217 \tan^5 \alpha, & \text{if } |\tan \alpha| < 0.3, \\ C_y &= -2.067 + 5.140 \tan \alpha, & \text{if } \tan \alpha > 0.3. \end{aligned} \quad (7)$$

The presence of end plates leads to the appearance of an inflection point on the curve $C_y(\tan \alpha)$ in the range of angles of attack $\tan \alpha < w$. It is impossible to obtain a satisfactory approximation taking the coefficient $A_3 = 0$.

Figure-2 illustrates the effect of the end plates, which are disks attached to the ends of the cylinders. The diameter of end plate is equal to $3.63 H$. End plates change the dependence C_y on $\tan \alpha$ for small angles of attack. Without disks, the values of the normal component of the aerodynamic force at low angles of attack are small, but normal force acting on the cylinder with end plates significantly differs from zero. After joining the endplates absolute value of the coefficient A_1 dramatically increases (see Eq.8). The dependence of $C_y(\tan \alpha)$ for small angles is similar to the dependence obtained in the work of Parkinson and Smith [2].

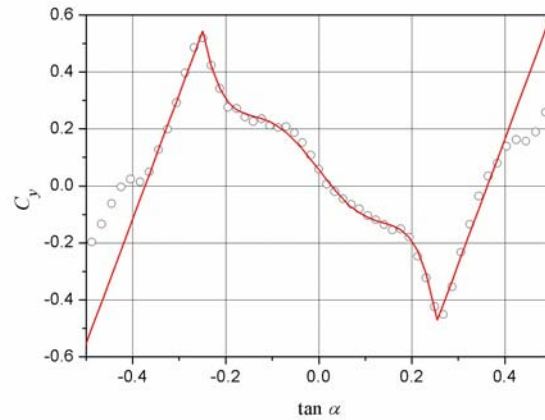


Figure-2. Aerodynamic coefficients C_y for cylinder $\lambda = 10$ with end plates. Experimental data and approximation.

$$\begin{aligned} C_y &= 1.637 + 4.383 \tan \alpha, & \text{if } \tan \alpha < -0.25, \\ C_y &= 0.057 - 2.127 \tan \alpha + 57.83 \tan^3 \alpha - \\ &894 \tan^5 \alpha, & \text{if } |\tan \alpha| < 0.25, \\ C_y &= -1.589 + 4.383 \tan \alpha, & \text{if } \tan \alpha > 0.25. \end{aligned} \quad (8)$$

One can see that in both tested cases there is the range of angles of attack, in which the sign of the angle of attack is opposite to C_y . In this range, the directions of the aerodynamic force and the velocity of the cylinder coincide. Thus, the aerodynamic force does positive work, and the vibrational energy of the system increases. The presence of such a range is the cause of galloping.

Figure-1 and Figure-2 show that aerodynamic normal force at $\alpha = 0$ is not equal to zero. Probably this phenomenon is caused by errors of manufacturing.

Dependence of amplitude of oscillation ρ on dimensionless velocity V presented on Figure-3. If the flow velocity exceeds the critical value V_* zero solutions $\rho = 0$ of Equation (6) becomes unstable. For the cylinder without end plates the oscillation amplitude increases abruptly. Equation (6) may have one, two, or three solutions with constant amplitude of oscillation. In the last two cases one solution is unstable. There is a hysteresis region characterized by the existence of three solutions, one of solution is unstable. For cylinder without end plates the critical velocity is the right border of the hysteresis range. The presence of end plates leads to appearance of an inflection point on the dependence of $C_y(\tan \alpha)$. The appearance of an inflection point, as it was shown in [8], changes the dependence of the amplitude of the steady-state oscillation on the free stream velocity (see Figure-3). The critical velocity V_* decreases, the hysteresis of flow velocities decreases too. The left boundary of the hysteresis range becomes greater than the critical velocity. Cylinder equipped with the end plates can vibrate with small amplitudes at a slight excess of the critical flow velocity.

