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SYNTHESIS OF INTEGRATED INERTIAL AND SATELLITE NAVIGATIONAL SYSTEMS ON THE BASIS OF STOCHASTIC FILTER, INVARIANT TO OBJECT MODEL

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ABSTRACT

It is shown the principal possibility of the problem solution of synthesis of tight integrated inertial and satellite navigational systems on the basis of theory of stochastic filtering without information engaging on object model, on character of its motion, etc. The offered suboptimal Kalman's algorithms of estimation, being invariant to object model, provide inconvertible high-precision estimation of navigational vector both in the presence of satellite measuring, and at their loss as well.

Keywords: synthesis, inertial, satellite, systems, stochastic filter, model.

INTRODUCTION

The existing algorithms of navigational parameters filtering in tight integrated satellite and strapdown inertial navigational systems (SINS) are formed, basically, either on the basis of known motion model of particular object [1], or on the basis of so-called equations of errors of INS [2]. In both cases it is assumed the existence of a priori information on character of object motion in time that for overwhelming majority of the mobile objects possible only with very restricted precision and on small intervals. It is in essence complicates the use of navigational algorithms on the basis of methods of non-linear filtering in majority of practical cases when it is not unknown neither a motion trajectory, nor a kind of physical analog not a character of perturbations acting on object, etc. At the same time, it is obvious that an application for processing of inertial and satellite measuring of non-linear filtering methods providing the solution of problem on estimation in most general case of prescription of stochastic object and its perturbing actions, will allow to increase considerably a precision of determination of navigational parameters owing to withdrawal from various simplifying assumptions (linearization, additional information on object, on interferences, etc.), used in existing algorithms of satellite navigation.

Task definition

Let's analyze the key possibility of a posteriori stochastic estimation of navigational parameters of the mobile objects (MO) on inertial and satellite measuring for objects of any class. Thereat the algorithms of filtering shall be invariant to kind of object physical analog, trajectory of its motion, character of perturbations and so forth. For this purpose at prescription of object motion we will use the following dextral coordinate systems (CS) [2, 3]:

- instrumental CS (Inst.CS) $J0xyz$, which beginning is arranged in center of mass (CM) of object, and axes are guided on crossly orthogonal axes of sensitivity of SINS measuring complex instruments,
- inertial CS (ICS) $I_{O\eta_1\xi_1\zeta_1}$ with beginning in the center of Earth,
- the Greenwich CS (GCS) rotating together with Earth $G_{O\eta\xi\zeta}$,
- accompanying (ACS) S_{0xyz}, which beginning coincides with MO center of mass, the axis Y coincides with direction of aboriginal meridian, the axis Z is guided up on plumb line, and the axis X supplements the system to dextral.

We assume also that into measuring complex of the integrated NS it is included the receiver of satellite navigational system (SNS), the Doppler speed sensor (DSS) and SINS consisting of three accelerometers and three angular velocity sensors (AVS). As a noise model of measuring the sensing elements (SE) we will accept the white Gaussian noise (WGN). Such approach does not superimpose the principal limiting on the objective solution, as by state vector dilating at the expense of introduction of formatting filters it appears being possible to gain a model of SE noises not just with the given time response statistical characteristics (mathematical expectation, correlative function and so on), but also with the required distribution law.

At the first stage we will consider a possibility on solving the problem on a posteriori estimation of navigational parameters vector at loss of satellite conferring, i.e. on the basis of only inertial component of tight integrated NS.

Estimation of navigational parameters in the absence of satellite measuring

The combined equations of SINS state vector, invariant to the kind of object physical analog, as shown in [3], have the following appearance:

$$\begin{vmatrix}\dot\alpha\\ \dot\beta\\ \dot\gamma\end{vmatrix} = \begin{vmatrix}\frac{\sin\gamma}{\cos\beta} & \frac{\cos\gamma}{\cos\beta} & 0\\ \cos\gamma & -\sin\gamma & 0\\ \sin\gamma\, tg\beta & \cos\gamma\, tg\beta & 1\end{vmatrix}(Z_d - W_d) =$$

$$= \Phi(\beta,\gamma)(Z_d - W_d),$$

$$\alpha_0 = \alpha(0), \beta_0 = \beta(0), \gamma_0 = \gamma(0),$$

$$\begin{vmatrix}\dot\lambda\\ \dot\varphi\end{vmatrix} = \begin{vmatrix}0 & (\cos\varphi)^{-1}\\ -1 & 0\end{vmatrix}\begin{vmatrix}-V_Y\\ V_X\end{vmatrix}(r+h)^{-1},$$

$$\lambda_0 = \lambda(0), \varphi_0 = \varphi(0), \quad (1)$$

$$\begin{vmatrix}\dot V_X\\ \dot V_Y\\ \dot V_Z\end{vmatrix} = C^T(\alpha,\beta,\gamma,\lambda,\varphi)Z_a +$$

$$+\left(\left(2\begin{vmatrix}0\\ \Omega\cos\varphi\\ \Omega\sin\varphi\end{vmatrix} + (r+h)^{-1}\begin{vmatrix}-V_Y\\ V_X\\ V_X tg\varphi\end{vmatrix}\right)\times\begin{vmatrix}V_X\\ V_Y\\ V_Z\end{vmatrix}\right) -$$

