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THRESHOLD RESULTS OF A HOST-MORTAL COMMENSAL ECOSYSTEM WITH A CONSTANT HARVESTING OF THE COMMENSAL SPECIES

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ABSTRACT

This paper focuses on phase plane diagrams for two species ecological model comprising a mortal commensal and the host species with a constant harvesting of the commensal species. This model is characterized by a couple of first order non-linear ordinary differential equations. The possible four existing equilibrium points of the system are identified and the nature of the ecological interaction between the species is discussed based on the equilibrium points of the model equations through the phase plane diagrams for specific values to the parameters in the basic model equations.

Keywords: commensalism interaction, commensal species, host species, null clines, trajectories, phase plane diagrams.

1. INTRODUCTION

In many cases we will be able to reduce a system of differential equation to two independent variables in which case we have a planer system. Such systems have many advantages rather higher dimensional ecological models. In particular, it is possible to qualitatively sketch solutions without ever computing them. By phase plane mean a sketch of the system in coordinates and is different from a sketch against time. Phase planes can provide much more useful information such as a global way of looking at the system. The pair of curves when plotted against time is not obviously a circle however if you plot then parametrically in the plane the geometry is clear. Several authors have been contributed to their work in phase plane analysis of ecological models of various interactions between the species. The general concepts of modeling have been presented in the treatises of Meyer [1], chusing [2], Paul Colinvaux [3], Freedman [4], Kapur [5, 6]. N.C. Srinivas [7] studied the competitive ecosystems of two species and three species with limited and unlimited resources. Lakshminarayan [8], Acharyulu K.V.L.N and Pattabhi Ramacharyulu [9-13] investigated Ammensal Ecological models and prey- predator ecological models with cover for the prey and alternative food for the predator and time delay. Recently Seshagiri Rao and Pattabhi Ramacharyulu [14] studied on the stability of host-a decaying commensal species pair with limited resources.

To construct the ecological model here we use the following notations:

$N_1(t)$ = The population of the commensal (S_1) at time t.

$N_2(t)$ = The population of the host (S_2) at time t.

d_1 = The mortal rate of the commensal (S_1).

a_2 = The rate of natural growth of the host (S_2).

a_{11} = The rate of decrease of the commensal (S_1) due to the limitations of its natural resources.

a_{22} = The rate of decrease of the host (S_2) due to the limitations of its natural resources.

a_{12} = The rate of increase of the commensal (S_1) due to the support given by the host (S_2).

$k_2(=a_2/a_{22})$ = The carrying capacity of S_2.

$c(=a_{12}/a_{11})$ = The coefficient of the commensal.

$e_1(=d_1/a_{11})$ = The mortality coefficient of S_1.

$h_1(=a_{11}H_1)$ = The coefficient of harvesting /migration of the commensal.

H_1 = The harvesting/migration of S_1 per unit time.

2. BASIC MODEL EQUATIONS

Employing the above terminology, the basic model equations for the growth rates of the mortal commensal and the host species with commensal harvesting at a constant rate are as follows

(i) Growth rate equation for the Mortal commensal species (S_1) **:**

$$\frac{dN_1(t)}{dt} = a_{11}\left[-e_1N_1(t) - N_1^2(t) + cN_1(t)N_2(t) - H_1\right] \quad (1)$$

(ii) Growth rate equation for the Host species (S_2) **:**

$$\frac{dN_2(t)}{dt} = a_{22}N_2(t)\left[k_2 - N_2(t)\right] \quad (2)$$

The equilibrium points are the turning points in the variation of N_1 and N_2 with respect to time t. The

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null clines non over lapping intersections of the hyperbola $e_1N_1 + N_1^2 - cN_1N_2 + H_1 = 0$ and the lines $N_2 = 0$, $N_2 = k_2$ given by $\frac{dN_1}{dt} = 0$ and $\frac{dN_2}{dt} = 0$ divide the first quadrant of the $N_1 - N_2$ plane into regions are called the threshold regions. The diagram showing the threshold lines and regions is called the threshold/phase-plane diagram. This diagram shows the direction of variations of the species around the stable/unstable equilibrium points.

The system under investigation has the following four equilibrium states (E_1)-(E_4) and these can be classified in two categories A and B.

