



SERVICE CONTRACT MANAGEMENT WITH AVAILABILITY IMPROVEMENT AND COST REDUCTION

H. Husniah¹, U.S. Pasaribu² and B. P. Iskandar³

¹Department of Industrial Engineering, Langlangbuana University, Bandung, Indonesia

²Department of Mathematics and Nature Science, Bandung Institute of Technology, Bandung, Indonesia

³Department of Industrial Engineering, Bandung Institute of Technology, Bandung, Indonesia

E-Mail: hennie.husniah@gmail.com

ABSTRACT

This paper deals with a maintenance service contract for a warranted product. A situation where an agent offers more than one service contract options is considered and the optimal option is selected by the owner of equipment (a company). This case is typically found in the mining industry where the Original Equipment Manufacturer (OEM) is the only maintenance service provider. As the availability of the equipment is directly influenced the revenue of the company, hence the availability target of the equipment needs to be included. In this paper, the maintenance service contract considering the availability target is studied from both the owner and OEM point of views and uses a non-cooperative game formulation to determine the optimal strategy (pricing structure) for the OEM, and the optimal option for the owner.

Keywords: service contract, availability, nNash equilibrium.

1. INTRODUCTION

We consider an equipment (such as dump trucks, excavators, etc.) used in a mining industry to support its business. The equipment deteriorates with usage and age and finally fails to operate as intended. If the equipment is in failed state, no revenue is generated. High availability of the equipment is needed for achieving the revenue of the company. To keep the equipment in high availability, preventive Maintenance (PM) actions are performed using age based or conditioned base maintenance - to reduce the likelihood of failure and down time. Corrective Maintenance (CM) actions are taken after failure occurs, which restores the failed equipment to the operational state.

For a complex and expensive equipment such as dump trucks, it is not economical to do all maintenance actions (PM and CM) in house and hence the owner of the equipment needs to outsource the maintenance actions (PM and/or CM). The maintenance programs is aimed at not only to sustain the performance (e.g. reliability) of the equipment according to the intended function but also to obtain optimum business profitability. In a mining industry, availability of dump trucks is a key measure, which influences significantly the revenue of a company. If the maintenance actions are outsourced then the owner wants to select an maintenance contract option that assures the availability of the equipment with reasonable maintenance cost.

Maintenance service contract has received attention in the literature. Authors in [1, 2, 3 and 4] studied maintenance service contract and decision problems are formulated as a Stackelberg game theory model to obtain an optimal cost strategy with the agent as a leader and consumer as the follower. However, this strategy did not consider any preventive maintenance action. Further, [5], [6], and [7], developed a similar model for the case of repairable items and involved a preventive maintenance policy. [8] studied a maintenance service contract model

that considers the periodic inspection and corrective maintenance, and the OEM incurs a penalty cost whenever the downtime greater than a threshold value. Three contract options were considered and the optimal strategy for selecting the options is obtained which maximizes the expected profit for both the agent and the owner. All maintenance service contract studied consider a penalty based on down time for each failure - i.e. a penalty cost incurs the agent (or OEM) when the actual down time to fix the failed equipment is greater than the target value.

In most mining companies, an availability of heavy equipment used is a critical measure to support their business. Consequently, study of the maintenance service contract needs to incorporate the availability of the equipment as a performance measure to be achieved if the owner buys the service contract from an agent.

In this paper, we study maintenance service contracts from the manufacturer's perspective and the customer's perspective, which considers the availability target of the equipment. In addition, we introduce a reduction cost as the penalty based on the availability per period (usually one year) -i.e. if it is lower than the target availability the OEM incurs the penalty cost. We consider that the availability target decreases each year as the equipment deteriorates with usage and age.

In section 2 of the paper we give model formulation for the service contract studied. Section 3 deals with model analysis to obtain the optimal price structure for the OEM and the optimal service option for the owner, and in Section 4 we present the numerical results and discussion of the results. Finally, we conclude with topics for further research in Section 5.