$$-\begin{vmatrix}0\\ -\Omega^2(r+h)\cos\varphi\sin\varphi\\ \Omega^2(r+h)\cos^2\varphi + g\end{vmatrix} -$$

$$-C^T(\alpha,\beta,\gamma,\lambda,\varphi)W_a,$$

$$V_{X0} = V_X(0), V_{Y0} = V_Y(0), V_{Z0} = V_Z(0),$$

$$\dot h = V_Z, \quad h_0 = h(0),$$

where α, β, γ - the angles of Euler - Krylov defining the orientation of Inst.CS regarding the In.CS, $Z_d = \left|Z_x \quad Z_y \quad Z_z\right|^T$ - a vector of measuring of three orthogonal AVSs, $W_d = \left|W_x \quad W_y \quad W_z\right|^T$ - a vector of the additive interferences of AVSs measuring (WGN with zero average and array of intensity D_d), $\lambda -$ a longitude, $\varphi -$ a latitude, *h* - the object altitude, V_X, V_Y, V_Z - the projections of the object linear speed to axes of accompanying CS, r - the Earth radius, Ω - an angular speed of Earth rotation, g - a gravitational acceleration, $Z_a = \left|Z_{ax} \quad Z_{ay} \quad Z_{az}\right|^T$ - a vector of accelerometers outlet signals, $W_a = \left|W_{a_x} \quad W_{a_y} \quad W_{a_z}\right|^T$ - a vector of accelerometers interferences (WGN with zero mathematical expectation and array of intensity D_a), $C(\alpha,\beta,\gamma,\lambda,\varphi) = D(\alpha,\beta,\gamma)B^T(\lambda,\varphi)$ - the array of cosines direction defining the orientation of Inst.CS regarding ACS, $D(\alpha,\beta,\gamma)$ - an array of rotational displacement of 2nd kind [4], defining the Inst.CS orientation regarding to In.CS, $B = D(\lambda + \Omega t = \psi, -\varphi, 0)$ - the array of 2nd kind defining the ACS orientation regarding to In.CS. In Lanzheven's canonical form the initial for the subsequent synthesis of filtering algorithms, the given stochastic equations, invariant to a character of object motion, it is possible to provide as:

$$\dot Y = F(Y,t) + F_1(Y,t)\xi, \quad (2)$$

where

$$Y = \left|\alpha \quad \beta \quad \gamma \quad \lambda \quad \varphi \quad V_X \quad V_Y \quad V_Z \quad h\right|^T, \quad Y(0) = Y_0$$

$\xi = \left|W_d^T \quad W_a^T\right|^T$, the vector and the matrix of functions $F(Y,t)$, $F_1(Y,t)$ are given in Application 1.

For final build-up of estimation algorithm of navigational parameters it is necessary to form the equation of their observation [5]. Here it is necessary to underline that the monitoring of the given complete vector of navigational parameters is possible only by means of no inertial measurers since the informational models of all inertial are already used in the equations of SINS state vector. For this purpose we are using further DSS information, assuming that the outlet signal *of* Z_D *DSS*, which measuring axis is guided on the Inst.CS conforming axis (for definiteness we assume further that it is per axis *Ox*), generally looks like: $Z_D = V_x + U_D$, where V_x - the vector projection of MO motion relative speed onto axis *Ox* Inst.CS, U_D - WGN with zero average and known intensity D_U.

Since the projection of velocity vector V_x can be expressed through the vector of relative MO motion speed in ACS $V_S = \left|V_X \ V_Y \ V_Z\right|^T$ as $V_x = C_{(1)}(\alpha,\beta,\gamma,\lambda,\varphi)\, V_S$ where $C_{(1)}(\alpha,\beta,\gamma,\lambda,\varphi)$ - the 1st row of a matrix $C(\alpha,\beta,\gamma,\lambda,\varphi)$, the informational model of DSS outlet signal becomes:

$$Z_D = C_{(1)}(\alpha,\beta,\gamma,\lambda,\varphi)\ V_S + U_D =$$
$$H_D(\alpha,\beta,\gamma,\lambda,\varphi,V_X,V_Y,V_Z) + U_D, \quad (3)$$

and the observation signal appears dependent on both angular and linear motions parameters. It is obvious that the magnification of DSS measuring canals will only increase the informativeness of observation and, thereby, will increase the general precision of navigational vector estimation. Existence of stochastic equations (2,3) in shape "object - observer" allows carrying out the principal rigorous problem solution of a posteriori estimation of navigational parameters vector at loss of satellite conferring, having constructed its a posteriori probability density and having created on its basis the estimation algorithms per conforming criterions of optimality [5]. Today the most preferable per criterion «precision - computing expenditures» estimation algorithm is the generalized Kalman's filter [5] ensuring a suboptimal estimation by mean squared measure and having in considered case an appearance follows:

$$\hat{Y} = F(\hat{Y},t) + K(\hat{Y},t)\left[Z_D - H_D(\hat{Y},t)\right]$$
$$K(\hat{Y},t) = R\frac{\partial H_D(\hat{Y},t)^T}{\partial \hat{Y}} D_U^{-1}, \quad (4)$$

$$\dot{R}(\hat{Y},t) = \frac{\partial F(\hat{Y},t)}{\partial \hat{Y}} R(\hat{Y},t) + R(\hat{Y},t)\frac{\partial F^T(\hat{Y},t)}{\partial \hat{Y}} +$$
$$+ F_1(\hat{Y})\begin{vmatrix} D_d & 0 \\ 0 & D_a \end{vmatrix} F_1^T(\hat{Y}) - K(\hat{Y},t) D_U\ K^T(\hat{Y},t),$$

where $\hat{Y}$ - a vector of current estimation of state vector of object Y*(t),* $R(\hat{Y},t)$ - an a posteriori dispersion matrix $\hat{Y}_0 = M(Y_0)R_0 = = M\{(Y_0 - \hat{Y}_0)(Y_0 - \hat{Y}_0)^T\}$.

