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CHOOSING THE RIGHT THRESHOLD FOR CROSS-ENTROPY-BASED STOPPING CRITERIA

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ABSTRACT
Cross-entropy (CE)-based stopping criteria for turbo iterative decoding are known to outperform fixed-iteration stopping criteria at high signal-to-noise ratios (SNRs). While CE-based stopping criteria have a range of thresholds, a high-value threshold for small frame sizes, and vice versa, should be used. It is difficult to advocate the value that can be categorized as either a small or large frame size. Moreover, thresholds may be specific for different SNRs. Hence, this paper provides a systematic analysis of threshold selection for the respective frame sizes of well-known CE-based stopping criteria. In this work, a range of thresholds was simulated for their error performance and required average number of iterations. To reduce complexity in the average iteration number, these results are thoroughly analysed and a suitable threshold for each CE-based stopping criterion in the specific SNR region is proposed.

Keywords: cross-entropy, stopping criteria, turbo codes, iterative decoding.

INTRODUCTION
A cross-entropy (CE) stopping criterion was introduced by the authors in [1] to stop turbo decoder iteration by calculating the improvement of mutual information. The CE algorithm operates by finding the closeness between two distributions of the estimated decoder output. The amount of mutual information improvement is used as a threshold for stopping criteria, and the iterations stop if the mutual information is less than the specific threshold. The complexity of mutual information calculation in [1] led to the enhancement of the CE stopping criterion using the assumption in [2]. This assumption stated that the CE algorithm can be simplified by measuring the sign changes between two distributions of a posteriori log-likelihood ratio (Lr) or extrinsic value (Le) outputs. This criterion is known as the sign change ratio (SCR). A simplification in the SCR criterion to sign difference ratio (SDR) was presented in [3] by converting the sign difference ratio between Lr or Le.

The authors in [2] suggested guidelines for choosing a threshold for the SCR. These include choosing a low threshold to maintain error performance, but with small savings in the number of iterations. The authors also suggested a different threshold value for the respective high and low SNR regions. In their simulation, a high threshold value was chosen for smaller frame sizes, and vice versa. In [3], the authors concluded that the lower threshold selection was the larger average number of required iterations. Despite this larger number of iterations, the authors suggested a low threshold for higher SNR to maintain its error performance. However, the effect of frame size on threshold determination was not considered.

CE-based stopping criteria mostly require a subjective threshold for decision making to stop the iteration process [4]. It is imperative for one to accurately determine the thresholds subject to the SNR. Moreover, the threshold determination can also change to the frame size and stopping criteria. Recent research regarding CE-based stopping criteria used a threshold based on the assumption discussed earlier without considering a suitable threshold related to the specific frame sizes [4]-[9]. To the best of the authors' knowledge, there are currently no studies on choosing the right threshold for respective frame size in varying SNR environments.

To properly choose the right threshold for the CE-based stopping criteria, this paper provides a systematic analysis of threshold selection for the respective frame sizes. We first discussed the CE-based stopping criteria for iterative turbo decoding. Then the performance analysis of CE-based stopping criteria, are presented, which include the CE, SCR, and SDR methods. These results will assist in choosing the right threshold for the iterative decoding process. Finally, we tabulate the suggested threshold for the respective frame sizes and SNR regions for CE-based stopping criteria and draw conclusions.

CE-BASED STOPPING CRITERIA
Figure-1 shows the structure of a turbo decoder rate 1/n, which consists of two soft output component decoders. Suppose the information bits, $u = (u_1, u_2, \ldots u_N)$ of length N.

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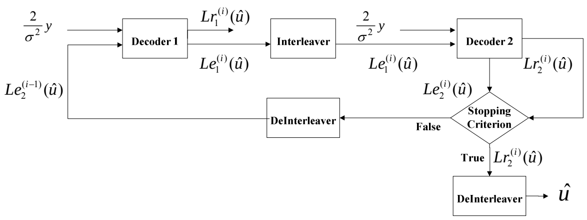

Figure-1. Turbo iterative decoder with a stopping criterion.