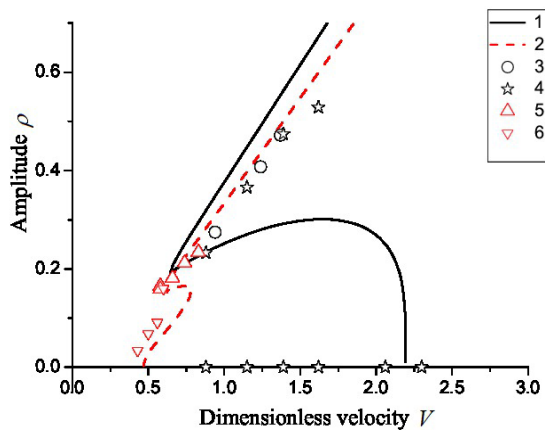


Figure-3. The dependence of the dimensionless oscillation amplitude ρ on the dimensionless flow velocity V . The calculation results and experimental data. **1** - calculation, the cylinder without end plates, **2**- calculation, the cylinder with end plates, **3**, **4**- experiment with a cylinder without end plates with different damping, **5**, **6**- experiment with a cylinder with end plates with different damping.

EXPERIMENTAL VERIFICATION

Despite the fact that different authors have frequently used the classical model of aeroelastic galloping, it certainly should be verified by comparison with the experimental results. Aerodynamic tests were performed in a subsonic wind tunnel. Cylinder was manufactured from wood. Its dimensions were $L = 0.6$ m, $H = 0.06$ m. Cylinder was suspended on two springs across the flow generated in the wind tunnel working section (see Figure-4).

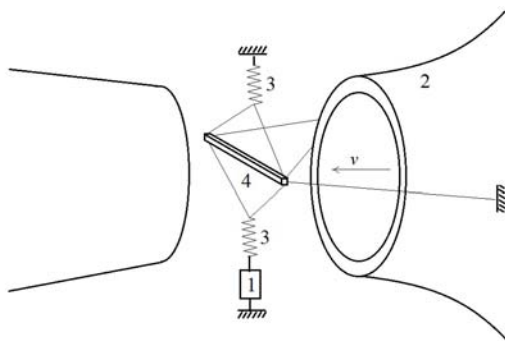


Figure-4. Scheme of the experiment in a wind tunnel. **1** - semiconductor tensoconverter, **2** - confuser, **3** - spring, **4** - cylinder.

The threads let the cylinder to move only along the arc of a circle with radius of two meters approximately along single direction normal to the flow velocity. The forces acting on the cylinder were measured using a semiconductor integral tensoconverter S-50, located between the lower spring and rigidly fixed holder. Tensometer is designed for measurement of the forces

within the range from 0 to 50 N. The tensometer output signal fed an analog-to-digital converter L-205, performed as a PC card. A program written in Pascal language controls the measurement process.

The measurement results had been recorded in a file for later processing. Calibration of the instrument was carried out by loading the known weights, which were placed on top face of the cylinder. In addition, the displacement of the cylinder caused by the known load was measured.

Laser beam spot was established on the surface of the cylinder. After loading a known weight, the spot of the laser beam moved over the surface of the cylinder. The measured displacement of spot is used to calculate the stiffness of the system. It should be noted that in this experiment the friction force played the role of damping force. To increase the frictional force we inserted a synthetic fibrous material in the springs. The oscillation amplitude was calculated using the experimental force data. To find the damping coefficient experiment was performed in the absence of airflow.

In a series of experiments, which were carried out at different flow velocities, some modes of steady-state oscillation were discovered. At the same time in some cases the excitement of oscillation was realized by putting the initial moment to the cylinder. In other cases, the oscillations developed from equilibrium position. The experiment was carried out with the cylinder without end plates and with the same cylinder equipped with the end plates. The results were compared with calculations according a mathematical model of galloping. The calculation results and the experimental data are presented in Figure-3.