The basic advantages of algorithm (4) are the estimation possibilities, at first, of navigational parameters at loss of satellite signals, and secondly, of object angular motion parameters, unobservable in satellite measuring. But, as the experiments [1, 2] results show the precision of linear motion parameters estimation vector here appears worse, than when handling the satellite measuring. In this regard, at presence of SNS conferring the vector of linear motion parameters is necessary to be estimated per satellite measuring. The existing algorithms of satellite measuring processing are used for this purpose either the various modifications of the least-squares method or manifold modifications of Kalman's filter [1, 2] which, in their turn, require binding knowledge of motion equations *of each particular* object. As it has been admired above, it in essence complicates the use of existing Kalman's navigational algorithms in overwhelming majority of the mobile objects. Therefore we will consider a possibility of synthesis of such Kalman's algorithm of processing of satellite measuring which is, on the one hand, would be invariant to object kind and to character of its motion, and on another, will allow a possibility of tight connection with Kalman's algorithm of inertial measuring processing for estimation of object angular motion parameters.

A posteriori estimation of motion parameters per satellite measuring

For build-up of this algorithm we use only the coded and Doppler measuring, as providing fully the objective solution. In standard mode the informational signal of coded measuring (pseudo-distance) in general case can be noted as [1, 6, 7]:

$$Z_R = \sqrt{(\xi_c - \xi)^2 + (\eta_c - \eta)^2 + (\zeta_c - \zeta)^2} + W_{Z_R}, \quad (5)$$

where ξ_c, η_c, ζ_c - the known coordinates of satellite in GCS ξ η , ζ - the current coordinates of object in GCS, W_{Z_R} - WGN with zero average and known intensity $D_{Z_R}(t)$, conditioned by algorithmically uncompensated errors of clocks of satellites and receiver, by signal delays at ionosphere and troposphere transiting, by errors of multi beemal and some other errors.

In turn, the informational signal of Doppler measuring (pseudo-velocity) Z_V in autonomous mode can be provided as follows [1, 6, 7]:

$$Z_V =$$
$$[\,(\xi_c - \xi)(V_{\xi c} - V_\xi) + (\eta_c - \eta)\ (V_{\eta c} - V_\eta) + (\zeta_c - \zeta)(V_{\zeta c} - V_\zeta)\,] \times$$
$$\times \left(\sqrt{(\xi_c - \xi)^2 + (\eta_c - \eta)^2 + (\zeta_c - \zeta)^2}\right)^{-1} + W_{Z_V}, \quad (6)$$

where $V_{\xi c}, V_{\eta c}, V_{\zeta c}$ - the projections of satellite velocity vector onto GCS axes, V_ξ, V_η, V_ζ - the projections of object velocity vector onto *GCS,* W_V - non-linear Markov's process with known performances*,* stipulated by errors of Doppler measuring.

For possibility of theoretically rigorous solution of problem of a posteriori estimation of object state vector on the basis of satellite measuring it is necessary, as well as earlier, to have its equations of state written down in the stochastic form of Lanzheven's. For solution of this problem we will consider beforehand the equation (6). Concerning the velocity vector of object V it can be re-written in a view:

$$[\,(\xi_c - \xi)V_{\xi c} + (\eta_c - \eta)V_{\eta c} + (\zeta_c - \zeta)V_{\zeta c}\,] -$$
$$\sqrt{(\xi_c - \xi)^2 + (\eta_c - \eta)^2 + (\zeta_c - \zeta)^2}\,(Z_V - W_{Z_V}) =$$
$$= (\xi_c - \xi)V_\xi + (\eta_c - \eta)V_\eta + (\zeta_c - \zeta)V_\zeta, \quad (7)$$

or in the vector shape:

$$(\varepsilon_c - \varepsilon)^T V_c - \left[(\varepsilon_c - \varepsilon)^T(\varepsilon_c - \varepsilon)\right]^{\frac{1}{2}}(Z_V - W_{Z_V}) = (\varepsilon_c - \varepsilon)^T V, \quad (8)$$

where $\varepsilon_c = |\xi_c\ \ \eta_c\ \ \zeta_c|^T$, $\varepsilon = |\xi\ \ \eta\ \ \zeta|^T$.

