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AN EFFICIENT ANALYTICAL METHOD FOR VIBRATION PROBLEMS

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ABSTRACT

In this paper, we implement a new analytical technique, variational iteration method for solving the some vibration problems. The variational iteration algorithm leads to analytical solutions in the present study. The results show that the present method can be easily extended to other nonlinear oscillations and it can be predicted that variational iteration method can be found widely applicable in engineering and physics.

Keywords: vibration problems, nonlinear vibration, variational iteration method.

1. INTRODUCTION

Vibration of dynamical systems can be divided into two main classes like discrete and distributed. The variables in discrete systems depend on time only, whereas in distributed systems such as beams, plates etc. variables depend on time and space. Therefore, equations of motion of discrete systems are described by ordinary differential equations, while equations of motion of distributed systems are described by partial differential equations (Meirovitch [1]).

On the other hand, in the last decades, scientists have proposed and applied some analytical methods to nonlinear equations. For example; the vibrational behavior of quintic nonlinear in extensional beam on two-parameter elastic substrate based on the three mode assumptions is investigated by Sedighi [2]. He employed parameter expansion method to obtain the approximate expressions of nonlinear frequency-amplitude relationship for the first, second, and third modes of vibrations. Hamiltonian approach is applied to the analysis of the nonlinear free vibration of a tapered beam by Pakar and Bayat [3]. Ghaffarzadeh and Nikkar [4] applied a new analytical method called the variational iteration method-II (VIM-II) for the differential equation of the large deformation of a cantilever beam under point load at the free tip. Askari *et al*. [5] applied He’s energy balance method and He’s variational approach to frequency analysis of nonlinear oscillators with rational restoring force. Sedighi and Daneshmand [6] studied nonlinear transversely vibrating beams by the homotopy perturbation method with an auxiliary term. Jodeiri and Tabrizi [7] employed variational approach to obtain approximate analytical solutions for nonlinear vibrations of a thin laminated composite plate. They also applied Hamiltonian approach to analyze the non-linear problem of an elastically restrained tapered cantilever beam [8]. Bagheri *et al*. [9] studied the nonlinear responses of clamped-clamped buckled beam. They used two efficient mathematical techniques called He’s variational approach and Laplace iteration method in order to obtain the responses of the beam vibrations. Salehi *et al*. [10] applied two efficient methods to consider large deformation of cantilever beams under point load. A novel approach is applied to the analytic treatment of nonlinear fifth-order Equations by Saravi and Nikkar [11]. Askari *et al*. [12] applied higher order Hamilton approach to nonlinear vibrating systems, and many other problems solved by these methods [13-16].

The main goal of this paper is to present an alternative approach, namely variational iteration method, for constructing highly accurate analytical approximations to the nonlinear oscillation problems.

2. BASIC CONCEPT OF THE PROPOSED METHOD

The variational iteration method was first proposed by He [17] used to obtain an approximate analytical solutions for nonlinear problems. To clarify the idea of the proposed method for solution of the large deformation of string with large amplitudes, the basic concept of Variational Iteration Method is firstly treated. We consider the following general differential equation,

$$Lu + Nu = g(t) \tag{1}$$

where, L is a linear operator, and N a nonlinear operator, $g(t)$ an inhomogeneous or forcing term. According to the variational iteration method, we can construct a correct functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau))d\tau \tag{2}$$

Where λ is a general Lagrange multiplier, which can be identified optimally via the variational theory, the subscript n denotes the nth approximation, $\tilde{u}_n$ is considered as a restricted variation, i.e. $\delta\tilde{u}_n = 0$.

For linear problems, its exact solution can be obtained by only one iteration step due to the fact that the Lagrange multiplier can be exactly identified. In this method, the problems are initially approximated with possible unknowns and it can be applied in non-linear problems without linearization or small parameters. The approximate solutions obtained by the proposed method rapidly converge to the exact solution.

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3. APPLICATION OF THE PROPOSED METHOD

Example-1

In this example, Mathematical Pendulum that was studied by He [18] is considered.