Let $\mathbf{y} = (y_1, y_2,\ldots, y_N)$ be the received sequence after experiencing additive white Gaussian noise (AWGN), i.e. $\mathcal{N}(0, \sigma^2)$. Then, $\mathbf{y} = u + \mathbf{v}$, where $\mathbf{v}$ is the AWGN. Then, let the output of $y_t = (y_{t,1}, y_{t,2}, \ldots, y_{t,n})$ and $\hat{u}_t$ be the estimate of the information bit, u_t, at time t. At the i-th iteration of two decoders, the relation between Lr and Le values at each decoder is defined as follows:

$$Lr_1^{(i)}(\hat{u}_t) = Le_2^{(i-1)}(\hat{u}_t) + Le_1^{(i)}(\hat{u}_t) + \frac{2}{\sigma^2} y_t \quad (1)$$

and

$$Lr_2^{(i)}(\hat{u}_t) = Le_1^{(i)}(\hat{u}_t) + Le_2^{(i)}(\hat{u}_t) + \frac{2}{\sigma^2} y_t \quad (2)$$

Then, $\hat{u}_t$ is estimated as follows:

$$\hat{u}_t = \begin{cases} 0 & if \quad Lr_2^{(i)}(\hat{u}_t) < 0 \\ 1 & if \quad Lr_2^{(i)}(\hat{u}_t) \geq 0 \end{cases} \quad (3)$$

In a CE stopping criterion, improvement of the mutual information between two distributions of estimated decoders is computed as follows:

$$T(i) \approx \sum_t \frac{\left|\Delta Le_2^{(i)}(\hat{u}_t)\right|^2}{\exp(Lr_1^{(i)}(\hat{u}_t))} \quad (4)$$

where

$$\Delta Le_2^{(i)}(\hat{u}_t) = Le_2^{(i)}(\hat{u}_t) - Le_2^{(i-1)}(\hat{u}_t) \quad (5)$$

At each iteration, this technique computes the approximate CE between Le of the component decoders, as in (4). As the iteration process continues, the decoding outputs between the two consecutive iterations tend to converge. The iteration can be stopped if

$$T(i) < \left(10^{-2} \sim 10^{-4}\right) T(1) \quad (6)$$

The SCR approach based on the CE concept was presented in [2] in an attempt to reduce the computational complexity and memory space requirement. Based on the assumption that the differences between the magnitude of $Le_2^{(i)}(\hat{u}_t)$ and $Le_2^{(i-1)}(\hat{u}_t)$ are very small and negligible, i.e. less than 1.0, the CE algorithm is simplified by measuring the sign changes, C(i), of $Le_2(\hat{u}_t)$ from iteration (i-1) to i. Then,

$$T(i) \approx \sum_{t \in \Lambda/\Lambda_s} \frac{\left|\Delta Le_2^{(i)}(\hat{u}_t)\right|^2}{\exp(Lr_1^{(i)}(\hat{u}_t))} \approx \delta_i C(i) \quad (7)$$

where δ_i is defined as the average value, that is

$$\delta_i = \left|\Delta Le_2^{(i)}(\hat{u}_t)\right|^2 \Big/ \exp(\left|Lr_1^{(i)}(\hat{u}_t)\right|) \quad (8)$$

C(i) is computed at each iteration and stops when $C(i) \leq (0.005{\sim}0.03)N$. However, the SCR method requires storage to keep the values from the previous iteration. Thus, an improved SCR approach (called SDR) was proposed in [3] with attempts to alleviate the storage requirements along with minimal computation. As the iteration converged, the authors observed that the sign values of $Le_2^{(i)}(\hat{u}_t)$ and $Le_1^{(i)}(\hat{u}_t)$ tended to be correlated with each other. Let D(i) be the number of sign differences in a frame, such as in equation (9). The SDR criterion terminates the iteration when $D(i) \leq (0.001{\sim}0.01)N$.

$$D(i) = \begin{cases} \sum_{t=1}^{N} a_t,\ a_t = 1, \text{if } \operatorname{sign}\left(Le_2^{(i)}(\hat{u}_t)\right) \neq \operatorname{sign}\left(Le_1^{(i)}(\hat{u}_t)\right) \\ \sum_{t=1}^{N} a_t,\ a_t = 0, \text{if } \operatorname{sign}\left(Le_2^{(i)}(\hat{u}_t)\right) = \operatorname{sign}\left(Le_1^{(i)}(\hat{u}_t)\right) \end{cases} \quad (9)$$