It is obvious, that for definition of all components of object velocity vector $V = \dot{\varepsilon}$ of reduced equation, gained per Doppler measuring, the one satellite is not enough. For forming of missing equations beforehand we will

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induct the following labels: $\varepsilon_{ci} = \left|\xi_{c_i} \ \eta_{c_i} \ \zeta_{c_i}\right|^T$, $i = 1,2,3,$ – a vector of known coordinates *of* i - th satellite in GCS, $V_{ci} = \left|V_{\xi c_i} \ V_{\eta c_i} \ V_{\zeta c_i}\right|^T$, $i = 1,2,3,$ - velocity vector of i - th satellite in GCS, Z_{Vi} - a signal of Doppler measuring *of* i - th satellite, $W_{Z_{Vi}}$ - the errors of Doppler measuring *of* i - th satellite. Further we will note the combined equations, similar to (8), but build-up this time per *three* satellites of Doppler measuring:

$$(\varepsilon_{c_1}-\varepsilon)^T V_{c_1} - \left[(\varepsilon_{c_1}-\varepsilon)^T(\varepsilon_{c_1}-\varepsilon)\right]^{\frac{1}{2}}(Z_{V_1}-W_{Z_{V1}}) = (\varepsilon_{c_1}-\varepsilon)^T\dot{\varepsilon},$$

$$(\varepsilon_{c_2}-\varepsilon)^T V_{c_2} - \left[(\varepsilon_{c_2}-\varepsilon)^T(\varepsilon_{c_2}-\varepsilon)\right]^{\frac{1}{2}}(Z_{V_2}-W_{Z_{V2}}) = (\varepsilon_{c_2}-\varepsilon)^T\dot{\varepsilon}, \qquad (9)$$

$$(\varepsilon_{c_3}-\varepsilon)^T V_{c_3} - \left[(\varepsilon_{c_3}-\varepsilon)^T(\varepsilon_{c_3}-\varepsilon)\right]^{\frac{1}{2}}(Z_{V_3}-W_{Z_{V3}}) = (\varepsilon_{c_3}-\varepsilon)^T\dot{\varepsilon}.$$

Having designated further for record cutting $\left[(\varepsilon_{c_i}-\varepsilon)^T(\varepsilon_{c_i}-\varepsilon)\right]^{\frac{1}{2}} = \rho_i$, $i = 1,2,3,$

$$\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon) = \begin{vmatrix}\rho_1 & 0 & 0\\ 0 & \rho_2 & 0\\ 0 & 0 & \rho_3\end{vmatrix}, \quad Z_{V0} = \begin{vmatrix}Z_{V_1}\\ Z_{V_2}\\ Z_{V_3}\end{vmatrix}, \quad W_{Z_{V0}} = \begin{vmatrix}W_{Z_{V1}}\\ W_{Z_{V2}}\\ W_{Z_{V3}}\end{vmatrix},$$

let's write down the gained combined equations in the vector view:

$$\begin{vmatrix}(\varepsilon_{c_1}-\varepsilon)^T V_{c_1}\\ (\varepsilon_{c_2}-\varepsilon)^T V_{c_2}\\ (\varepsilon_{c_3}-\varepsilon)^T V_{c_3}\end{vmatrix} - \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, Z_{V0} + .$$

$$+\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, W_{Z_{V0}} = \begin{vmatrix}(\varepsilon_{c_1}-\varepsilon)^T\\ (\varepsilon_{c_2}-\varepsilon)^T\\ (\varepsilon_{c_3}-\varepsilon)^T\end{vmatrix}\dot{\varepsilon}$$

This system easily allows the resolution concerning the object velocity vector $V = \dot{\varepsilon}$:

$$\dot{\varepsilon} = \begin{vmatrix}(\varepsilon_{c_1}-\varepsilon)^T\\ (\varepsilon_{c_2}-\varepsilon)^T\\ (\varepsilon_{c_3}-\varepsilon)^T\end{vmatrix}^{-1} \left(\begin{vmatrix}(\varepsilon_{c_1}-\varepsilon)^T V_{c_1}\\ (\varepsilon_{c_2}-\varepsilon)^T V_{c_2}\\ (\varepsilon_{c_3}-\varepsilon)^T V_{c_3}\end{vmatrix} - \right.,$$

$$\left. -\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, Z_{V0} + \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, W_{Z_{V0}}\right)$$

$$\varepsilon_0 = \varepsilon(0), \qquad (10)$$

where $\begin{vmatrix}(\varepsilon_{c_1}-\varepsilon)^T\\ (\varepsilon_{c_2}-\varepsilon)^T\\ (\varepsilon_{c_3}-\varepsilon)^T\end{vmatrix}^{-1} = \Phi(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)$ - an array, inverse to array $\begin{vmatrix}(\varepsilon_{c_1}-\varepsilon)^T\\ (\varepsilon_{c_2}-\varepsilon)^T\\ (\varepsilon_{c_3}-\varepsilon)^T\end{vmatrix}$

(Application 2).

The gained equations prescribe the feature dynamics of modification just for object coordinates vector while in the majority of practical applications it is required to evaluate its velocity as well. For synthesis of object linear speed equations let's write down the equation (10) in the view follows:

$$V = \Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\left(\begin{vmatrix}(\varepsilon_{c_1}-\varepsilon)^T V_{c_1}\\ (\varepsilon_{c_2}-\varepsilon)^T V_{c_2}\\ (\varepsilon_{c_3}-\varepsilon)^T V_{c_3}\end{vmatrix} - \right.$$

$$\left. -\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, Z_{V0} + \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, W_{Z_{V0}}\right) =$$

$$= \Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\{\Xi(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},\varepsilon) -$$

$$-\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, Z_{V0} + \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, W_{Z_{V0}}\}$$

and differentiate the two units of this system on time. Let's use thus the Markov's representation of interference vector of SNS Doppler measuring in the form of system of non-linear stochastic equations in the form of Lanzheven's:

$$\dot{W}_{Z_{V0}} = \phi(W_{Z_{V0}}, t) + \varsigma, \qquad (11)$$

where $\phi(W_{Z_{V0}}, t)$ - the known non-linear vector- function, ς - centered by WGN with known array of intensity D_ς.

