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# FAULT TOLERANT CONTROL DESIGN WITH ACCEPTABLE PERFORMANCE DEGRADATION

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## ABSTRACT

The main objective of this work is to design a fault tolerant control approach using multiple model technique and with acceptable performance degradation due to faults in actuators, sensors and system dynamics. The achievable performance under various component failures is represented in the form of reference models, known as acceptable performance reference models. These models are used to synthesize a set of controllers. Under a specific fault condition, proper controller is reconfigured and revised command inputs are selected automatically to achieve desired performance. The aircraft model is chosen to demonstrate the effectiveness of the model.

Keywords: command input strategy, fault diagnosis, multiple model, performance degradation, reference models.

## INTRODUCTION

Modern technological systems rely on sophisticated control systems to meet increased performance and safety requirements. This is particularly important for safety-critical systems, such as aircrafts, spacecrafts, nuclear power plants, and chemical plants processing hazardous materials. In such systems, the consequences of a minor fault in a system component can be catastrophic. It is necessary to design control systems which are capable of tolerating potential faults in these systems in order to improve the reliability and availability while providing a desirable performance. These types of control systems are often known as Fault Tolerant Control Systems (FTCS). More precisely, FTCS are control systems which possess the ability to accommodate component failures automatically. They are capable of maintaining overall system stability and acceptable performance in the event of failures.

Adaptive control was proposed as a way for dealing with wide range of flight conditions. Adaptive control is used in order to automatically adjust the controller parameters to achieve the desired performance. Model - reference adaptive control (MRAC) and Self Tuning Control (STC) are two popular methodologies. In STC, online parameter estimation is required for the controller adaptation and controller reconfiguration may or may not be an online process. In MRAC, the unknown parameters are not perfectly estimated, but rather are tuned and adjusted so that the output of the plant follows the desired trajectory. Also, MRAC does not support actuator failures or actuator constraints [1]. Adaptive control strategies are most suitable fault tolerant control techniques when the system dynamics are slow varying.

In flight control system, failures of actuator or sensor may cause serious problems and has to be taken care such that the system is stable. Even though it is a common sense to accept a certain degree of performance degradation in the presence of system component failures, the fault-tolerant control system design which considers the fault-inflicted physical constraints for maintaining achievable performance has mostly been ignored until recently [2].

It is straight forward to design a post-fault controller to recover the pre-fault system performance. In practice, once the fault occurs, the degree of system redundancy and the available actuator capabilities can be greatly reduced. If the design objective of FTCS is still to achieve the pre-fault performance of the system, the system actuators may have to provide extra efforts to compensate for the changes caused by fault, especially during the initial fault recovery period. This may be undesirable for a practical system due to physical limitations of its actuators. The consequence of the designed FTCS may lead to actuator saturation, or worse still, to cause further damage to the system and even result in loss of the system stability. Therefore, trade-off between achievable performance and the available control capability should be carefully examined in FTCS design [3, 4]. Using multiple-model schemes is one way to ensure that the controller can be designed so that the stability and performance can be guaranteed for a wide flight envelope.

In the recent work, two reference models were used: one for the normal system operation and the other for the system under contingencies with actuator failures, respectively, where the magnitude of the fault is estimated and controller is reconfigured accordingly. It soon becomes evident that a twin model approach is not comprehensive enough to represent all potential system malfunctions. Different faults in a system can exhibit distinctive characteristics; a single performance reduced model cannot simply represent all of them. Naturally, a multiple-model approach offers a logical extension to the concept in dealing with multiple type of faults in actuators, sensors and system dynamics, which could not easily be done under the framework of [5] because of the difficulty faced when estimating all fault parameters associated to different types of failures in actuators, sensors and system dynamics on-line. By representing each fault type with a separate model, it has been shown that the overall fault handling capability for different types of faults in the control system can be enhanced considerably. The control



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system performance also becomes less conservative, because each controller only needs to deal with a single fault scenario [6]. Furthermore, the same failure type but at different severities may be represented by different performance reduced models under the very same framework.