The differential equation of motion of the undamped mathematical pendulum is given by,

$$\ddot{y} + \omega^2 \sin y = 0 \tag{3}$$

The initial conditions for this problem are as follows:

$$y(0) = A \tag{4a}$$

$$\dot{y}(0) = 0 \tag{4b}$$

The sin*y* term in Equation (3) is a nonlinear term and it can be expanded as

$$\sin y \approx y - \frac{1}{6} y^3 \tag{5}$$

Substituting Equation (89) into Equation (87) gives

$$\ddot{y} + \omega^2 y - \frac{\omega^2}{6} y^3 = 0 \tag{6}$$

The Lagrange multiplier of this problem is

$$\lambda = \frac{1}{\omega} \sin\left[\omega(\tau - t)\right] \tag{7}$$

Hence the iteration formula is

$$y_{n+1}(t) = y_n(t) + \frac{1}{\omega}\int_0^t \sin\left[\omega(\tau - t)\right][y''(\tau) + \omega^2 y(\tau) - \frac{\omega^2}{6} y^3(\tau)]d\tau \tag{8}$$

The complementary solution of this problem that is used as an initial approximation is given by

$$y_0(t) = A\cos(\alpha\omega t) \tag{9}$$

where α is an unknown constant.

Substituting the initial approximation into Equation (6), the following residual is obtained

$$R_0(t) \approx \ddot{y} + \omega^2 y - \frac{\omega^2}{6} y^3 = A(1 - \frac{1}{8}A^2 - \alpha^2)\omega^2 \cos(\alpha\omega t) - \frac{1}{24}A^3\omega^2 \cos(3\alpha\omega t) \tag{10}$$

The coefficient of the cos($\alpha\omega t$) term is set to zero in order to eliminate the secular term which may occur in the next iteration. Doing so, the expression of α is found as follows

$$\alpha = \sqrt{1 - \frac{A^2}{8}} \tag{11}$$

Hence,

$$y_1(t) = A\cos\alpha\omega t - \frac{A^3}{24(9\alpha^2 - 1)\omega^2}(\cos 3\alpha\omega t - \cos\omega t) \tag{12}$$

where α defined in Equation (11).
The period can be expressed as follows

$$T = \frac{2\pi}{\omega\sqrt{1 - \frac{1}{8}A^2}} \tag{13}$$

If $A=\pi/2$, then $T=1.20T_0$. On the other hand He's [18, 19] approximation gives $T=1.17T_0$, while the exact period is $T_{ex}=1.16T_0$, where $T_0=2\pi/\omega$.

Example-2

In this example, the problem that was studied by Nayfeh and Mook [20] is considered.

The differential equation of motion is given by,

$$\ddot{u} + \omega^2 u + \varepsilon u^2 \ddot{u} = 0 \tag{14}$$

The initial conditions for this problem are as follows:

$$y(0) = A \tag{15a}$$

$$\dot{y}(0) = 0 \tag{15b}$$

The Lagrange multiplier of this problem is

$$\lambda = \frac{1}{\omega} \sin\left[\omega(\tau - t)\right] \tag{16}$$

The iteration formula is given by

$$u_{n+1}(t) = u_n(t) + \frac{1}{\omega}\int_0^t \sin\left[\omega(\tau - t)\right][u''(\tau) + \omega^2 u(\tau) + \varepsilon u^2(\tau) u''(\tau)]d\tau \tag{17}$$

The complementary solution of this problem that is used as an initial approximation is given by

$$u_0(t) = A\cos(\alpha\omega t) \tag{18}$$

where α is an unknown constant.