Differentiating the given above equation, we have:

$$\dot{V} = \theta(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},$$
$$\dot{V}_{c_1},\dot{V}_{c_2},\dot{V}_{c_3},W_{Z_{V0}},Z_{V0},\dot{Z}_{V0},\varepsilon,V) \qquad (12)$$
$$+\Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\,\varsigma,$$

where the vector-function

$$\theta(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},\dot{V}_{c_1},$$
$$\dot{V}_{c_2},\dot{V}_{c_3},W_{Z_{V0}},Z_{V0},\dot{Z}_{V0},\varepsilon,V)$$

and deduction of the equations (12) are given in Application 3.

For possibility of subsequent synthesis of algorithms of a posteriori estimation of vector of parameters of linear motion per satellite measuring it is

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necessary to construct the uniform system of its equations, having'em united into combined equations (10) - (12):

$$\dot{\varepsilon} = \Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)(\Xi(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},\varepsilon) - \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, Z_{V0} + \rho\,(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\; W_{Z_{V0}}),$$
$$\dot{V} = \theta(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},W_{Z_{V0}},Z_{V0},\dot{Z}_{V0},\varepsilon,V) + \Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\,\varsigma,$$

$$\dot{W}_{Z_{V0}} = \phi(W_{Z_{V0}},t) + \varsigma. \qquad (13)$$

For the purpose of further shaping of observer vector of linear motion parameters we will analyze the informational model of coded measuring (5). Obviously the measuring of coded distances even of one satellite provide an observation of linear motion parameters (object's coordinates), but since the measuring of all three satellites are anyways necessary for shaping the state vector (13) then it is expedient to use for magnification of informativeness of the observer at synthesis of the equations of observation of coded distances measuring also from three satellites:

$$Z_{R1} = \sqrt{(\xi_{c_1}-\xi)^2 + (\eta_{c_1}-\eta)^2 + (\zeta_{c_1}-\zeta)^2} + W_{Z_{R1}} = H_1(\varepsilon_{c_1},\varepsilon) + W_{Z_{R1}}, \qquad (14)$$

$$Z_{R2} = \sqrt{(\xi_{c_2}-\xi)^2 + (\eta_{c_2}-\eta)^2 + (\zeta_{c_2}-\zeta)^2} + W_{Z_{R2}} = H_2(\varepsilon_{c_2},\varepsilon) + W_{Z_{R2}},$$

$$Z_{R3} = \sqrt{(\xi_{c_3}-\xi)^2 + (\eta_{c_3}-\eta)^2 + (\zeta_{c_3}-\zeta)^2} + W_{Z_{R3}} = H_3(\varepsilon_{c_3},\varepsilon) + W_{Z_{R3}},$$

where Z_{Ri} - the code measuring *of* i - th satellite, $W_{Z_{Ri}}$ - the errors of measuring *of* i - th satellite.

The gained equations (13, 14) in the classical form "object-observer" allow solving in essence per SNS information the problem of optimum estimation of vector of linear motion parameters of any object, irrespective of kind of its model and motion character. But thereat they do not allow evaluating its angular motion parameters. For possibility of pin-point precision estimation of all navigational state vectors we will consider a tight integrated SINS and SNS.

The solution of navigational problem on the basis of tight integrated NS

When forming a tight connection of SINS and SNS we will consider that parameters determination of linear object motion can be more precisely carried out at utilization of satellite measuring, and the its angular position definition - only with use of autonomous (inertial and no inertial) measuring. Since at determination of the current coordinates of object on satellite measuring (the equations (13, 14)) there is no need of use for equations of SINS state (1) equations featuring the object linear motion, then the equations of state vector of tight integrated system become:

$$\begin{vmatrix}\dot{\alpha}\\ \dot{\beta}\\ \dot{\gamma}\end{vmatrix} = \begin{vmatrix}\dfrac{\sin\gamma}{\cos\beta} & \dfrac{\cos\gamma}{\cos\beta} & 0\\ \cos\gamma & -\sin\gamma & 0\\ \sin\gamma tg\beta & \cos\gamma tg\beta & 1\end{vmatrix}(Z_d - W_d) = \Phi(\beta,\gamma)(Z_d - W_d),$$
$$\dot{\varepsilon} = \Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)(\Xi(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},\varepsilon) - \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, Z_{V0} + \rho\,(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\; W_{Z_{V0}}),$$

$$\dot{V} = \theta(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},W_{Z_{V0}},Z_{V0},\dot{Z}_{V0},\varepsilon,V) + \Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\,\varsigma,$$

$$\dot{W}_{Z_{V0}} = \phi(W_{Z_{V0}},t) + \varsigma\,,$$

or in the form of Lanzheven's:

$$\dot{Y}_H = F_H(Y_H,t) + F_{1H}(Y_H,t)\xi_H, \qquad (15)$$

where

$$Y_H = \begin{vmatrix}\alpha & \beta & \gamma & \varepsilon^T & V^T & W_{Z_{V0}}{}^T\end{vmatrix}^T, \qquad Y_H(0) = Y_{H0}$$
$$\xi_H = \begin{vmatrix}W_d^T & \varsigma^T\end{vmatrix}^T,$$
$$F_H(Y_H,t) = \begin{vmatrix}\Phi(\beta,\gamma)\, Z_d\\ \Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},Y)(\Xi(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},Y) - \\ -\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},Y)\, Z_{V0} + \\ +\rho\,(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},Y)\; W_{Z_{V0}})\\ \theta(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},Z_{V0},\dot{Z}_{V0},Y)\\ \phi(Y,t)\end{vmatrix},$$
$$F_{1H}(Y_H,t) = \begin{vmatrix}-\Phi(\beta,\gamma) & 0\\ & 0\\ \Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},Y)\,\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},Y) & \\ & E_3\end{vmatrix},$$

E_3 - unit dimensiality matrix 3.