The objective of this brief paper is to present an approach to incorporate performance limitations under different fault conditions using multiple-model technique. The current work differs significantly from that of [2] as a completely different control structure is used, in which the controller for each failure scenario is designed individually. Under the assumption that all potential faults in the system can be represented in terms of a finite set of models, a performance reduced reference model is synthesized for each failure scenario with due consideration of system performance limitations. There are three unique advantages associated with the current approach: a) it can handle multiple type of faults; b) it is able to isolate faults quickly by performing a simple statistical test on the multiple-model residuals; and c) it results in a less conservative control system for a specific fault situation by using the corresponding performance reduced reference model.

This paper is organised as follows. Modelling of multiple failures are presented in section 2. The overall scheme is presented in section 3. In section 4 the controller design is discussed. In section 5, the results for aircraft model are discussed. Conclusion is given in section 6.

## MODELLING OF MULTIPLE FAULTS

#### System and fault models

Faults are those system malfunctions, which could lead to undesirable consequences if left unattended. In practice, faults may occur in actuators, sensors and system dynamics. Therefore, all three types of faults are considered in this paper. Each fault type can be represented by one or more models depending on the nature and severity of the fault.

Assume that a finite set of N models are used to represent the system under normal and the (N - 1) failure modes. Thus, the system can be represented as  $x(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + x(t)$ 

$$= A_{0}(t) + B_{1}(t) + w(t) \tag{1}$$

 $z(t) = (C + \Delta G)x(t) + z(t)$ 

$$= C_{j} x(t) + v(t) \qquad j = 1, \dots, N$$
 (2)

where  $\mathbf{x} \in \mathbb{R}^n$  is the state vector,  $\mathbf{z} \in \mathbb{R}^m$  is the measurement vector,  $\mathbf{u} \in \mathbb{R}^d$  is the control input vector.  $\mathbf{w} \in \mathbb{R}^n$  is a zero-mean white Gaussian sequence with covariance  $\mathbf{Q}$  to represent modelling uncertainties.  $\mathbf{v} \in \mathbb{R}^m$  is a zero-mean white Gaussian sequence with covariance **R** to represent measurement noise. The initial state is assumed to have mean  $\bar{\mathbf{x}}_0$  and covariance  $\overline{\mathbf{P}}_0$  and to be independent from **w** and **v**. Furthermore,  $\Delta \mathbf{A}_j$ ,  $\Delta \mathbf{B}_j$  and  $\Delta \mathbf{C}_j$  **j** = 2..., **N** represent the fault induced changes in the system dynamics, actuators and sensors respectively. When **j** = 1,  $\Delta \mathbf{A}_j$ ,  $\Delta \mathbf{B}_j$  and  $\Delta \mathbf{C}_j$  are null matrices and represents the normal condition. The subscript **j** denotes the quantities pertaining to the model,  $\mathbf{m}_4 \subset \mathbf{M}$ . **M** =  $[\mathbf{m}_4, \mathbf{m}_5, \mathbf{m}_{N_1}]$  is a set containing system models for all conditions. Matrices  $\mathbf{A}_j$ ,  $\mathbf{B}_j$  and  $\mathbf{C}_j$  (**j** = 1, ..., **N**) correspond to the jth post-fault model of the system.

#### Multiple acceptable performance reference model

To capture and specify the characteristics of the faulty system under each fault scenario, a corresponding acceptable performance reference model need to be synthesized. These models will represent the desirable dynamic behaviours of the closed-loop system under specific fault conditions. To handle different type of faults, different models are often needed. Several models may even be needed for a single failure type if the characteristics of the system changes significantly at different fault severities. In particular, the dynamic behaviour of the post-fault system is governed by the characteristics of the designed reference model, which takes into consideration the allowable performance limits under a given fault condition without violating the physical constraints in any system variables. The reference model is taken from [7].