PERFORMANCE OF CE-BASED STOPPING CRITERIA

The simulation was done for generator polynomial, g = (7, 5), the turbo decoder rate R = 1/2 with an interleave size of N. The maximum iteration was set to 7, i.e. $i_{max} = 7$. The turbo encoder was modulated to binary phase shift keying (BPSK). The BPSK demodulate signal was passed to the turbo decoder to adopt the log-maximum a posteriori probability (log-MAP). We compared the simulation of CE-based stopping criteria for turbo decoding (CE, SCR and SDR) discussed in the previous section and show the bit error rate (BER) and the average iteration number of each CE-based stopping criterion in Figures 2 to 7 for a frame size of 50 and 2500. Note that this analysis was done based on the SNR range suggested in [10]. That is, the SNR range was divided into three regions, as follows:

Low SNR region, $SNR < 0.8dB$
Turning point (TP) SNR region, $0.8dB \leq SNR < 1.2dB$
High SNR region, $SNR \geq 1.2dB$

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Figure-2 depicts the average iteration number required for the CE simulated with a predefined threshold (Th) values of 0.01, 0.001, and 0.0001, which represent high, medium, and low threshold values, respectively. The average numbers of iterations at 1.5 dB were in the range of 4.0 to 4.7 and 5.3 to 5.9 for the frame sizes of 50 and 2500, respectively, and the high threshold values of each range required fewer iterations. In comparing the BER for all predefined Th exhibits, both frame sizes had the same curves, as shown in Figure-3. Thus, it is best to choose the Th value of 0.01 in all SNR regions for both frame sizes in the CE stopping criteria. The Th value depends on the frame size of the larger value, which will be discussed in the next section.

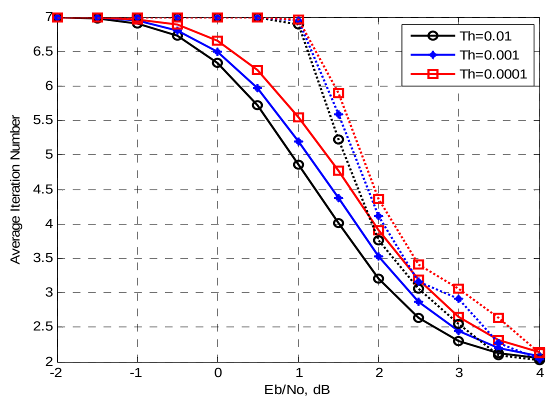

Figure-2. The average iteration number of the (7,5) code with the CE stopping criterion (solid line, frame 50; dotted line, frame 2500).

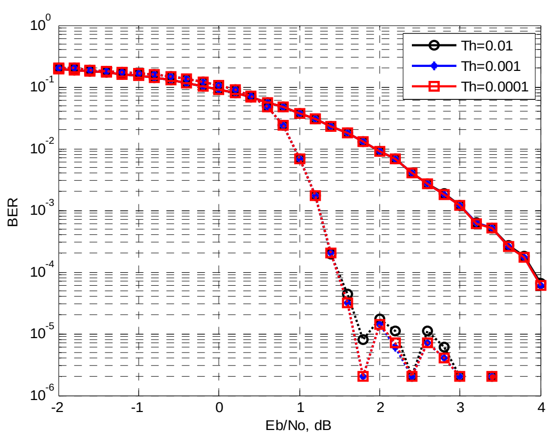

Figure-3. BER of the (7,5) code with the CE stopping criterion (solid line, frame 50; dotted line, frame 2500).

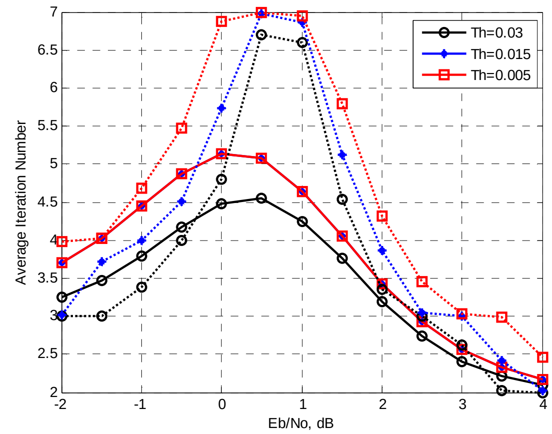

Figure-4. Average iteration number of the (7,5) code with the SCR stopping criterion (solid line, frame 50; dotted line, frame 2500).

In the simulation of the SCR, the predefined Th was set to 0.03, 0.015, and 0.005, as suggested in [2]. For a frame size of 50, the medium and low Th values had the same average number of iterations, as depicted in Figure-4, while the high Th values gave the least average number of iterations. Again, the BER of SCR exhibited the same curves as those shown in Figure-5.

