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SIMULTANEOUS COMPUTATION OF MODEL ORDER AND PARAMETER ESTIMATION OF A HEATING SYSTEM BASED ON GRAVITATIONAL SEARCH ALGORITHM FOR AUTOREGRESSIVE WITH EXOGENOUS INPUTS

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ABSTRACT

System identification is a class of control system engineering that determines physical functionality of a plant and represents them in the form of mathematical expression by utilizing real experimental data. It is a process of acquiring, formatting, processing, and identifying mathematical models by considering raw data from the real-world system. Once the mathematical model is chosen, it can be characterized in terms of suitable descriptions such as transfer function that can be used for controller design. Most essential stages of model identification process can be summarized into four main stages. The first stage is collecting experimental data. Then, the model order and structure are chosen. The next stage is to estimate the parameters of the model and finally, the mathematical model is validated. Model order selection and parameter estimation are two significant aspects of determining the mathematical model for system identification. In this paper, an approach termed as Simultaneous Model Order and Parameter Estimation (SMOPE), which is basically based on Gravitational Search Algorithm (GSA), is proposed to combine these two parts into a simultaneous solution. In this technique, both the model order and the parameters of the system are computed simultaneously to obtain the best mathematical model of a system. According to heating system case study, it is proven that the proposed method is outstanding in comparison with some other approaches in literature.

Keywords: gravitational search algorithm, system identification, model order selection, parameter estimation.

INTRODUCTION

Conventionally, least mean square (LMS) as well as other algorithms has been discovered for the identification of linear and static systems [1]. The goal of system identification is to adjust parameters of mathematical model in order to approximate actual parameters of an unidentified system from its inputs and outputs. This is implemented by varying the parameters of the developed model so that for a set of assigned inputs, its output match with actual system. In such cases, minimization of an objective function (generally the mean square error between actual output of unidentified system and predicted output) is usually followed by gradient based iterative search algorithms.

However, in cases where the error surface (objective function) is multimodal, gradient-based approaches typically unable to succeed in converging to the global minimum. Therefore conventional approaches of parameter approximation are unsuccessful since they get trapped into local minimum and consequently unable to reach the global minimum [2]. In this situation, heuristic optimization techniques that require no gradient which enables them to attain a global optimal solution deliver significant capabilities in dealing with these particular challenging system identification problems. For that reason the problem of system identification can be considered as optimization problem. System identification implementing these heuristic algorithms is mentioned in some researches [3-6]. In spite of this, identification of systems without prior structural information is still challenging, hence new and innovative algorithms are increasingly being researched.

A schematic of system identification problem utilizing the heuristic search algorithms is shown in Figure-1. The difference of the output from the actual system with the modeled system gives the error $e(k)$. This error is used by the heuristic algorithm to adjust and tunes the parameters of the ARX model, which are the pole-zero coefficients and thus minimize the error in a number of iterations to effectively identify the actual system.

Five years ago, a heuristic search algorithm, known as Gravitational Search Algorithm (GSA), has been introduced inspired by the gravitational law and laws of motion [7]. It is indicated as a simple idea that is both convenient to execute and computationally effective. GSA has a functional and well-balanced mechanism to improve exploration and exploitation capabilities.

System identification can be categorized based on their technique such as parametric model identification and non-parametric model identification. Non parametric identification technique in comparison with parametric identification technique is relatively simple but less accurate. However, parametric model identification provides complete model description that truly describes the physical dynamics of a system. Examples of model structures for parametric identification are Auto-

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Regressive Model with Exogenous Inputs (ARX) [8], Auto-Regressive Moving Average with Exogenous Inputs (ARMAX) [9], and Box-Jenkins (BJ) model [10].

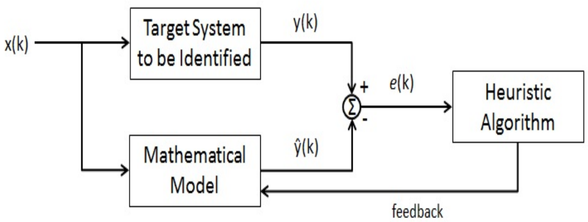

Figure-1. A schematic of system identification problem utilizing the heuristic search algorithms.