A). Commensal washed out state

$E_1 : (0, k_2)$, when $ck_2 = e_1$

B). Coexistence States

Case: B.1

E_2: $\overline{N_1} = \frac{ck_2 - e_1}{2}$; $\overline{N_2} = k_2$, when $(ck_2 - e_1)^2 = 4H_1$

This would exist only when $ck_2 > e_1$ which is independent of H_1.

Case: B.2

$$E_3: \overline{N_1} = \frac{(ck_2 - e_1) + \sqrt{(ck_2 - e_1)^2 - 4H_1}}{2} \quad ; \quad \overline{N_2} = k_2$$

$$E_4: \overline{N_1} = \frac{(ck_2 - e_1) - \sqrt{(ck_2 - e_1)^2 - 4H_1}}{2} \quad ; \quad \overline{N_2} = k_2$$

both these states would exist only when $(ck_2 - e_1)^2 > 4H_1$ and $ck_2 > e_1$.

Note: (i). As H_1 increases and ultimately approaches to $\frac{(ck_2 - e_1)^2}{4}$, the two equilibrium points E_3 and E_4 coincide with E_2.

(ii). When $ck_2 < e_1$ there would be no equilibrium states since harvesting rate $H_1 > 0$.

(iii). When $ck_2 = e_1$, evidently E_3 and E_4 do not exists. However E_2 exist, which is independent of the harvesting rate H_1 of the commensal species.

A. The threshold Diagram for equilibrium point E_1

The threshold lines/ null clines divide the phase plane into four regions I, II, III and IV in the first quadrant (i.e., $N_1 \geq 0$, $N_2 \geq 0$). The corresponding threshold regions for $a_{11} = 0.1$, $e_1 = 0.9$, $c = 0.3$, $H_1 = 8$, $a_{22} = 0.1$, $k_2 = 3$ are shown in Figure-1.

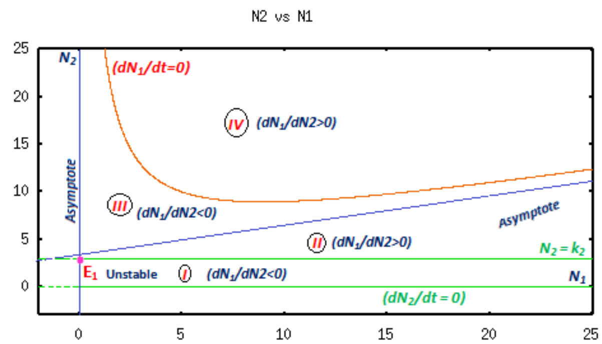

Figure-1. Threshold regions

Region I: Here $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} > 0 \Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$.

Region II: Here $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} < 0 \Rightarrow \frac{dN_1}{dN_2} > 0$ then $N_1(t)$ is an increasing function of $N_2(t)$.

Region III: Here $\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} < 0 \Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$.

Region IV: Here $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} < 0 \Rightarrow \frac{dN_1}{dN_2} > 0$ then $N_1(t)$ is an increasing function of $N_2(t)$.

Figure-2 shows the direction of the field lines in the threshold regions.

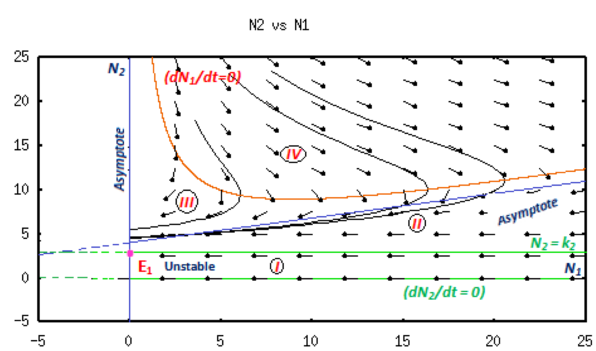

Figure -2. Threshold diagram for E_1

B.1 The threshold Diagram for equilibrium point E_2

The threshold lines/ null clines given by $\frac{dN_1}{dt} = 0$, $\frac{dN_2}{dt} = 0$ divide the phase plane into five regions I, II, III, IV and V in the first quadrant (i.e.,

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$N_1 \geq 0$, $N_2 \geq 0$). The threshold regions for $a_{11} = 0.1$, $e_1 = 2$, $c = 3$, $H_1 = 12.25$, $a_{22} = 0.1$, $k_2 = 3$ are shown in Figure-3.