2. MODEL FORMULATION

2.1. Notation



The following notations will be used to formulate mathematical models needed to study the proposed service contract:

W	- Warranty period
X	- Product age
L	- life cycle
\tilde{A}	- Availability target
ζ	- Total downtime target
$Y(t)$	- Total downtime in $(0, t]$
$EP(t)$	- Expected penalty cost
$F(x)$	- Distribution function of downtime
$F^{(k)}(x)$	- The k -fold Stieltjes convolution of $F(x)$.
$r(x)$	- Hazard function
$R(x)$	- Cumulative hazard function
$F(x), f(x)$	- Distribution function, density function X
α	- Scale parameter
β	- Shape parameter
P_G	- Service contract cost
P_0	- PM cost done by owner
K	- Revenue
C_m	- Repair cost done by OEM
C_s	- Repair cost option O_0 owner
C_{pm}	- Preventive maintenance cost per unit time
C_p	- Penalty cost
C_b	- Price of the product

2.2. Equipment failures and repairs

We use a black-box approach to model equipment failure. Every failure is fixed by a minimal repair or the failure rate after repair is the same as that before it fails. It is assumed that the repair time is very small compared to its mean time between failures, hence it can be ignored. As a result, the failure occurs as a Non-Homogenous Poisson process (NHPP) with the intensity function [9].

To keep the equipment in good condition, PM is conducted regularly. PM can be done in-house or by the OEM or an agent. We consider that PM done in-house is less effective than that of the OEM. We model the effect of PM through the failure rate function as follows. If $r_0(x)$ represents the failure rate function for the equipment with PM done in-house, then it is given by

$$r_0(x) = \begin{cases} r(x) & 0 \leq x < W \\ r(W) + \eta r(x) & W \leq x < L \end{cases} \quad (1)$$

where $\eta > 1$. For PM done by OEM, the failure rate function is given by

$$r_1(x) = r(x) \quad 0 \leq x < L \quad (2)$$

Note that $\eta > 1$ meaning that the failure rate function increases with a higher rate or the PM done in-house is less effective than PM by OEM ($\eta = 1$).

2.3. PM and repair options

As mentioned in the earlier section, the equipment under consideration is sold with warranty and the warranty also covers PM. The manufacturer will rectify all failures and preventive actions during the warranty period without any charge to the consumer. The consumer will be responsible for all CM and PM actions, after the warranty ceases. Hence, a comprehensive maintenance program of the equipment over a life cycle, L , is needed by the company to give a maximum availability of the equipment.

We consider that an agent offers two options to the customer after the warranty ends –ie. Option O_0 and O_1 will be defined in part D.

2.4. OEM's decision problem

We define service contracts studied as follows.

Option O_0 : After the expiry of warranty or in $[W, L)$, the consumer carries PM in-house. If the equipment fails, the owner calls the OEM to fix the equipment. The OEM will charge the consumer for the full cost of each repair.

There is no penalty cost to the OEM if the availability falls below the target. The OEM will charge the higher cost of repair under this option.

Option O_1 : For a fixed price of service contract P_G , the OEM agrees to carry out PM and CM in $[W, L)$. The owner is required to pay an additional cost if the repair cost is greater than a threshold value \mathcal{G} . If the availability falls below the target, the OEM should pay a penalty cost. Here, the OEM performs PM and CM over the life cycle of the equipment. Generally, several packages of maintenance service contract offers by the OEM - e.g. full coverage or partial coverage. Here PM is full coverage.

Under Option O_1 , the OEM provides a service covering PM and CM to the consumer with a fixed cost P_G within the contract period -e.g. 3 to 5 years. If the availability of the equipment for period j , A_j is less than



the availability target \tilde{A}_j , then the OEM should pay a penalty cost. The amount of the penalty cost is proportional to $\delta_j = \tilde{A}_j - A_j$. The penalty cost, C^p is viewed as a compensation given by the OEM.

The OEM needs to determine the optimal price structure (i.e. service contract cost for option and repair cost for option) to maximize the expected profit.