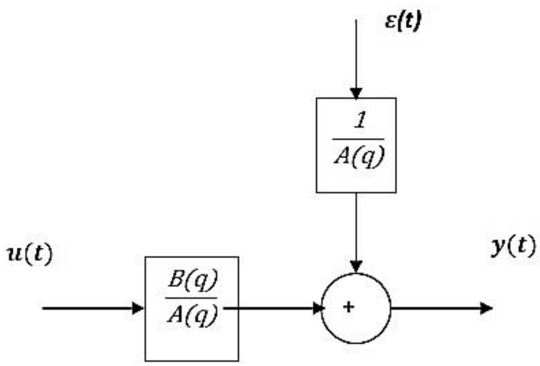

Figure-2. ARX model structure

ARX is the most basic model in linear black box identification. In dealing with the linear black box identification problem of the ARX model, the model order selection and parameter estimation part is solved with separated process. This separation may result the identified mathematical model not offer the best performance. The process needs to be repeated with the nearest model order to ensure that the best performance improvements are attained. Due to this fact, both of these parts should be merged into individual stage to obtain the ideal performance of the model.

In this research project, we explore the possibility to perform model order selection and parameter estimation simultaneously. We found that the combination is possible considering the fact that the process of system identification can also be recognized as optimization problem. In mathematics and computer science, optimization is the problem of acquiring the best solution from all feasible solutions.

In this paper, GSA is proposed to simultaneously identify model order and pole zero parameters of heating system without prior structural information. To validate the effectiveness of the proposed GSA, the performance of GSA is also compared with other existing techniques in literature.

The rest of the paper is organized as follows. The proposed SMOPE-GSA is presented in the next section to give a proper background. This section is followed by experimental results and comparison with other methods in Section 3. Lastly, the paper is concluded in Section 4.

SIMULTANEOUS COMPUTATION OF MODEL ORDER AND PARAMETER ESTIMATION

In this part, an alternative approach for solving an ARX model will be highlighted. The technique is called the simultaneous model order and parameter estimation utilizing GSA as an optimizer (SMOPE-GSA). The strategy in SMOPE incorporates both solution of model order selection and parameter estimation in identification problem simultaneously by using GSA.

The simplest form of time-domain system identification is ARX model, which has the following structure as shown in Figure-2. The figure shows input and output variables, $u(t)$ and $y(t)$ respectively. $\varepsilon(t)$ is a Gaussian white noise process. $A(q)$ and $B(q)$ are polynomials in the backward shift operator, q^{-1}. Based on the ARX model structure, the single-input single-output (SISO) ARX mathematical model can be defined as:

$$y(t) = \frac{B(q)}{A(q)}u(t) + \frac{1}{A}\varepsilon(t) \quad (1)$$

A linear differential equation and transfer function can be derived from that model as follows:

$$y(t) + a_1 y(t-1) + a_2 y(t-2) + \dots + a_n y(t-n) = b_1 u(t-1) + b_2 u(t-2) + \dots + b_m u(t-m) + \varepsilon(t) \quad (2)$$

$$G(z) = \frac{Y(z)}{U(z)} = \frac{b_1 z^{-1} + b_2 z^{-2} + \dots + b_m z^{-m}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}} \quad (3)$$

where m and n are the number of numerator and denominator orders of the transfer function respectively and a_n and b_m are the pole and zero parameters that will be tuned by GSA as well as model order.

In SMOPE-GSA, maximum order of 9th is considered. To determine the parameter 'a' and 'b', the constraint $n \geq m$ is taken into account. This is based on the transfer function form which the order value of poles (n value) must be the same or more than the order of zeroes (m value).

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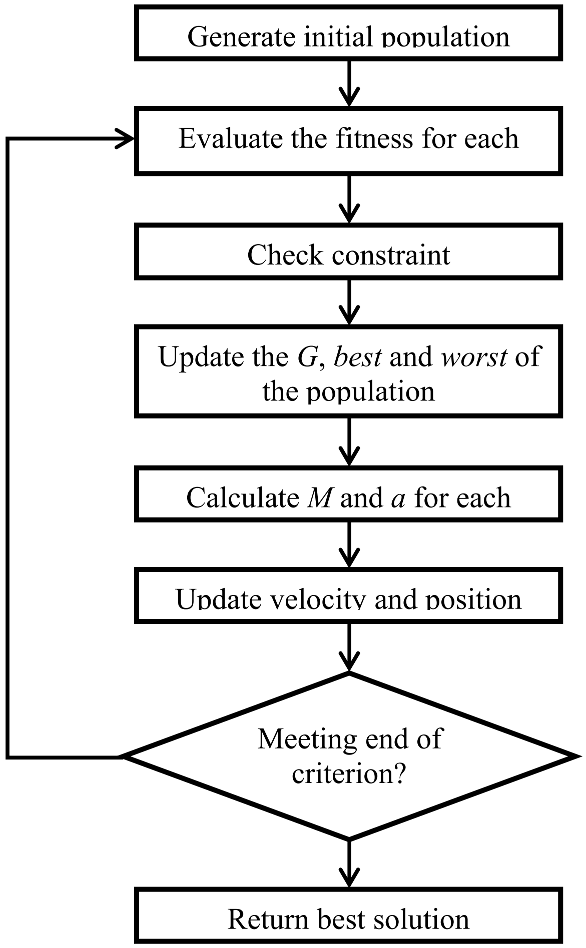