Observation of combined state vector (15) is expedient for carrying out with employment of all available measurers both: autonomous (3), and exterior (14). But thus it is necessary to consider the circumstance that parameters of object linear motion here are set in GCS and as appropriate to transform the equation of DSS informational signal (3). This, in turn, results in need on preliminary definition of array of direction cosines C_G defining the Inst. CS orientation regarding the GCS. The given array is formed as the array of cross rotational displacement of Inst. CS regarding the GCS taking into

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account the array of rotational displacement G of the Greenwich CS regarding to the In.CS

$$G = \begin{vmatrix} \cos\Omega t & 0 & -\sin\Omega t \\ 0 & 1 & 0 \\ \sin\Omega t & 0 & \cos\Omega t \end{vmatrix}$$

and of given above array of rotational displacement $D(\alpha,\beta,\gamma)$ of Inst. CS regarding to the In. CS: $C_G\ (\alpha,\beta,\gamma,\Omega t) = DGT\ (t)(\alpha,\beta,\gamma)\ \Omega$.

Then the velocity vector projection V_x in Inst.CS can be expressed through the vector of relative speed V of object motion in *GCS* as $V_x = C_{G(1)}(\alpha,\beta,\gamma,\Omega t)\ V$, where $C_{G(1)}(\alpha,\beta,\gamma,\Omega t)$ - the 1st row of a matrix $C_G(\alpha,\beta,\gamma,\Omega t)$.

In this case the informational model of outlet signal of DSS becomes:

$$Z_D = C_{G(1)}(\alpha,\beta,\gamma,\Omega t)\ V + U_D = H_{D1}(\alpha,\beta,\gamma,\Omega t, V) + U_D$$

and the signal of observation appears dependent *explicitly* same as from parameters of linear motion (velocity of object) and angles of turn of instrumental trihedron. The gained informational models of code measuring (14) and DSS allow providing an explicit observation of all navigational parameters of object at their integration and, thereby, completely to solve the problem of their inconvertible a posteriori estimation. The combined observer has in this case the appearance follows:

$$Z_H = \begin{vmatrix} Z_{R_1} \\ Z_{R_2} \\ Z_{R_3} \\ Z_D \end{vmatrix} = \begin{vmatrix} H_1(\varepsilon_{c_1}, Y_H) \\ H_2(\varepsilon_{c_2}, Y_H) \\ H_3(\varepsilon_{c_3}, Y_H) \\ H_{D1}(Y_H, t) \end{vmatrix} + \begin{vmatrix} W_{Z_{R1}} \\ W_{Z_{R2}} \\ W_{Z_{R3}} \\ U_D \end{vmatrix} = H_H(Y_H, t) + W_H, \qquad (16)$$

and the equations of estimation can be noted as follows:

$$\dot{\hat{Y}}_H = F_H(\hat{Y}_H, t) + K(\hat{Y}_H, t)\left[Z_H - H_H(\hat{Y}_H, t)\right] \qquad (17)$$

$$K(\hat{Y}_H, t) = R(\hat{Y}_H, t)\frac{\partial H_H^{\ T}(\hat{Y}_H, t)}{\partial \hat{Y}_H}\begin{vmatrix} D_{Z_{R0}}^{\ -1} & 0 \\ 0 & D_U^{-1} \end{vmatrix},$$

$$D_{Z_{R0}} = \begin{vmatrix} D_{Z_{R1}} & 0 & 0 \\ 0 & D_{Z_{R2}} & 0 \\ 0 & 0 & D_{Z_{R3}} \end{vmatrix},$$

$$\dot{R}(\hat{Y}_H, t) = \frac{\partial F_H(\hat{Y}_H, t)}{\partial \hat{Y}_H}R(\hat{Y}_H, t) + $$

$$+ R(\hat{Y}_H, t)\frac{\partial F_H^{\ T}(\hat{Y}_H, t)}{\partial \hat{Y}_H} + F_{1H}(\hat{Y}_H)\begin{vmatrix} D_d & 0 \\ 0 & D_{Z_{V0}} \end{vmatrix}F_{1H}^T(\hat{Y}_H) -$$

$$- K(\hat{Y}_H, t)\begin{vmatrix} D_{Z_{R0}} & 0 \\ 0 & D_U \end{vmatrix}K^T(\hat{Y}_H, t),\ \hat{Y}_H = M(Y_H),$$

$$R_0 = M\left\{(Y_H - \hat{Y}_H)(Y_H - \hat{Y}_H)^T\right\}.$$

The given algorithm of estimation can be used only in the presence of satellite measuring, but unlike "exclusively satellite" the navigational algorithm, at first, allows to estimate the parameters of object angular motion, and secondly, more precisely evaluates the vector of linear motion parameters at the expense of use of DSS additional observations. At loss of satellite navigational conferring the tight integrated NS passes to autonomous mode and uses the algorithm (4).