Assume that a reference model of the system under the normal condition is represented by

$$\mathbf{y}_{\mathbf{n}}^{\mathbf{n}} = \mathbf{U}_{\mathbf{n}}^{\mathbf{n}} \mathbf{x}_{\mathbf{n}}^{\mathbf{n}} \tag{3}$$

where  $\mathbf{x}_{1}^{\mathbf{p}} \in \mathbf{x}_{2}^{\mathbf{p}}$  is the state vector of the reference model,  $\mathbf{y}_{1}^{\mathbf{p}} \in \mathbf{R}^{\mathbf{p}^{\mathbf{m}}}$  is the output vector and  $\mathbf{r}_{2}^{\mathbf{f}} \in \mathbf{R}^{\mathbf{p}^{\mathbf{m}}}$  is the command input vector. The above model, known as the desired reference model, specifies the desired dynamic characteristics of the system under the normal condition. Let the eigen values of this system be represented as

$$\Delta_1 = \operatorname{diag}\left[\lambda_1^*, \lambda_2^*, \dots, \lambda_n^*\right] \tag{4}$$

In the presence of a fault, based on failure models represented in (1) and (2), it is expected that the eigen values of the achievable performance reference models would shift toward the imaginary axis to reflect the loss of system dynamic performance. This can be achieved by simply selecting a mode degradation matrix,  $\Psi_i^j$ , (1 - 2, ..., N), for each fault condition. Suppose that the eigen values of the acceptable performance reference model under each fault condition are related to those under normal condition by

$$\Lambda_j = \Psi_j^{-1} \Lambda_{j,\ell} \qquad j = 1, \dots, N \qquad (5)$$

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Where

$$\Psi_{j} = dag[\Psi_{1}^{i}, \Psi_{2}^{i}, \dots, \Psi_{n}^{i}]$$
(6)

and  $\Psi_{i}^{1} \geq 1, \forall i = 1, \dots, n^{m}, j = 2, \dots, N$ .

The transfer function matrix of the reference model for the system in the failure mode can then be obtained as

$$T_{i}(e) = C_{i}^{m}(Ie\Psi_{i} - A_{i}^{m})^{-1}B_{i}^{m}$$
$$= C_{i}^{m}(Ie - \Psi_{i}^{-1}A_{i}^{m})^{-1}B_{i}^{m}$$
$$= C_{i}^{m}(Ie - A_{i}^{m})^{-1}B_{i}^{m}, \quad j = 2, ..., N$$
(7)

Hence, a set of acceptable performance reference models can be obtained as

$$\dot{\mathbf{x}}_{l}^{m} = \mathbf{A}_{l}^{m} \mathbf{x}_{l}^{m} + \mathbf{B}_{l}^{m} \mathbf{v}_{l}^{l}$$

$$\mathbf{y}_{l}^{m} = \mathbf{C}_{l}^{m} \mathbf{x}_{l}^{m}, \quad \mathbf{j} = 2, \dots, \mathbf{N}$$
(8)

where,  $\mathbf{A}^{\mathbf{m}} = \Psi_{\mathbf{1}}^{\mathbf{m}} \mathbf{A}^{\mathbf{m}}_{\mathbf{1}}, \mathbf{B}^{\mathbf{m}}_{\mathbf{1}} = \Psi_{\mathbf{1}}^{\mathbf{m}} \mathbf{E}^{\mathbf{m}}_{\mathbf{1}}$ 

 $C_{l}^{m} = \Psi_{l}^{-1}C_{l}^{m}, l = 2, ..., N$ 

The matrix triplets  $\{M_{j}, P_{j}, Q_{j}, P_{j}, Q_{j}, P_{j}, Q_{j}, P_{j}, Q_{j}, Q_{j},$ 

## **Command input adjustment**

To ensure that all system variables are within the safe operating range and that all of the control effectors are free from saturation in the event of failures, one has to make appropriate adjustments to the level of control commands as well. A command governor is used just for this purpose. Essentially, it performs two functions to determine: 1) which output variables the closed-loop system has to track and 2) what is the appropriate reduced level of command inputs for a given fault scenario.