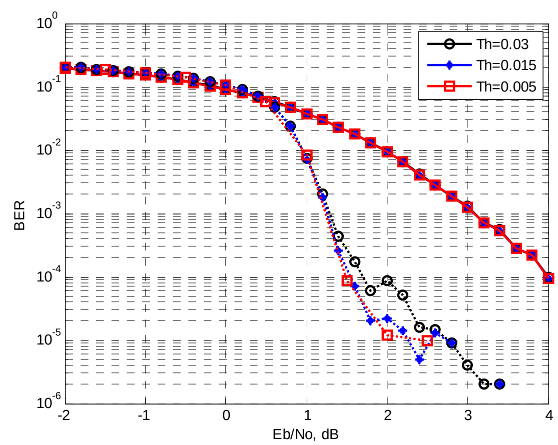

Figure-5. BER of the (7,5) code with the SCR stopping criterion (solid line, frame 50; dotted line, frame 2500).

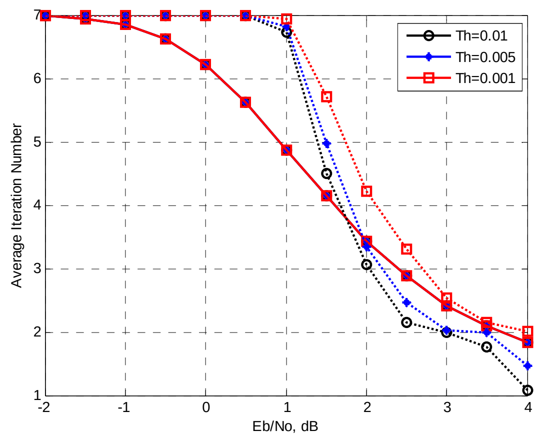

Figure-6. Average iteration number of the (7,5) code with the SDR stopping criterion (solid line, frame 50; dotted line, frame 2500).

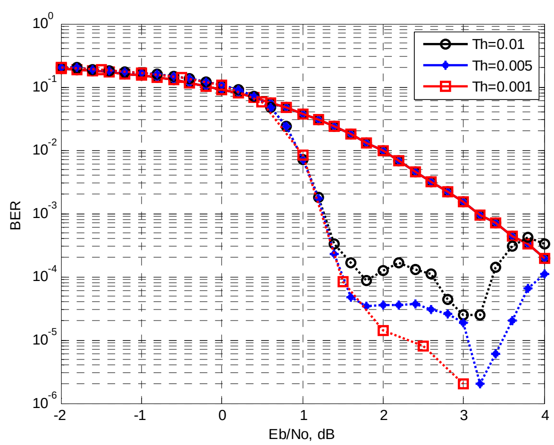

Figure-7. BER of the (7, 5) code with the SDR stopping criterion (solid line, frame 50; dotted line, frame 2500).

Therefore, the high Th value of 0.03 is the most suitable choice for small frame sizes. However, for frame sizes of 2500, a Th value of 0.015 is the best choice for high SNR regions as it shows better error performance with a moderate average number of iterations. The same trend can also be observed for SDR, in which a Th value of 0.01 is chosen for frame sizes of 50 in all SNR regions. For frame sizes of 2500, a low Th value of 0.001 gives the best BER at a high SNR region with a reasonable average number of iterations, as shown in Figure 6 and 7.

CHOOSING THE RIGHT THRESHOLD FOR CE-BASED STOPPING CRITERIA

The results obtained and discussed earlier will help in the proper threshold determination for respective SNR regions and frame sizes. Note that the range of predefined Th used in this simulation is as suggested by the respective authors of CE, SCR, and SDR. We further simulated the average number of iterations for each stopping criterion and suggested Th values for frame sizes ranging between 50 and 10, 000. Then, we computed the difference of BER between each Th value at each SNR, as follows:

$$\Delta BER_{1,SNR} = \frac{BER_{H,SNR} - BER_{M,SNR}}{BER_{H,SNR}} \times 100\% \tag{10}$$

$$\Delta BER_{2,SNR} = \frac{BER_{M,SNR} - BER_{L,SNR}}{BER_{M,SNR}} \times 100\% \tag{11}$$

where $BER_{H,SNR}$, $BER_{M,SNR}$ and $BER_{L,SNR}$ are the BER of high, medium and low Th values, respectively, at SNR dB. $\Delta BER_{1,\ SNR}$ is the percentage difference between high and medium Th values, while $\Delta BER_{2,SNR}$ is the percentage difference between medium and low Th values. Therefore, choosing the right threshold based on below assumptions