Figure-3. Flowchart of GSA for SMOPE.

Table-1. Particle representation.

Dimension	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
Variable in ARX	Order, *n*	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9

The function of GSA, which is presented in Figure-3, is to identify the best mathematical model. The combination of model order and parameter for ARX equation are viewed in particle representation, which is shown in Table-1 while Table-2 and Table-3 indicate which ARX parameters should be decided for any assigned number of order, *n*. Hence, a set of 45 mathematical models are tested according to *n* value.

As an example, if the model order is 2, therefore '*n*' value is 2 and all possible mathematical models are subjected to fitness calculation. In that case, the computations focus on two mathematical models, which are $\frac{b_1z^{-1}}{1+a_1z^{-1}+a_2z^{-2}}$ and $\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}$.

Another example, if the model order is 3, then the computations involve three mathematical models, which are $\frac{b_1z^{-1}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$, $\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$ and $\frac{b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}{1+a_1z^{-1}+a_2z^{-2}+a_3z^{-3}}$. Note that up to ninth order mathematical model of ARX is taken into account for the purpose of this research.

In the beginning phase of GSA, certain parameters are initialized. The GSA parameter values used in this research is shown in Table-4. The GSA parameter includes the number of agents, initial value, G_0, α, and the maximum number of iterations, *k*. The initial placement of agents is randomly positioned in a search space. After the initialization stage is complete, the fitness function is computed as follows:

$$best\ fit = 100\left[1 - \frac{norm(y_{actual} - \hat{y}_{estimated})}{norm(y_{actual} - y_{mean})}\right]\% \quad (4)$$

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Table-2. ARX parameters selected for the calculation of best fit (*n*=1, 2, 3, 4, 5, 6).

Order, *n*	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
1	X									X								
2	X	X								X								
2	X	X								X	X							
3	X	X	X							X								
3	X	X	X							X	X							
3	X	X	X							X	X	X						
4	X	X	X	X						X								
4	X	X	X	X						X	X							
4	X	X	X	X						X	X	X						
4	X	X	X	X						X	X	X	X					
5	X	X	X	X	X					X								
5	X	X	X	X	X					X	X							
5	X	X	X	X	X					X	X	X						
5	X	X	X	X	X					X	X	X	X					
5	X	X	X	X	X					X	X	X	X	X				
6	X	X	X	X	X	X				X								
6	X	X	X	X	X	X				X	X							
6	X	X	X	X	X	X				X	X	X						
6	X	X	X	X	X	X				X	X	X	X					
6	X	X	X	X	X	X				X	X	X	X	X				
6	X	X	X	X	X	X				X	X	X	X	X	X			

Note that the GSA assigns floating values to every dimension of an agent although the value of model order is discrete. Therefore, for the first dimension, the floating value is converted to discrete value by rounding its value. The search space at first dimension is limited between 0.5 and 9.4. We put this constrain for maintaining stability.

In order to maximize the best fit, the fitness evolution is performed by evaluating the best and worst fitness for all agents at each of iteration.

$$best(t) = \max_{j\in\{1,\ldots,N\}} fit_j(t) \quad (5)$$

$$worst(t) = \min_{j\in\{1,\ldots,N\}} fit_j(t) \quad (6)$$

where $fit_j(t)$ represents the fitness value of the j^{th} agent at iteration *t*, *best*(*t*) and *worst*(*t*) represents the best and worst fitness at iteration *t*. Then, gravitational constant *G* is computed at iteration *t*.

$$G(t) = G_0 e^{\left(-\frac{\alpha t}{T}\right)} \quad (7)$$

where *G0* and *α* are initialized at the beginning. *T* is the total number of iterations.