The objective of the command input management is to determine appropriate command inputs in the presence of actuator faults for avoiding potential saturation in actuators. Adjustment of command input includes two parts: 1) selection of a new command input to the system at the steady-state, and 2) adjustment of the command input during the initial period of control reconfiguration.

## **Overall structure**

Based on the above description, the overall configuration of the proposed FTCS [1] is depicted in Figure-1, which includes the following modules: 1) an interacting multiple-model (IMM) based fault detection and diagnosis (FDD) 2) multiple acceptable performance reference models and the associated controllers 3) a reconfigurable control mechanism, and 4) a command governor.



Figure-1. Block diagram of IMM estimator.

## Reconfigurable controller design

Eigen structure Assignment (EA) is one of the most popular controller design techniques for Multi-Input and Multi Output systems. The method gives a control designer extra freedom over merely assigning the closedloop eigen values of the system. This freedom is in the form of the specification of elements of the closed-loop eigenvectors of the system. The advantage of EA is that when the performance specifications are given in terms of system eigen structure, the eigen structure can be achieved exactly for the stability and desired dynamic performance.

#### Feedback controller design

Suppose that the dynamics of the system have undergone some changes due to faults in system components, actuators, and sensors, and the system models with faulty conditions have become

$$x(k + 1) = P_{i}(k)x(k) + G_{i}(k)u(k), j = 2...N$$
 (9)

The aim of reconfigurable control system design is to synthesize a set of new feedback gain matrices so that the closed-loop eigen values of the reconfigured system are nearly same as those of the pre-fault system,

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(10)

$$\lambda_{i}^{1} = \lambda_{i}^{1} \mathbb{P}_{i} + \mathbb{G}_{i} \mathbb{K}_{i} = \lambda^{1} = \lambda_{i}^{1} \mathbb{P}_{i} + \mathbb{G}_{i} \mathbb{K}_{i},$$
  
$$i = 1, \dots, n, j = 2, \dots, N$$

where  $\mathbb{K}_1$  denotes a control gain matrix designed for the nominal (fault-free) system and  $\mathbb{K}_1 = 2 \dots \mathbb{N}$  represents the new feedback control gain matrices under different fault conditions.  $\mathbb{M}_1$  denotes the eigen values of the system.

The closed-loop system eigenvectors of the reconfigured system,  $\{v_1, l = 1, ..., n_i\} = 2$  .... N} with the feedback gain matrix  $K_i$  will satisfy

$$(\mathbf{F}_{l} + \mathbf{G}_{l}\mathbf{K}_{l})\mathbf{v}_{l}^{l} = \lambda_{l}\mathbf{v}_{l}^{l}$$
(11)

or 
$$\mathbf{v}_{1}^{i} = (\lambda_{1}^{i}\mathbf{I} - \mathbf{F}_{1})^{-1}\mathbf{G}_{1}\mathbf{K}_{1}\mathbf{v}_{1}^{i}$$
 (12)

Then, the objective of the reconfigurable control system is to synthesize a feedback gain matrix  $\mathbf{K}_{i}$  such that the eigen vectors of reconfigured closed-loop system  $\mathbf{v}_{j}^{i}$  is as close to the corresponding eigenvectors of the pre-fault system  $\mathbf{v}_{i}^{i}$  as possible. Because of the variations in system dynamics, in general,  $\mathbf{v}_{j}^{i}$  does not lie in the same subspace as  $\mathbf{v}_{i}^{i}$ , which may be viewed as the desired eigenvector for  $\mathbf{v}_{j}^{i}$ . The best possible closed-loop system eigenvector can be obtained by the projection of the desired eigenvector onto the subspace spanned by the columns of  $(\lambda_{i}^{i}\mathbf{1} - \mathbf{F}_{i}^{i})^{-1}\mathbf{G}_{i}$ . In the context of reconfigurable control system design, the best choice of  $\mathbf{v}_{j}^{i}$  can be obtained by projecting the corresponding  $\mathbf{v}_{i}^{i}$  onto  $\mathbf{E}_{j}^{i}$  orthogonally, where  $\mathbf{E}_{i}^{i}$  and a new vector  $\mathbf{w}_{i}^{i}$  can be defined as