$$\begin{aligned} &\text{If } \Delta BER_{1,SNR} < 25\% \text{ and } \Delta BER_{2,SNR} < 25\% \\ &\therefore \Delta BER = \Delta BER_{1,SNR} \Rightarrow \text{Choose high Th} \\ &\text{If } \Delta BER_{1,SNR} > 25\% \text{ and } \Delta BER_{2,SNR} < 25\% \\ &\therefore \Delta BER = \Delta BER_{1,SNR} \Rightarrow \text{Choose medium Th} \\ &\text{If } \Delta BER_{2,SNR} > 25\% \\ &\therefore \Delta BER = \Delta BER_{2,SNR} \Rightarrow \text{Choose low pH} \end{aligned} \tag{12}$$

Table-1 suggests Th values for CE, and there is no difference in error performance between the Th values of 0.01 and 0.001 for all frame sizes at low SNR. Hence, a high Th value is suggested for small to large frame sizes. The highest Th value is suitable for both SCR and SDR at low SNR regions, as suggested in Tables 2 and 3, respectively. However, a frame size of 5000 shows that ΔBER approaching 75% started occurring at 2.6 dB. Hence, a lower Th value of 0.001 was chosen for the CE stopping criteria at high SNR regions for frame sizes larger than 5,000. In the SCR stopping criteria, a high Th was chosen for smaller frame sizes (and vice versa) in the high SNR region. However, for SDR, high and medium Th selections occurred at small frame sizes of 50 and 100, respectively, while low Th values were suitable for frame sizes of 250 and larger.

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Table-1. Suggested Th for the CE stopping criterion.

Frame size	BER characteristic			Suggested Th	
	Low SNR	High SNR		Low SNR	High SNR
	ΔBER	ΔBER	Starting pPoint (dB)		
50	≈0%	≈0%	-	0.01	0.01
100	≈0%	≈0%	-	0.01	0.01
250	≈0%	<25%	-	0.01	0.01
500	≈0%	<25%	-	0.01	0.01
1000	≈0%	<25%	-	0.01	0.01
2500	≈0%	<25%	-	0.01	0.01
5000	≈0%	<75%	2.6	0.01	0.001
7500	≈0%	<75%	2	0.01	0.001
10000	≈0%	<75%	1.6	0.01	0.001

Table-2. Suggested Th for the SCR stopping criterion.

Frame size	BER characteristic			Suggested Th	
	Low SNR	High SNR		Low SNR	High SNR
	ΔBER	ΔBER	Starting Point (dB)		
50	≈0%	≈0%	-	0.03	0.03
100	≈0%	≈0%	-	0.03	0.03
250	≈0%	<50%	3.4	0.03	0.015
500	≈0%	<50%	3.2	0.03	0.015
1000	≈0%	<100%	3.0	0.03	0.015
2500	≈0%	<100%	1.6	0.03	0.015
5000	≈0%	<100%	1.6	0.03	0.005
7500	≈0%	<100%	1.4	0.03	0.005
10000	≈0%	<100%	1.2	0.03	0.005

Table-3. Suggested Th for the SDR stopping criterion.

Frame size	BER characteristic			Suggested Th	
	Low SNR	High SNR		Low SNR	High SNR
	ΔBER	ΔBER	Starting Point (dB)		
50	≈0%	≈0%	-	0.01	0.01
100	≈0%	<50%	2.5	0.01	0.005
250	≈0%	<75%	2.8	0.01	0.001
500	≈0%	<75%	2.6	0.01	0.001
1000	≈0%	<150%	2	0.01	0.001
2500	≈0%	<100%	1.8	0.01	0.001
5000	≈0%	<150%	1.4	0.01	0.001
7500	≈0%	<150%	1.4	0.01	0.001
10000	≈0%	<175%	1.4	0.01	0.001

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CONCLUSIONS

We investigated the CE-based stopping criteria for iterative turbo decoding to aid in choosing the right threshold for the respective frame size. Through systematic analysis, we suggested the proper threshold selections for each CE, SCR, and SDR stopping criteria. The selection of Th values was found to depend on the SNR regions and frame sizes. Given a certain frame size, a suitable selection of Th values for CE-based stopping criteria provide an optimal average number of iterations, thereby resulting in negligible performance degradation.

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