After that, mass of each agent is calculated as follows:

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \quad (8)$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \quad (9)$$

To calculate acceleration of an agent, total forces from a set of heavier masses that apply on it should be considered based on law of gravity (Equation 10).

$$F_i^d(t) = \sum_{j\in Kbest, j\neq i} rand_j G(t) \frac{M_i(t) \times M_j(t)}{R_{ij}(t) + \varepsilon} \left(x_j^d(t) - x_i^d(t)\right) \quad (10)$$

where $rand_j$ is a uniform random variable in the interval [0,1], ε is a small constant, $R_{ij}(t)$ is the Euclidian distance between two agents *i* and *j* (Equation 11), and *Kbest* is the set of first *K* agents with the best fitness value and biggest mass. *Kbest* will decrease linearly with time and at the end there will be only one agent applying force to the others.

$$R_{ij}(t) = \left\| X_i(t), X_j(t) \right\|_2 \quad (11)$$

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Based on the law of motion, the acceleration of the ith agent is computed by:

$$a_i^d(t) = \frac{F_i^d(t)}{M_i(t)} \tag{12}$$

Finally, velocity and the position of the agents at next iteration (t+1) are computed based on the following equations:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{13}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{14}$$

Table-3. ARX parameters selected for the calculation of best fit (n=7, 8, 9).

Order, *n*	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9
7	X	X	X	X	X	X	X			X								
7	X	X	X	X	X	X	X			X	X							
7	X	X	X	X	X	X	X			X	X	X						
7	X	X	X	X	X	X	X			X	X	X	X					
7	X	X	X	X	X	X	X			X	X	X	X	X				
7	X	X	X	X	X	X	X			X	X	X	X	X	X			
7	X	X	X	X	X	X	X			X	X	X	X	X	X	X		
8	X	X	X	X	X	X	X	X		X								
8	X	X	X	X	X	X	X	X		X	X							
8	X	X	X	X	X	X	X	X		X	X	X						
8	X	X	X	X	X	X	X	X		X	X	X	X					
8	X	X	X	X	X	X	X	X		X	X	X	X	X				
8	X	X	X	X	X	X	X	X		X	X	X	X	X	X			
8	X	X	X	X	X	X	X	X		X	X	X	X	X	X	X		
8	X	X	X	X	X	X	X	X		X	X	X	X	X	X	X	X	
9	X	X	X	X	X	X	X	X	X	X								
9	X	X	X	X	X	X	X	X	X	X	X							
9	X	X	X	X	X	X	X	X	X	X	X	X						
9	X	X	X	X	X	X	X	X	X	X	X	X	X					
9	X	X	X	X	X	X	X	X	X	X	X	X	X	X				
9	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X			
9	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		
9	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	
9	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X

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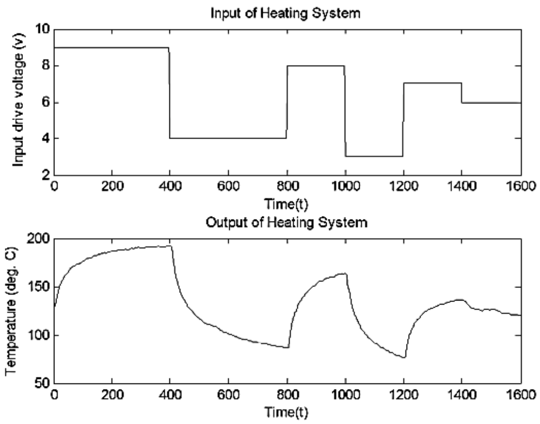

Figure-4. Input and output behavior of heating system.

where $rand_i$ is a uniform random variable in the interval [0,1]. The purpose of this random number is to give stochastic characteristic to the search strategy.

EXPERIMENT

Heating system

In heating system, the input drives a 300 Watt Halogen lamp, suspended several inches above a thin steel plate. The output is a thermocouple measurement taken from the back of the plate. The input and output of the experiment is shown in Figure-4, which is taken from http://www.esat.kuleuven.ac.be/sista/daisy. The data collect from the system is separated for training and testing. This training data is used to calculate the model order and parameter of the ARX while the testing data is used to validate the model.

Table-4. GSA parameter values.