Substituting the initial approximation given by Equation (18), the following residual is obtained as follows

$$R_0(t) \approx \ddot{u} + \omega^2 u + \varepsilon u^2 \ddot{u} = A\omega^2(1-\alpha^2 - \frac{3}{4}A^2\varepsilon)\cos(\alpha\omega t) - \frac{1}{4}\varepsilon A^3\alpha^2\omega^2\cos(3\alpha\omega t) \quad (19)$$

The coefficient of the $\cos(\alpha\omega t)$ term is set to zero in order to eliminate the secular term which may occur in the next iteration. Doing so, the expression for α is obtained as follows

$$\alpha = \frac{2}{\sqrt{4+3\varepsilon A^2}} \quad (20)$$

Hence,

$$y_1(t) = A\cos\alpha\omega t + \frac{\varepsilon A^3\alpha^2}{4(9\alpha^2-1)}(\cos\omega t - \cos 3\alpha\omega t) \quad (21)$$

where α defined in Equation (20).
The new frequency is defined as follows

$$\omega_1 = \alpha\omega \Rightarrow \omega_1 = \frac{2}{\sqrt{4+3\varepsilon A^2}}\omega \quad (22)$$

The frequency that is obtained by Nayfeh and Mook [20] using the perturbation method is

$$\omega_1 = \omega(1 - \frac{3}{8}\varepsilon A^2) \quad (23)$$

Note that Equation (23) is valid only for small ε values. However, the frequency expression given by Equation (22) is valid for all ε values and takes the following form for small ε values

$$\omega_1 \approx 1 - \frac{3}{8}\varepsilon A^2 + \frac{27}{128}\varepsilon^2 A^4 + \cdots \quad (24)$$

Example-3

In this example, the Duffing-harmonic oscillator that was studied by Lim and Wu [21] and Mickens [22] is considered.

The differential equation of motion is given by,

$$\frac{d^2y}{dt^2} + \frac{y^3}{1+y^2} = 0 \quad (25)$$

The initial conditions for this problem are as follows:

$$y(0) = A \quad (26a)$$

$$\dot{y}(0) = 0 \quad (26b)$$

with initial conditions $y(0)=A$ and $y'(0)=0$.

The following form of Equation (25) is going to be studied in this example

$$(1+y^2)\frac{d^2y}{dt^2} + y^3 = 0 \quad (27)$$

He's technique is going to be used to overcome seculer terms that appear in the iterations. The initial approximation is,

$$y_0(t) = A\cos(\alpha t) \quad (28)$$

where α is an unknown constant.

Substituting the initial approximation into Equation (27), the following residual is obtained

$$R_0(t) \approx (1+y^2)\ddot{y} + y^3 = (\frac{3}{4}A^2 - \alpha^2 - \frac{3}{4}A^2\alpha^2)\cos(\alpha t) + \frac{A^3}{4}(1-\alpha^2)\cos(3\alpha t) \quad (29)$$

In oreder to discard the seculer terms, the coefficient of $\cos(\alpha t)$ is set to zero which gives the expression of α as follows

$$\alpha = \sqrt{\frac{\frac{3}{4}A^2}{1+\frac{3}{4}A^2}} \quad (30)$$

Hence the new frequency is defined as follows

$$\omega = \sqrt{\frac{\frac{3}{4}A^2}{1+\frac{3}{4}A^2}} \quad (31)$$

which is the same with the one found by Lim and Wu [21] and Mickens [22].
The iteration formula is given by

$$u_1(t) = u_0(t) + \int_0^t (\tau - t)[\frac{A^3}{4}(1-\alpha^2)\cos(3\alpha\tau)]d\tau \quad (32)$$

Hence,

$$y_1(t) = \cos\omega t + \frac{A}{27}(\cos 3\omega t - 1) \quad (33)$$

where ω defined in Equation (31).

4. CONCLUSIONS

In this paper, the main purpose was to illustrate the application of VIM in solving nonlinear oscillator systems. The variational iteration algorithm leads to analytical solutions in the present study. Additionally, the procedure presented in this paper can be simply extended to solve more complex vibration problems; such as aeroelasticity, random vibrations etc.

REFERENCES

[1] L. Meirovitch. 1980. Computational Methods in Structural Dynamics. Sijthoff and Noordhoff International Publishers. Alphen aan den Rijn, The Netherlands.

[2] Sedighi Hamid. M. Nonlinear free vibrations of quintic inextensional beams lying on Winkler elastic substrate based on three-mode assumptions. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, DOI: 10.1177/1464419314522780.