2.5. Owner's decision problem

The owner needs to decide which options best fit to maintain the equipment over (W, L) – i.e. to decide whether a PM is done in house and CM by the OEM or both PM and CM is fully done by the OEM. As a result, after the expiry of warranty, the consumer must choose the option O^* taken from the set $\{O_0, O_1\}$. As the equipment is used to generate income, then the owner has to select the optimal option that maximizes the expected profit.

3. MODEL ANALYSIS

We consider a common practice in mining industries where OEM and consumer will negotiate the pricing of service contract and the cost of repair. Consequently, the optimal values of service contract price and repair cost can be obtained using a non-cooperative game theory.

3.1. OEM's decision problem

Let C denote the repair cost to fix the failed equipment. As in many cases, the repair cost varies, then C is considered as a random variable with distribution function $G(c)$. Since every failure is fixed by a minimal repair then the failure process follows the Non Homogeneous Poisson Process [9], and hence the expected of OEM revenue for option O_0 is given by

$$E[\pi(O_0)] = R_0(W, L) \left[\int_0^\infty c g(c) dc \right]$$

where $R_0(W, L) = \int_w^L r_0(x) dx$. The expected of the OEM revenue for option O_1 is given by

$$E[\pi(O_1)] = P_G - E[\text{Penalty cost}] - E[\text{CM cost}] - E[\text{PM cost}]$$

We first obtain the expected of repair cost, expected of penalty cost and then expected of PM cost in (W, L) .

Under Option 1, if repair cost, C is greater than \mathcal{G} then the owner has to pay $(C - \mathcal{G})$. As a result the expected additional repair cost paid by the owner is given

by $\int_{\mathcal{G}}^\infty (c - \mathcal{G}) g(c) dc$. Then, the expected of repair cost incurred by the OEM is

$$EC_m = R_1(W, L) p C_m \quad (4)$$

Where $C_m = \int_0^\infty c g(c) dc$ and $R_1(W, L) = \int_w^L r(x) dx$.

Expected of penalty Cost: Let $Y(t)$ and $A(t)$ denote the total of down time and availability of the equipment in $(0, t)$. If \tilde{A} is the availability target, then a penalty occurs if $A(t) < \tilde{A}$ (or $1 - Y(t)/t < \tilde{A}$). If $\zeta = (1 - \tilde{A})t$, then the penalty incurs the OEM if $Y(t) > \zeta$ or the total down time in $(0, t)$ is greater than ζ . The expected penalty cost is given by $EP(t) = c_p E[\text{Max}\{0, Y(t) - \zeta\}] / t$ where c_p is the penalty cost and

$$E[\text{Max}\{0, Y(t) - \zeta\}] = \int_\zeta^\infty (y - \zeta) g(y) dy \quad (5)$$

Hence, we have

$$EP(t) = \frac{c_p \left\{ \int_\zeta^\infty (y - \zeta) g(y) dy \right\}}{t} \quad (6)$$

In most cases, availability target is given for each year. As the equipment deteriorates with age and usage, the availability target decreases from year to year. Let $\tilde{A}_j, j = 1, \dots, (L - W)$ denote the availability target at t_j , then $\tilde{A}_1 < \tilde{A}_2 < \dots < \tilde{A}_{L-W}$. Let $Y(t_{j-1}, t_j)$ and $A(t_{j-1}, t_j)$ denote the total of down time and availability of the equipment in (t_{j-1}, t_j) , then $A(t_{j-1}, t_j) = 1 - Y(t_{j-1}, t_j) / (t_j - t_{j-1})$. The penalty incurs the OEM at time t_j if $A(t_{j-1}, t_j) < \tilde{A}_j$ or $Y(t_{j-1}, t_j) > \zeta_j$ (the total down time in (t_{j-1}, t_j) is greater than ζ_j), where $\zeta_j = (1 - \tilde{A}_j)(t_j - t_{j-1})$. Hence, the probability that the penalty incurs at t_j is given by



$$P\{Y(t_{j-1}, t_j) > \zeta_j\} = \sum_{k=1}^{\infty} \left\{ 1 - F^{[k]}(\zeta_j) \right\} \frac{R(t_{j-1}, t_j)^k e^{-R(t_{j-1}, t_j)}}{k!}$$

$$(7) \quad EP(W, L) = \begin{cases} \sum_{j=1}^{L-W} \frac{c_p \int_{\zeta_j}^{\infty} [1 - G_j(y)] dy}{(t_j - t_{j-1})} & \text{for } A_j < \tilde{A}_j \\ 0 & \text{otherwise} \end{cases}$$