$$\mathbf{E}_{\mathbf{i}} = (\mathbf{i}_{\mathbf{i}} \mathbf{I} - \mathbf{F}_{\mathbf{i}})^{-1} \mathbf{G}_{\mathbf{i}}$$
(13)

$$\mathbf{W} = \mathbf{K}_{\mathbf{V}} \tag{14}$$

Consequently, (12) can be rewritten as

The desired eigenvector  $\mathbf{v}_j^i$  in the achievable subspace can be found by the following least-squares minimization

$$\min_{i} (v_{1}^{i}) = \min \left\{ (v_{1}^{i} - v_{1}^{i})^{T} W_{1}^{i} (v_{1}^{i} - v_{1}^{i}) \right\}$$

$$= \min \left\{ (\mathbb{E}_{1}^{i} w_{1}^{i} - v_{1}^{i})^{T} W_{1}^{i} (\mathbb{E}_{1}^{i} w_{1}^{i} - v_{1}^{i}) \right\},$$

$$i = 1, ..., i = 2 ..., N$$

$$(16)$$

and

$$\mathbf{w}_{i}^{I} = \mathbf{R}_{i}^{I} \left( \mathbf{R}_{i}^{T} \mathbf{W}_{i}^{IT} \mathbf{W}_{i}^{T} \mathbf{R}_{i}^{I} \right)^{-1} \mathbf{R}_{i}^{T} \mathbf{W}_{i}^{T} \mathbf{w}_{i}^{I}$$
(17)

where  $W^{1} = \mathbb{R}^{n \times n}$  is a positive definite weighting matrix.

Consider the following closed-loop reconfigured system equation

$$\mathbf{x}_{k+1} = (\mathbf{F}_1 + \mathbf{G}_1 \mathbf{X}_1) \mathbf{x}_k \tag{18}$$

To simplify the procedure in calculating the matrix  $\mathbb{K}_i$ , a linear transformation matrix

$$\Omega_{j} = \begin{bmatrix} G_{j} & \Theta_{j} \end{bmatrix} \in \mathbb{R}^{n \times n}$$
(19)

is chosen, where  $\theta_1 \in \mathbb{R}^{n \times (n-1)}$  is an arbitrary matrix such that rank  $(\Omega_1) = n$ .

Applying the linear transformation  $\Omega_1$  to (18), a new set of state variables  $\mathbb{R}_{\mathbb{R}} \cong \mathbb{R}^n$  can be obtained

$$\mathbf{x}_{\mathbf{k}} = \mathbf{x}_{\mathbf{j}}^{-1} \mathbf{x}_{\mathbf{k}} \tag{20}$$

Thus (18) is transformed to

$$\mathbf{\overline{x}_{k+1}} = (\mathbf{\overline{P}_{j}} + \mathbf{\overline{G}_{j}}\mathbf{\overline{x}_{j}}\mathbf{\Omega_{j}})\mathbf{\overline{x}_{k}}$$
(21)

where,  $\mathbf{F}_{j} = \hat{\mathbf{a}}_{j}^{-1} \mathbf{F}_{j} \hat{\mathbf{a}}_{j}$ ,  $\mathbf{G} = \hat{\mathbf{a}}_{j}^{-1} \mathbf{G}_{j} \hat{\mathbf{a}}_{j} = \begin{bmatrix} \mathbf{f}_{j} \\ \mathbf{g}_{j} \end{bmatrix}$ 

The corresponding eigenvectors under this transformation are related by

$$\mathbf{v}_{l}^{i} = \Omega_{l}^{-1}\mathbf{v}_{l}^{i} = \begin{bmatrix} \mathbf{s}^{i} \\ \mathbf{g}^{i} \end{bmatrix}$$

Clearly, the eigen values, eigen vectors and system matrices satisfy

$$(\mathbf{F}_{1} + \mathbf{G}_{1}\mathbf{K}_{1}\Omega_{1})\nabla_{1}^{1} = \lambda_{1}\nabla_{1}^{1}, \ i = 1...n, \ i = 2...N$$
(22)