[3] Pakar I. and Bayat M. 2012. Analytical study on the non-linear vibration of euler-bernoulli beams. Journal of vibroengineering. 14(1): 216-224.

[4] Ghaffarzadeh H. and Nikkar A. 2013. Explicit solution to the large deformation of a cantilever beam under point load at the free tip using the variational iteration method-II. Journal of Mechanical Science and Technology. 27(11): 3433-3438.

[5] Askari H., Yazdi M.K. and Saadatnia Z. 2010. Frequency analysis of nonlinear oscillators with rational restoring force via He's energy balance method and He's variational approach. Nonlinear Science Letters A. 1(4): 425-430.

[6] Sedighi H. M. and Daneshmand F. 2014. Nonlinear transversely vibrating beams by the homotopy perturbation method with an auxiliary term. Journal of Applied and Computational Mechanics. 1(1): 1-9.

[7] Jodeiri H. and Tabrizi S. 2014. Study of nonlinear vibration of laminated composite plates using variational approach. ARPN Journal of Engineering and Applied Sciences. 10(1): 122-127.

[8] Jodeiri H. and Tabrizi S. 2014. Study of nonlinear vibration of an elastically restrained tapered beam using Hamiltonian approach. ARPN Journal of Engineering and Applied Sciences. 10(1): 251-255.

[9] Bagheri S., Nikkar A. and Ghaffarzadeh H. 2014. Study of nonlinear vibration of Euler-Bernoulli beams by using analytical approximate techniques. Latin American Journal of Solids and Structures. 11: 157-168.

[10] Salehi P., Yaghoobi H. and Torabi M. 2012. Application of the differential transformation method and variational iteration method to large deformation of cantilever beams under point load. Journal of mechanical science and technology. 26(9): 2879-2887.

[11] Saravi M. and Nikkar A. 2014. Promising Technique for Analytic Treatment of Nonlinear Fifth-order Equations. World Journal of Modelling and Simulation. 10(1): 27-33.

[12] Askari H., Nia Z. S., Yildirim A., Yazdi M. K. and Khan Y. 2013. Application of higher order Hamilton approach to nonlinear vibrating systems. Journal of Theoretical and Applied Mechanics. 51(2): 287-296.

[13] Hashemi M.H. and Nikkar A. 2014. On the approximate analytical solution for parametrically nonlinear excited oscillators. Geomechanics and Geoengineering. DOI:10.1080/17486025.2014.969787.

[14] Khan Y., Taghipour R., Falahian M. and Nikkar A. 2013. A new approach to modify regularized long wave equation. Neural Computing and Applications. 23: 1335-1341.

[15] Nikkar A., Bagheri S. and Saravi M. 2014. Study of nonlinear vibration of Euler-Bernoulli beams by using analytical approximate techniques. Latin American Journal of Solids and Structures. 11: 320-329.

[16] Taghipor R., Akhlaghi T. and Nikkar A. 2014. Explicit solution of the large amplitude transverse vibrations of a flexible string under constant tension. Latin American Journal of Solids and Structures. 11: 545-555.

[17] He J.H. 1999. Variational iteration method-a kind of non-linear analytical technique: some examples, Internat. J. Nonlinear Mech. 34: 699-708.

[18] He J.H. 1998. Approximate solution of nonlinear differential equations with convolution product nonlinearities, Comput. Methods Appl. Mech. Eng. 167: 69-73.

[19] He J.H. 2005. A generalized variational principle in micromorphic thermoelasticity. Mech. Res. Comm. 32(1): 93-98.

[20] Nayfeh A.H. and Mook D.T. 1979. Nonlinear Oscillation. John Wiley and Sons.

[21] Lim C.W. and Wu B.S. 2003. A new analytical approach to the Duffing-harmonic oscillator. Physics Letters A. 311: 365-373.

[22] Mickens R.E. 2001. Mathematical and Numerical Study of the Duffing Harmonic Oscillator. Journal of Sound and Vibration. 244: 563-567.