Define,

$$G_j(\zeta_j) = P\{Y(t_{j-1}, t_j) \leq \zeta_j\}$$

From (7), we have

$$G_j(\zeta_j) = \sum_{k=1}^{\infty} F^{[k]}(\zeta_j) \frac{R(t_{j-1}, t_j)^k e^{-R(t_{j-1}, t_j)}}{k!} \quad (8)$$

We assume that $F(x)$ has Exponential distribution with parameter λ , then we have after simplification,

$$G_j(\zeta_j) = P\{Y(t_{j-1}, t_j) \leq \zeta_j\} \\ = \sum_{k=1}^{\infty} \frac{R(t_{j-1}, t_j)^k e^{-R(t_{j-1}, t_j)}}{k!} \frac{(\lambda \zeta_j)^k}{k!} e^{-\lambda \zeta_j}$$

And the density function of $G_j(\zeta_j)$ is given by

$$g_j(\zeta_j) = \frac{dG_j(\zeta_j)}{d\zeta_j}$$

Now, we obtain the expected penalty cost in (t_{j-1}, t_j) and than in (W, L) . The expected penalty cost in (t_{j-1}, t_j) is given by

$$EP_j(t_{j-1}, t_j) = \frac{c_p E[\text{Max}\{0, Y(t_{j-1}, t_j) - \zeta_j\}]}{(t_j - t_{j-1})}$$

$$EP_j(t_{j-1}, t_j) = \frac{c_p \int_{\zeta_j}^{\infty} [1 - G_j(y)] dy}{(t_j - t_{j-1})} \quad (9)$$

As a result, the expected penalty cost in (W, L) is given by

The expected of PM cost is

$$EC_{pm} = C_{pm}(L - W) \quad (10)$$

As a result, the total expected revenue of the OEM is

$$E[\pi(O_1)] = P_G - C_p \sum_{j=1}^{L-W} \frac{\int_{\zeta_j}^{\infty} [1 - G_j(y)] dy}{(t_j - t_{j-1})} - R_1(W, L) C_m - C_{pm}(L - W) \quad (11)$$

3.2. Customer's decision problem

The expected profit of the consumer upon choosing the O_1

option, $E[\omega(O_1)]$ is given by

$$E[\omega(O_1)] = K[L - W - (R_1(L - W) - R_1(W))E[U_i]] \quad (12)$$

$$+ C_p \sum_{j=1}^{L-W} \frac{\int_{\zeta_j}^{\infty} [1 - G_j(y)] dy}{(t_j - t_{j-1})}$$

$$- R_1(W, L) \int_0^{\infty} (c - \vartheta) g(c) dc - P_G - C_b$$

The expected profit of the consumer upon choosing the

option O_0 , $E[\omega(O_0)]$, is given by

$$E[\omega(O_0; P_0, \vartheta)] = K[L - W - (R_0(L - W) - R_0(W))E[U_i]] \quad (13)$$

$$- R_0(W, L)(C_m - C_s) - P_0 - C_b$$

4. RESULT

We first obtain P_G^* and then C_s^* . In the presence of negotiation between the two parties for every option offered, the consumer and the OEM will receive the same profit. Then, for Option O_1 we have P_G^* given by



$$P_G^* = \frac{1}{2} \left[K \left[L - W - \sum_{j=1}^{L-W} E[Y(t_{j-1}, t_j)] \right] + 2C_p \sum_{j=1}^{L-W} \frac{\int_{\zeta_j}^{\infty} [1 - G_j(y)] dy}{(t_j - t_{j-1})} + R_1(W, L) \left(C_m - \int_g^{\infty} (c - g)g(c)dc \right) \right] + C_{pm}(L - W) - P_G - C_b \quad (14)$$