Equation (22) can be rearranged as follows

$$[\lambda_{j}]I - F_{j}[w] = \overline{G}_{j}E_{j}\Omega_{j}w]A = 1...n, j = 2...N$$
 (23)

By exploiting the special structure of  $\overline{\mathbf{G}}_{j}$ , we can rewrite (23) as

$$\begin{bmatrix} \lambda_{1}^{i} \mathbf{h}_{1} - \mathbf{F}_{11} & -\mathbf{F}_{12} \\ -\mathbf{F}_{21} & \lambda_{1}^{i} \mathbf{I}_{n-1} - \mathbf{F}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{s}^{1} \\ \mathbf{s}^{1} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{1} \\ \mathbf{h}_{1} \end{bmatrix} \mathbf{K}_{1} \mathbf{\Omega}_{1} \begin{bmatrix} \mathbf{s}^{1} \\ \mathbf{g}^{1} \end{bmatrix}$$
(24)

where

$$E_j = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} = \Omega_j^{-1} E_j \Omega_j$$

The first matrix equation in the partitioned form in (24) can be written as



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$$\begin{pmatrix} \lambda_j^t I_l - F_{11} \end{pmatrix} s^t - F_{22} g^t = K_j \Omega_j \begin{bmatrix} s^t \\ g^t \end{bmatrix},$$

$$t = 1 \dots n_t t = 2 \dots N$$

$$(25)$$

Further, by letting  $\mathbf{R} = [\mathbf{R}_{11} \ \mathbf{R}_{12}]$ , (25) becomes  $[\mathbf{R}_1 + \mathbf{N}_1 \mathbf{\Omega}_1] \mathbf{\mathcal{O}}_1^{\mathsf{f}} = \lambda_1^{\mathsf{f}} \mathbf{s}_1^{\mathsf{f}}, t = 1...n_t t = 2...N$  (26) or, in a compact form

$$\left[ \mathbf{A}_{i} + \mathbf{K}_{j} \mathbf{\Omega}_{j} \right] \mathbf{V}_{j} = \mathbf{S}_{j} \tag{27}$$

where,  $\vec{R} = [\vec{v}_1^{\dagger} \cdot \vec{v}_2^{\dagger} \dots \cdot \vec{v}_l^{th}] \in \mathbb{R}^{n \times n}$ 

and 
$$S_t = [\lambda_1 s^{\perp} \lambda_1^{\dagger} s^{\perp} \dots \lambda_1^{\eta} s^{\eta}] \in \mathbb{R}^{l \times n}$$

It should be noted that  $\overline{V_j}$  and  $S_j$  are often complex. To alleviate the need for complex arithmetic, a transformation is needed to transform  $\overline{V_j}$  and  $S_j$  to real matrices. Assume that  $\lambda_j^1 = (\lambda_j^{i+1})^n$  and  $\overline{V_j} = (\overline{V_j}^{i+1})^n$  and assuming all remaining eigen values are real, we can define a transformation matrix

$$\varphi_{f} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0.5 & -0.5f & 0 \\ 0 & 0.5 & 0.5f & 0 \\ 0 & 0 & 0 & i \end{bmatrix} \in \mathbb{R}^{n \times n}$$
(28)

Multiplying both sides of (27) by the transformation matrix  $\varphi_i$ , i.e.,

$$[\mathbf{R}_{i} + \mathbf{K}_{j}\mathbf{\Omega}_{j}]\mathbf{F}_{j}\boldsymbol{\varphi}_{j} = \mathbf{S}_{j}\boldsymbol{\varphi}_{j}$$
(29)

will transform  $V_j$  and  $S_j$  to real matrices and at the same time not alter the calculation of the feedback gain matrix. Note that for the case of more than one pair of selfconjugate eigen values, the corresponding rows and columns in the  $\varphi_j$  matrix need to be assigned by the transformation matrix

$$\varphi_0 = \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

in place of unity matrix **I**.