Parameters	Value
Population size	100
Dimension	19
Initial value, *G0*	100
α	20
Maximum iterations	2000
Number of run	50

Experimental setup for SMOPE-GSA

In the SMOPE-GSA technique, each particle chooses a suitable model order and parameters of the ARX model from 1st order up to 9th order. The GSA parameter values used in this experiment is shown in Table-4. In each execution, the numbers of data points are divided equally into the proportions of 50% for training samples and 50% for testing samples from the entire dataset. The portion is described by Ljung (1999) in solving conventional ARX [8].

To validate this SMOPE-GSA technique, Equation 4 is taken into consideration. The output from identified mathematical model will be compared with actual output based on testing samples. The best fit for validation must be in good range to make sure other response from the system will give similar results to the model built. High best fit in the range of 80-100% is considered a good model.

Table-5. Result of Best fit using SMOPE-GSA technique in heating system identification.

Data set	**Best fit (Training)** (%)		**Average best fit (Training)** (%)	**STDEV** (Training)	**Best fit (Testing)** (%)		**Average best fit (Testing)** (%)	**STDEV** (Testing)
	Min	Max			Min	Max		
Heating system	96.68	99.26	98.85	0.55	95.83	98.40	97.31	0.78

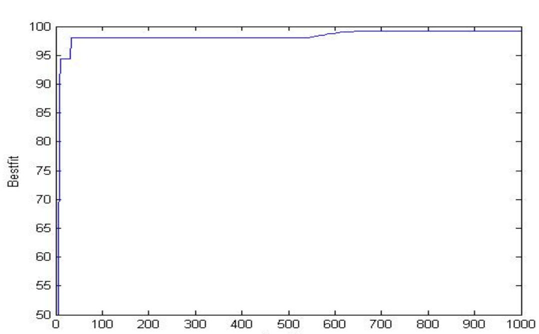

Figure-5. Convergence curve for the maximum best fit value of the heating system converge at 660th iteration.

RESULT

All the results are shown in Table-5. The minimum and maximum value of best fit obtained in training is 96.68% and 99.26%, respectively. Average value of best fit is 98.85%. In testing, the minimum, maximum, and average best fits are 95.83%, 98.40%, and 97.31%, respectively.

Based on 50 experiments, Equation (15), Equation (16) and Equation (17) are the best three mathematical equation selected by SMOPE-GSA.

$$G_{HS}(z) = \frac{0.0005z^{-1}+0.4897z^{-2}}{1-1.2180z^{-1}+0.2814z^{-2}} \quad (15)$$

where the model order value is 2 and parameter value, b_1 = 0.0005, b_2 = 0.4587, a_1 = -1.2100, and a_2 = 0.2314.

$$G_{HS}(z) = \frac{0.6490z^{-1}+0.6697z^{-2}}{1-0.8836z^{-1}+0.0244z^{-2}} \quad (16)$$

where the model order value is 2 and parameter value, b_1 = 0.3490, b_2 = 0.2277, a_1 = -0.9936, and a_2 = 0.2244.

$$G_{HS}(z) = \frac{0.8700z^{-1}}{1-0.8788z^{-1}} \quad (17)$$

where the model order value is 1 and parameter value, b_1 = 0.5700 and a_1 = -0.9735.

The convergence curve is shown in Figure-5. This convergence curve shows that all the GSA agents search and optimize the parameters in order to find the best combination of model order and pole zero parameters indicated by highest percentage of best fit value.

Table-6. Performance of SMOPE-GSA against existing methods applied to the heating system dataset.

Method	Best fit (%)
SMOPE-GSA	97.3
NOINSTR [11]	84.9
Nuclear norm [12]	84.7
IVM [11]	84.4
CVA [11]	84.3
N4SID [11]	82.5
MOESP [11]	82.5
NONE [11]	81.9

In this heating system experiment, the SMOPE-GSA converges at 660th iteration when finding the best mathematical model of Equation 15.

The performance of SMOPE-GSA against existing methods is shown in Table-6. The performance of SMOPE-GSA is 97.3%, which is the highest best fit in dataset. It followed by Hansson *et al.* with 84.9% using NOINSTR and Liu *et al.* with 84.7% using nuclear norm. Others are 84.4% by IVM, 84.3% by CVA, 82.5% by N4SID and MOESP, and 81.9% by NONE.

CONCLUSIONS

This paper presented to analyze the performances of mentioned SMOPE-GSA. The entire outcome is calculated based on best fit percentage taken over 50 run. From evaluation based on heating system data, the mentioned SMOPE-GSA offers the satisfactory results in comparison with others. For future work, more case studies shall be considered to further assess the effectiveness of the SMOPE-GSA.