$$\sum_{j=1}^{L-W} E[Y(t_{j-1}, t_j)] = (R_1(L - W) - R_1(W))E[U_i]$$

Substituting (14) to (11), the expected profit of the OEM on option O_1 , becomes

$$E[\pi(O_1; P_G)] = \frac{1}{2} \left[K \left[L - W - \sum_{j=1}^{L-W} E[Y(t_{j-1}, t_j)] \right] - R_1(W, L) \left(C_m - \int_g^{\infty} (c - g)g(c)dc \right) - C_{pm}(L - W) - C_b \right] \quad (15)$$

We have C_s^* for Option O_0 that satisfied

$$C_s^* = \frac{1}{2R_0(W, L)} \left[K \left\{ L - W - \sum_{j=1}^{L-W} E[Y(t_{j-1}, t_j)] \right\} + C_m R_0(W, L) - P_0 - C_b \right] \quad (16)$$

Then, the expected profit of the OEM on option O_0 become

$$E[\pi(O_0; P_0, C_s)] = \frac{1}{2} \left[K \left\{ L - W - \sum_{j=1}^{L-W} E[Y(t_{j-1}, t_j)] \right\} - P_0 - C_b - C_m R_0(W, L) \right] \quad (17)$$

Here, we can see that maximum expected profit for the OEM using option O1 is always greater than using option O0. It can be seen by the difference of (15) and (17), i.e.,

$$E[\pi(O_1; P_G)] - E[\pi(O_0; C_s)] = \frac{1}{2} \left[P_0 + C_m R_0(W, L) - R_1(W, L) \left(C_m - \int_g^{\infty} (c - g)g(c)dc \right) - C_{pm}(L - W) \right] > 0$$

since all parameter is positive, $P_0 > C_{pm}(L - W)$, and $R_0(W, L) > R_1(W, L)$.

5. CONCLUSIONS

We have studied a service maintenance contract for equipment with availability target and reduction cost. We have determined the optimal price structure for the OEM and the optimal service option for the owner of the equipment. In this paper, we consider only two player -i.e. OEM and owner. In many cases, owner has more than one agent included the OEM and offers more options -partial, moderate, and full coverage of service contract. Also, in our paper, every failure is fixed by the minimal repair. One can consider other servicing strategy -e.g. repair-replace strategy or servicing strategy involving imperfect repair. These further research topics are currently under investigation.

REFERENCES

- [1] D. N. P. Murthy and E. Ashgarizadeh. 1999. Optimal decision making in a maintenance service operation. *European Journal of Operational Research*. 62: 1-34.
- [2] E. Ashgarizadeh and D. N. P. Murthy. 2000. Service contracts. *Mathematical and Computer Modelling*. 31: 11-20.
- [3] K. Rinsaka and H. Sandoh. 2006. A stochastic model on an additional warranty service contract. *Computers and Mathematics with Applications*. 51: 179-188.
- [4] D. N. P. Murthy and E. Ashgarizadeh. 1998. A stochastic model for service contract. *International Journal of Reliability, Quality and Safety Engineering*. 5: 29-45.
- [5] C. Jackson and R. Pascual. 2008. Optimal maintenance service contract negotiation with aging equipment. *European Journal of Operational Research*. 189: 387-398.
- [6] I. Djameludin, D. N. P. Murthy and C.S. Kim. 2001. Warranty and preventive maintenance. *International Journal of Reliability, Quality and Safety Engineering*. 8(2): 89-107.



www.arnjournals.com

- [7] D. N. P. Murthy and V. Yeung. 1995. Modelling and analysis of maintenance service contracts. *Mathematical and Computer Modelling*. 22: 219-225.

- [8] Wang. 2010. A model for maintenance service contract design, negotiation and optimization. *European Journal of Operational Research*. 201(1): 239-246.

- [9] R. E. Barlow, and L. Hunter. 1960. Optimum preventive maintenance policies. *Operational Research*. 8: 90-100.