From (27) the desired feedback gain matrix can be calculated as

$$K_{f} = \left(S_{f} - \overline{F}_{k} \overline{F}_{f}\right) \left(\Omega_{f} \overline{F}_{f}\right)^{-1}, \quad f = 2 \dots N$$
(30)

or from (26), the desired feedback gain matrix for the case of complex eigen values can be obtained as

$$K_{j} = \left(S_{j}\varphi_{j} - F_{i}\varphi_{j}\right)\left(\Omega_{i}\varphi_{j}\right)^{-1}, \ j = 2...M$$
(31)

## Feedforward controller design

Since the feedback control can only guarantee the dynamic behaviour of the system, a reconfigurable feed forward controller is needed to achieve steady-state tracking of the reference input. For this reason, a feed forward control law based on a command input strategy is developed. The basic principle of the command input strategy is to make the system outputs track the command inputs via proper design of a feed forward controller based on the model-following principle.

The multiple-model reconfigurable control gains can be determined as follows

$$K_{f}^{p^{e_{1}}} = S_{f}^{p_{1}} + K_{f}^{p} S_{f}^{p_{1}}$$

$$K_{f}^{p^{e}} = S_{f}^{p_{2}} + K_{f}^{p} S_{f}^{p_{2}}$$
(32)

where the control gains  $K_i^{(m)}$  and  $K_i^{(m)}$  are functions of the feedback control gains  $K_i^{(m)}$ , and the constant gain matrices  $S_i^{(m)}$ ,  $k_i = 1, 2, j = 2 \dots N$  are calculated by

$$S_{f}^{ii} = \varphi_{f}^{ii} S_{f}^{ii} (F_{f}^{m} - I) + \varphi_{f}^{ii} H_{f}^{m}$$
 (33)

$$B_{i}^{p1} = \varphi_{i}^{p1} B_{i}^{p1} (\mathbf{F}_{i}^{m} - \mathbf{I}) + \varphi_{i}^{p2} \mathbf{H}_{i}^{m}$$
 (35)

$$\mathbf{S}_{\mathbf{j}}^{\mathsf{p}\mathsf{s}} = \boldsymbol{\varphi}_{\mathbf{j}}^{\mathsf{p}\mathsf{s}} \mathbf{S}_{\mathbf{j}}^{\mathsf{s}\mathsf{s}} \mathbf{G}_{\mathbf{j}}^{\mathsf{p}\mathsf{s}} \tag{36}$$

and gain matrices  $\varphi_{1}^{[4]}$ ,  $k_{1} = 1,2$  are given by  $\varphi_{1} = \begin{bmatrix} \varphi_{1}^{[4]} & \varphi_{1}^{[4]} \end{bmatrix} = \begin{bmatrix} \varphi_{1}^{[4]} & \varphi_{1}^{[4]} \end{bmatrix} = \begin{bmatrix} \varphi_{1}^{[4]} & \varphi_{1}^{[4]} \end{bmatrix}$ 

During the system operation, the most appropriate controller will automatically be selected based on the decision of the FDD scheme. Thus, based on the system models and the multiple reference models, the overall control signal is given as

$$u_{1}(\mathbf{k}) = -\mathbf{K}_{1}^{x} x_{1}(\mathbf{k}) + \mathbf{K}_{2}^{xm} x_{1}^{m}(\mathbf{k}) + \mathbf{K}_{1}^{x} v_{1}(\mathbf{k})$$
(37)

where,  $\mathbf{K}_{j}^{x}$  – Stabilising feedback controller,

 $K_j^{x^m}$  – Feed forward gain for reference model

**K** – Feedforward gain for command input

#### Illustrative example

#### Aircraft model

The linearized model of the aircraft under the normal condition can be described as  $k(t) = A_{t}(t) + B_{t}(t)$ 