Example

For illustration of possibility of effective use of the offered algorithm of tight integration the numerical model modeling of estimation equations (17) has been conducted. Modeling came true on time interval $t \in [0;1000]\ c$ with pitch Δt=0,01sec. by method of Runge-Kutty of 4- th order. The object motion was set on loxodromic curve with angle concerning the meridian $35°$ from the point with coordinates $\varepsilon_0 = |\ 2\ 254\ 963{,}52 \quad 4\ 509\ 927{,}05 \quad 3\ 905\ 711{,}39\ |^T$ m and the law of modification of object acceleration module: $\dot{V} = 3{,}5\exp(-0.01t) + 0.03\cos(0.03t)$ m/s2. As the model of noises the additive Gaussian vector-noise with zero mathematical expectation and LMS has been used for: accelerometers - 10^{-6} m/s^2, ASS - 10^{-8} 1/sec., DSS - 0.1 m/sec., code measuring - 15 m, the Doppler measuring - 0.5 m/sec. In Application 4 the most typical diagrams of errors of integrated navigational vector Y_H components estimation - angle α (Figure. P4.1), the projections of object velocity vector V_ξ (Figure. P4.2) and object coordinates in GCS are provided (fig. P4.3) gained as a result of numerical experiment. Character of modification and level of estimation errors of remaining navigational parameters is similar to the given ones (angular parameters - angle α, projections of linear speed of object - projection V_ξ) and it also reflects a stability and good convergence of estimation process.

As a whole, upon termination of time interval of modeling the peak error of navigational variables estimations have not exceeded: on orientation angles - 1 ang. Minute, on projections of linear speed - 0.8 m/sec., on object coordinates - 2, 4 m that is comparable to precision *of steady* conditions of differential measurements and testifies to possibility of effective practical use of the offered algorithm.

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Application-1

$$F(Y,t)=\left|\begin{array}{c} \Phi(\beta,\gamma)Z_d \\ \hline \left|\begin{matrix} 0 & (\cos\varphi)^{-1} \\ -1 & 0 \end{matrix}\right| \left|\begin{matrix} -V_Y \\ V_X \end{matrix}\right| (r+h)^{-1} \\ \hline C^T(\alpha,\beta,\gamma,\lambda,\varphi)Z_a + \left(2\left|\begin{matrix} 0 \\ \Omega\cos\varphi \\ \Omega\sin\varphi \end{matrix}\right| + (r+h)^{-1}\left|\begin{matrix} -V_Y \\ V_X \\ V_X tg\varphi \end{matrix}\right| \right) \times \left|\begin{matrix} V_X \\ V_Y \\ V_Z \end{matrix}\right| - \\ - \left|\begin{matrix} 0 \\ -\Omega^2(r+h)\cos\varphi\sin\varphi \\ \Omega^2(r+h)\cos^2\varphi + g \end{matrix}\right| \\ \hline V_Z \end{array}\right|$$

$$F_1(Y,t)=\left|\begin{array}{c|c} -\Phi(\beta,\gamma) & 0 \\ \hline 0 & 0 \\ \hline 0 & -C^T(\alpha,\beta,\gamma,\lambda,\varphi) \\ \hline 0 & 0 \end{array}\right|$$

Application 2

$$\Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)=\left\{\xi_1(\eta_2\zeta_3-\eta_3\zeta_2)-\eta_1(\xi_2\zeta_3-\xi_3\zeta_2)+\zeta_1(\xi_2\eta_3-\xi_3\eta_2)\right\}^{-1}\times$$
$$\times\left|\begin{matrix} \eta_2\zeta_3-\eta_3\zeta_2 & -\eta_1\zeta_3+\eta_3\zeta_1 & \eta_1\zeta_2-\eta_2\zeta_1 \\ \xi_3\zeta_2-\xi_2\zeta_3 & -\xi_3\zeta_1+\xi_1\zeta_3 & \xi_2\zeta_1-\xi_1\zeta_2 \\ \xi_2\eta_3-\xi_3\eta_2 & -\xi_1\eta_3+\xi_3\eta_1 & \xi_1\eta_2-\xi_2\eta_1 \end{matrix}\right|$$

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Application-3
Procedure of velocity vector equations differentiation:

$$\dot{V} = \left[\frac{\partial \Phi_{\varepsilon}(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)}{\partial \begin{vmatrix}\varepsilon_{c_1}\\ \varepsilon_{c_2}\\ \varepsilon_{c_3}\\ \varepsilon\end{vmatrix}} \hat{\otimes} \begin{vmatrix}\dot{\varepsilon}_{c_1}\\ \dot{\varepsilon}_{c_2}\\ \dot{\varepsilon}_{c_3}\\ \dot{\varepsilon}\end{vmatrix}\right]\times$$

$$\times\{\Xi(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},\varepsilon) - \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, Z_{V0} + \rho\,(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\; W_{Z_{V0}}\} +$$