As one can see on the graph, the results of calculation and experiment are in good agreement for all the stable modes of oscillation.

CONCLUSIONS

Based on a mathematical model of galloping we analyzed the stationary oscillations of a square cylinder. Experimentally determined aerodynamical coefficients are used in the model. Cylinders of small aspect ratio cannot oscillate with small amplitude at a slight excess of the critical flow velocity. The critical flow velocity for the cylinders of low aspect ratio is at the right border of the hysteresis range. The cylinder equipped by end plates that prevent air flowing through the ends, changes the dependence of the normal force on the angle of attack at low angles of attack. The critical velocity decreases and hysteresis range of flow velocities decreases. The left boundary of the hysteresis region is greater than the critical velocity. The cylinder equipped with the end plates can vibrate with small amplitudes at a slight excess of the critical flow velocity. These results were verified in experiments with elastically fixed cylinders in the wind tunnel.



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REFERENCES

- [1] Parkinson G. V. and Brooks N. P. 1961. On the aeroelastic instability of bluff cylinders. *Journal of Applied Mechanics*. 28: 252-258.
- [2] Parkinson G. V. and Smith J. D. 1964. The square prism as an aeroelastic non-linear oscillator. *Quarterly Journal of Mechanics and Applied Mathematics*. 17: 225-239.
- [3] Novak M. 1969. Aeroelastic galloping of prismatic bodies. *ASCE Journal of Engineering Mechanics Division*. 95: 115-142.
- [4] Alonso G., Meseguer J. and Perez-Grande I. 2005. Galloping instabilities of two-dimensional triangular cross-section bodies. *Experiments in Fluids*. 38: 789-795.
- [5] Alonso G. and Meseguer J. 2006. A parametric study of the galloping stability of two-dimensional triangular cross-section bodies. *Journal of Wind Engineering and Industrial Aerodynamics*. 94: 241-253.
- [6] Alonso G., Meseguer J. and Perez-Grande I. 2007. Galloping stability of triangular cross-sectional bodies: A systematic approach. *Journal of Wind Engineering and Industrial Aerodynamics*. 95: 928-940.
- [7] Alonso G., Meseguer J. and Valero E. 2009. An analysis on the dependence on cross section geometry of galloping stability of two-dimensional bodies having either biconvex or rhomboidal cross sections. *European Journal of Mechanics. B / Fluids*. 28: 328-334.
- [8] Barrero-Gil A., Sanz-Andres A. and Alonso G. 2009. Hysteresis in transverse galloping: The role of the inflection points. *Journal of Fluids and Structures*. 25: 1007-1020.
- [9] Barrero-Gil A., Sanz-Andres A. and Roura M. 2009. Transverse galloping at low Reynolds numbers. *Journal of Fluids and Structures*. 25: 1236-1242.
- [10] Hémon P., Santi F. 2002. On the aeroelastic behavior of rectangular cylinders in cross-flow. *Journal of Fluids and Structures*. 16: 855-889.
- [11] Petrova R. V., Hoffmann K. and Liehl R. 2007. Modelling and simulation of bicable ropeways under cross-wind influence. *Mathematical and Computer Modelling of Dynamical Systems*. 13(1): 63-81.
- [12] Thompson J.M.T. 1982. *Instabilities and catastrophes in science and engineering*. Chichester: John Wiley and Sons.
- [13] Lyusin V.D. and Ryabinin A.N. 2011. Investigation of aspect ratio of the prism on its aerodynamic characteristics and the vibration amplitude during prism galloping. *Vestnik Sankt-Peterburgskogo Universiteta. Seriya 1, No. 2*: 139-145 (In Russian).
- [14] Bogoliubov N. N. and Mitropolski Y. A. 1961. *Asymptotic Methods in the Theory of Non-Linear Oscillations*. New York: Gordon and Breach.