$$\mathbf{z}(\mathbf{t}) = \mathbf{C}\mathbf{x}(\mathbf{t}) \tag{38}$$

where the state and the input vectors are  $\mathbf{x} = \begin{bmatrix} \mathbf{p} & \mathbf{p} \end{bmatrix}^{\mathbf{r}}$  and  $\mathbf{u} = \begin{bmatrix} \mathbf{a}_{\mathbf{a}} & \mathbf{b}_{\mathbf{r}} \end{bmatrix}^{\mathbf{r}}$ , respectively, with  $\mathbf{p}$  representing the roll rate,  $\mathbf{r}$  the yaw rate,  $\boldsymbol{\beta}$  the side slip angle,  $\boldsymbol{\varphi}$  the bank angle,  $\boldsymbol{\delta}_{\mathbf{a}}$  the aileron deflection, and  $\boldsymbol{\delta}_{\mathbf{r}}$  the rudder deflection [9, 10].



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Here only two out of four state variables sideslip and bank angle, are measurable. For simplicity, these two variables will be designated as the controlled variables. Hence the output matrices,  $C_1$  become

į = 1, ...N

The specific faults are: 1) a system dynamic fault as a result of a partial loss of the rudder control surface, 2) a fault in either one of the two actuators, and 3) a fault in sideslip angle sensor. Therefore, there are total of 5 possible operating modes. In practice, if additional fault scenarios or the same fault type but with different severities need to be considered, more fault modes would have to be included in the model set.

## SIMULATION RESULTS

1.

G = [0

0 0

The actuator faults result in reduced values in the corresponding columns of the control matrix B, the sensor fault is represented also by a reduction in the corresponding row of the measurement matrix C and the loss of control surface is reflected as the changes in both A and B matrices.

The response of the normal system with controller is shown in Figure-2.



Figure-2. Normal mode output.

When fault occurs in the system, the controller should be reconfigured such that it produces acceptable performance. The system output with controller settles faster (at t=5 sec) and the oscillations are greatly reduced when compared to the system output without controller.

The response of the system with different fault conditions are given below. The response of the system with dynamic fault introduced at t=5 sec is shown in Fig 3. The dynamic fault is represented as 50% loss in effectiveness of rudder. Before the occurrence of the fault, the two system outputs have followed the desired reference trajectories specified by the desired reference model. Thus, when fault is introduced, the system initially tends to oscillate. Once the fault is detected, the system output starts to follow the revised reference trajectories

governed by the acceptable performance reduced reference model at the level of  $[4] \oplus [2,73] = [2,73] = [2,73] = [2,73]$ . The reconfigured system settles at t = 30 sec.



Figure-3. System output with abrupt Dynamic fault.

Next, the faults in actuators are shown. The faults in actuators usually represent the loss of effectiveness in rudder or aileron.

The response of the system with aileron fault introduced at t= 5 sec is shown in Figure-4. The revised reference trajectories are governed by the acceptable performance reduced reference model at the level of [9, 9] = [0.3] []. Initially the amplitude of oscillation is high but soon the system recovers and settles at t=30 sec. The reconfigured system follows the command input given.



Figure-4. System output with continuous aileron fault.



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Next, the response of the system with rudder fault introduced at t = 5 sec is shown in Figure-5. The revised reference trajectories are governed by the acceptable performance reduced reference model at the level of  $[p \ \varphi] - [0.01]$ . The amplitude of oscillation is high in both outputs. The sideslip angle output settles at t=20 sec and the bank angle output settles at t=50 sec. The reconfigured system follows the command input given for the particular fault mode and the acceptable performance is achieved.



Figure-5. System output with rudder fault introduced at t=5 sec.

From this, it can see that the dynamic fault and actuator fault are predominant and affects the system more. This fault has to be taken care before it results in catastrophic results.

## CONCLUSIONS

In this paper, a fault tolerant control using multiple model technique and with acceptable performance degradation due to faults in actuators, sensors and system dynamics is designed. The fault detection and diagnosis is done using IMM estimator and the controller reconfiguration is done using eigen structure assignment. In this paper, different types of faults are considered. Thus for each fault, a particular model is considered and the fault tolerant control is designed. Simulation results have demonstrated the effectiveness of the proposed methodology using an aircraft example and shown that if the performance limitations had not been considered, actuator saturation would have occurred.

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