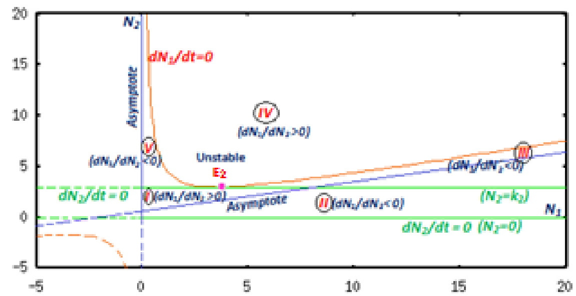

Figure-3. Threshold regions

Region I: In this region $\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} > 0 \Rightarrow \frac{dN_1}{dN_2} > 0$ then $N_1(t)$ is an increasing function of $N_2(t)$.

Region II: Here $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} > 0 \Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$.

Region III: Here $\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} < 0 \Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$.

Region IV: Here $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} < 0 \Rightarrow \frac{dN_1}{dN_2} > 0$ then $N_1(t)$ is an increasing function of $N_2(t)$.

Region V: Here $\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} < 0 \Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$.

Figure-4 shows the direction of the field lines in the threshold regions.

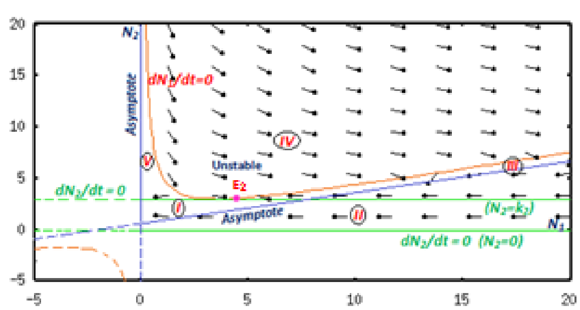

Figure-4. Threshold diagram for E_2

B.2 The threshold Diagram for equilibrium points E_3 and E_4

The threshold lines / null clines divide the phase plane into six regions I , II , III , IV , V and VI in the first quadrant (i.e., $N_1 \geq 0$, $N_2 \geq 0$) are shown in Figure-5.

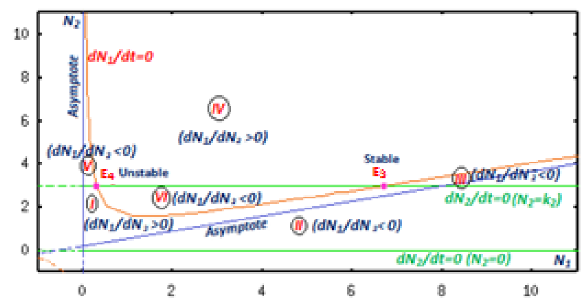

Figure-5. Threshold regions

Region I: In this region $\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} > 0$ $\Rightarrow \frac{dN_1}{dN_2} > 0$ then $N_1(t)$ is an increasing function of $N_2(t)$.

Region II: $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} > 0$ $\Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$.

Region III: $\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} < 0$ $\Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$.

Region IV: $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} < 0$ $\Rightarrow \frac{dN_1}{dN_2} > 0$ then $N_1(t)$ is an increasing function of $N_2(t)$.

Region V: $\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} < 0$ $\Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$.

Region VI: $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} > 0$ $\Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$.

Figure-6 shows the direction of the field lines in the threshold regions for $a_{11} = 0.1$, $e_1 = 2$, $c = 3$, $H_1 = 2$, $a_{22} = 0.1$, $k_2 = 3$

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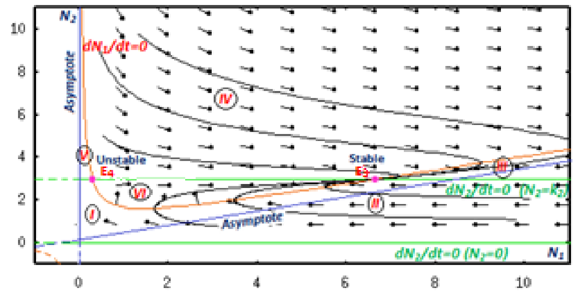

Figure-6. Threshold diagram for E_3 and E_4

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