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APPENDIX

Example of transfer function from 1st order up to 9th order ARX model

1st order

$$EQN\ 1 = G_1(z) = \frac{b_1 z^{-1}}{1 + a_1 z^{-1}}$$

2nd order

$$EQN\ 2 = G_2(z) = \frac{b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$EQN\ 3 = G_3(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

3rd order

$$EQN\ 4 = G_4(z) = \frac{b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$

$$EQN\ 5 = G_5(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$

$$EQN\ 6 = G_6(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$

4th order

$$EQN\ 7 = G_7(z) = \frac{b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

$$EQN\ 8 = G_8(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

$$EQN\ 9 = G_9(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

$$EQN\ 10 = G_{10}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

5th order

$$EQN\ 11 = G_{11}(z) = \frac{b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5}}$$

$$EQN\ 12 = G_{12}(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5}}$$

$$EQN\ 13 = G_{13}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5}}$$

$$EQN\ 14 = G_{14}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5}}$$

$$EQN\ 15 = G_{15}(z) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4}b_5z^{-5}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5}}$$

6th order

$$EQN\ 16 = G_{16}(z) = \frac{b_1z^{-1}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6}}$$

$$EQN\ 17 = G_{17}(z) = \frac{b_1z^{-1} + b_2z^{-2}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6}}$$

$$EQN\ 18 = G_{18}(z) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6}}$$

$$EQN\ 19 = G_{19}(z) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6}}$$

$$EQN\ 20 = G_{20}(z) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4} + b_5z^{-5}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6}}$$

$$EQN\ 21 = G_{21}(z) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4} + b_5z^{-5} + b_6z^{-6}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6}}$$

7th order

$$EQN\ 22 = G_{22}(z) = \frac{b_1z^{-1}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6} + a_7z^{-7}}$$

$$EQN\ 23 = G_{23}(z) = \frac{b_1z^{-1} + b_2z^{-2}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6} + a_7z^{-7}}$$

$$EQN\ 24 = G_{24}(z) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6} + a_7z^{-7}}$$

$$EQN\ 25 = G_{25}(z) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6} + a_7z^{-7}}$$

$$EQN\ 26 = G_{26}(z) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4} + b_5z^{-5}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6} + a_7z^{-7}}$$

$$EQN\ 27 = G_{27}(z) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4} + b_5z^{-5} + b_6z^{-6}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6} + a_7z^{-7}}$$

$$EQN\ 28 = G_{28}(z) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3} + b_4z^{-4} + b_5z^{-5} + b_6z^{-6} + b_7z^{-7}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6} + a_7z^{-7}}$$

8th order

$$EQN\ 29 = G_{29}(z) = \frac{b_1z^{-1}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6} + a_7z^{-7} + a_8z^{-8}}$$

$$EQN\ 30 = G_{30}(z) = \frac{b_1z^{-1} + b_2z^{-2}}{1 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5} + a_6z^{-6} + a_7z^{-7} + a_8z^{-8}}$$

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$$EQN\ 31 = G_{31}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8}}$$

$$EQN\ 32 = G_{32}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8}}$$

$$EQN\ 33 = G_{33}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8}}$$

$$EQN\ 34 = G_{34}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8}}$$

$$EQN\ 35 = G_{35}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6} + b_7 z^{-7}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8}}$$

$$EQN\ 36 = G_{36}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6} + b_7 z^{-7} + b_8 z^{-8}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8}}$$

9th order

$$EQN\ 37 = G_{37}(z) = \frac{b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8} + a_9 z^{-9}}$$

$$EQN\ 38 = G_{38}(z) = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8} + a_9 z^{-9}}$$

$$EQN\ 39 = G_{39}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8} + a_9 z^{-9}}$$

$$EQN\ 40 = G_{40}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8} + a_9 z^{-9}}$$

$$EQN\ 41 = G_{41}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8} + a_9 z^{-9}}$$

$$EQN\ 42 = G_{42}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8} + a_9 z^{-9}}$$

$$EQN\ 43 = G_{43}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6} + b_7 z^{-7}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8} + a_9 z^{-9}}$$

$$EQN\ 44 = G_{44}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6} + b_7 z^{-7} + b_8 z^{-8}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8} + a_9 z^{-9}}$$

$$EQN\ 45 = G_{45}(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6} + b_7 z^{-7} + b_8 z^{-8} + b_9 z^{-9}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8} + a_9 z^{-9}}$$