$$+ \quad \Phi_{\varepsilon}(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\; \{\frac{\partial \Xi(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},\varepsilon)}{\partial \begin{vmatrix}\varepsilon_{c_1}\\ \varepsilon_{c_2}\\ \varepsilon_{c_3}\\ \varepsilon\\ V_{c_1}\\ V_{c_2}\\ V_{c_3}\end{vmatrix}} \hat{\otimes} \begin{vmatrix}\dot{\varepsilon}_{c_1}\\ \dot{\varepsilon}_{c_2}\\ \dot{\varepsilon}_{c_3}\\ \dot{\varepsilon}\\ \dot{V}_{c_1}\\ \dot{V}_{c_2}\\ \dot{V}_{c_3}\end{vmatrix} -$$

$$-\left[\frac{\partial\,\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)}{\partial \begin{vmatrix}\varepsilon_{c_1}\\ \varepsilon_{c_2}\\ \varepsilon_{c_3}\\ \varepsilon\end{vmatrix}} \hat{\otimes} \begin{vmatrix}\dot{\varepsilon}_{c_1}\\ \dot{\varepsilon}_{c_2}\\ \dot{\varepsilon}_{c_3}\\ \dot{\varepsilon}\end{vmatrix}\right] Z_{V0} - \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, \dot{Z}_{V0} +$$

$$+\left[\frac{\partial\,\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)}{\partial \begin{vmatrix}\varepsilon_{c_1}\\ \varepsilon_{c_2}\\ \varepsilon_{c_3}\\ \varepsilon\end{vmatrix}} \hat{\otimes} \begin{vmatrix}\dot{\varepsilon}_{c_1}\\ \dot{\varepsilon}_{c_2}\\ \dot{\varepsilon}_{c_3}\\ \dot{\varepsilon}\end{vmatrix}\right] W_{Z_{V0}} + \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, \dot{W}_{Z_{V0}}\} =$$

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$$= [\frac{\partial \Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)}{\partial \begin{vmatrix}\varepsilon_{c_1}\\ \varepsilon_{c_2}\\ \varepsilon_{c_3}\\ \varepsilon\end{vmatrix}} \hat{\otimes} \begin{vmatrix}V_{c_1}\\ V_{c_2}\\ V_{c_3}\\ V\end{vmatrix}]\times$$

$$\times\{\Xi(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},\varepsilon) - \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, Z_{V0} + \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, W_{Z_{V0}}\} +$$

$$+\Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\{\frac{\partial \Xi(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},\varepsilon)}{\partial \begin{vmatrix}\varepsilon_{c_1}\\ \varepsilon_{c_2}\\ \varepsilon_{c_3}\\ \varepsilon\\ V_{c_1}\\ V_{c_2}\\ V_{c_3}\end{vmatrix}} \hat{\otimes} \begin{vmatrix}V_{c_1}\\ V_{c_2}\\ V_{c_3}\\ V\\ \dot{V}_{c_1}\\ \dot{V}_{c_2}\\ \dot{V}_{c_3}\end{vmatrix} -$$

$$-[\frac{\partial \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)}{\partial \begin{vmatrix}\varepsilon_{c_1}\\ \varepsilon_{c_2}\\ \varepsilon_{c_3}\\ \varepsilon\end{vmatrix}} \hat{\otimes} \begin{vmatrix}V_{c_1}\\ V_{c_2}\\ V_{c_3}\\ V\end{vmatrix}] Z_{V0} - \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\, \dot{Z}_{V0} +$$

$$+[\frac{\partial \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)}{\partial \begin{vmatrix}\varepsilon_{c_1}\\ \varepsilon_{c_2}\\ \varepsilon_{c_3}\\ \varepsilon\end{vmatrix}} \hat{\otimes} \begin{vmatrix}V_{c_1}\\ V_{c_2}\\ V_{c_3}\\ V\end{vmatrix}] W_{Z_{V0}} + \rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\,(\phi(W_{Z_{V0}},t)+\varsigma)\} =$$

$$=\theta(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},V_{c_1},V_{c_2},V_{c_3},\dot{V}_{c_1},\dot{V}_{c_2},\dot{V}_{c_3},W_{Z_{V0}},Z_{V0},\dot{Z}_{V0},\varepsilon,V) + \Phi_\varepsilon(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\rho(\varepsilon_{c_1},\varepsilon_{c_2},\varepsilon_{c_3},\varepsilon)\,\varsigma,$$

where $\hat{\otimes}$ - a character of unitized work of array onto vector.

Here the array derivative $\Lambda = \begin{vmatrix}\Lambda_1 & \Lambda_2 & \dots & \Lambda_m\end{vmatrix}$, where Λ_i $-$ i - th column of array, per vector $\chi = \begin{vmatrix}\chi_1\\ \chi_2\\ \vdots\\ \chi_n\end{vmatrix}$ where χ_i - i - th element of the vector, has the following unitized structure: $\frac{\partial \Lambda}{\partial \chi} = \begin{vmatrix}\frac{\partial \Lambda_1}{\partial \chi} & \frac{\partial \Lambda_2}{\partial \chi} & \dots & \frac{\partial \Lambda_m}{\partial \chi}\end{vmatrix}$,

and unitized work of array onto vector, respectively:

$$\frac{\partial \Lambda}{\partial \chi} \hat{\otimes} \dot{\chi} = \begin{vmatrix}\frac{\partial \Lambda_1}{\partial \chi}\dot{\chi} & \frac{\partial \Lambda_2}{\partial \chi}\dot{\chi} & \dots & \frac{\partial \Lambda_m}{\partial \chi}\dot{\chi}\end{vmatrix}.$$