



# DETERMINATION OF THE PROJECTIONS OF THE VELOCITY VECTOR OF THE COOLED AIR FROM THE REFRIGERATING COMPARTMENT WITH THE DOOR OPEN

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## ABSTRACT

The article considers the question of movement of cooled air by opening the cell doors of a domestic refrigerator. The object of research is the characteristics of the flow of cooling air, which include the velocity vector of the flow. The actual flow of air flowing out of the refrigerator compartment is three-dimensional, ie, it is characterized at each point in the flow values of the longitudinal, transverse and vertical projections of the velocity, pressure, temperature, viscosity, specific heat, thermal conductivity. To solve this problem, we used data of field tests, which led to some simplifications in the mathematical description of the process of movement of cooled air without compromising reliability and objectivity of the obtained patterns. The studies were obtained according to determination the projections of the velocity vector of the flow of cool air coming out of the refrigerating compartment of the refrigerator when the door is open at different temperatures within the chambers and the surrounding air, which allows to estimate the heat loss associated with the replacement of the cooled air warmer when you open the cell doors of domestic refrigerator.

**Keywords:** household refrigerator, the movement of cooled air, heat and mass transfer, energy.

## 1. INTRODUCTION

Improvement of household refrigeration appliances include tasks to increase their performance in the operation of thermal power. Consumption compression refrigerator, along with other operational factors, depends on the heat exchange and mass transfer processes internal chambers with the environment.

When you open the cell doors of the refrigerator, as it is known, are replaced by warm ambient air cooled air in the chambers of refrigeration and freezer compartments.

This substitution causes the cost of running a sealed unit of the refrigerator and its power consumption. Assess the extent of this substitution, we can study the process of heat and mass transfer of air in the chambers of the refrigerator with the ambient air. For this it is necessary to identify the characteristics of the flow of air flowing out of the open door of the refrigerator chamber and examine the changes in the characteristics of the flow in time. After studying the conditions and characteristics of the formation of the flow under consideration, it is possible to determine the heat loss caused by opening the door for a variety of design solutions cabinet of household refrigerator and develop recommendations to reduce them.

The object of research is the flow of cooled air flowing out of the chamber household refrigeration appliance when opening the door of a household refrigerator. The actual flow of air flowing out of the refrigerator compartment is three-dimensional, ie, it is characterized at each point in the flow values of the longitudinal, transverse and vertical projections of the velocity, pressure, temperature, viscosity, specific heat, thermal conductivity.

## 2.1. ANALYSIS OF THE STATIONARY MODE OF AIR MOVEMENT

Consider the characteristics of the cooled air during its free outlet from the chamber of the refrigerator. The cold air flows freely out of the refrigerator cabinet by gravity, displacing warmer air due to its higher density.

In stationary mode at the outlet of the flow velocity vector of the cabinet has a longitudinal (horizontal) component of the "u", and the vertical component of the "w" is equal to zero. Initial velocity component may be determined experimentally, and thus its output value will be given. Not given the resistance bottom cooling chamber air flow, it can be assumed that the output of the cabinet flow is evenly:

$$u(z;0) = u_0, \quad (1)$$

The value of the vertical projection of the velocity vector at the output of the refrigerator compartment will be equal to zero:

$$w(z;0) = 0, \quad (2)$$

Here the acceleration value of the outlet flow of the refrigerating compartment is equal to:

$$\frac{dw(z;0)}{dz} = g, \quad (3)$$

where g – acceleration of gravity force.

Emerging from the cabinet, the flow begins to decelerate in the direction of the OX-axis due to the viscosity of the medium, ie, the longitudinal velocity component has the following properties:

$$u(x;z) \rightarrow 0, \text{ when } x \rightarrow \infty \text{ and} \\ \text{And } u(x;z) \rightarrow 0 \text{ when } z \rightarrow \infty \quad (4)$$



Moreover, the function  $u(x, z)$  must be monotonically decreasing as the variable  $x$  and the variable  $z$ .

The vertical velocity component of flow at the outlet of the enclosure will initially increase due to the force of gravity acting on the flow which occurs due to its higher density (due to a low temperature  $T_x = 5^\circ C$ ) than the ambient air temperature  $T_k = 25^\circ C$ .

However, with an increase in the vertical component, " $w$ " increases and the resistance force of a viscous medium, respectively, after a while gravity force of the air flow directed vertically downwards is balanced by the resistance of the medium, respectively, the vertical component of the flow  $w$  tends to some constant value which can be determined experimentally:

$$w(x; z) \rightarrow 0 \text{ when } z \rightarrow \infty \quad (5)$$

In this case, the function  $w(x; z)$  must be monotonically increasing with respect to  $z$  and monotonically decreasing the variable  $x$ .

### 3. BASIC EQUATION FOR DETERMINING THE FLOW RATE PROJECTIONS AND ITS SOLUTION

To further solve the problem, use the basic equation describing the motion of the air flow, resulting in [1], [2], [3]:

$$\begin{cases} \frac{\partial u}{\partial x} u + \frac{\partial u}{\partial z} w = -\frac{\mu \cdot k}{\rho} u^2 \\ \frac{\partial w}{\partial x} u + \frac{\partial w}{\partial z} w = m \cdot g - \frac{\mu \cdot k}{\rho} w \end{cases}, \quad (6)$$

where:  $\mu$  – coefficient of dynamic viscosity;  $m$  – molecular mass of the gas;  $k$ – coefficient characterizing the shape of the flow determined experimentally;  $\rho$  – the density of the gas flow;  $u$  – horizontal component of the velocity;  $w$ – vertical velocity component.

System (6) is the original system in order to find the flow rates at the end of its free from the refrigerator compartment in the steady state.

$$\begin{cases} u(x; z) = u_0(z) \cdot e^{-x} \\ w(x; z) = w(z) + w_0(z) \cdot e^{-x} \end{cases} \quad (7)$$

Finding solutions for future system (6) can be conveniently converted to the form

$$\begin{cases} \frac{\partial u}{\partial x} u + \frac{\partial u}{\partial z} w + \frac{\mu \cdot k}{\rho} u^2 = 0; \\ \frac{\partial w}{\partial x} u + \frac{\partial w}{\partial z} w - m \cdot g + \frac{\mu \cdot k}{\rho} w^2 = 0 \end{cases}. \quad (8)$$

Substitute (7) into (8), we find the necessary derivatives and group terms in the left-hand side of each equation in powers  $e^{-lx}$  for  $l=0,1,2$ .

After simplification we get:

$$\begin{cases} \left( \frac{\partial u_0}{\partial z} w_1 + \left( \frac{\mu \cdot k}{\rho} - 1 \right) \cdot u_0^2 \right) \cdot e^{-2x} + \\ + \frac{\partial u_0}{\partial z} w_0 e^{-x} = 0; \\ \left( \frac{\partial w_1}{\partial z} w_1 - w_1 u_0 + \frac{\mu \cdot k}{\rho} - w_1^2 \right) \cdot e^{-2x} + \\ + \left( \frac{\partial w_0}{\partial z} w_1 + \frac{\partial w_1}{\partial z} w_0 + \frac{2\mu \cdot k}{\rho} w_1 w_0 \right) \cdot e^{-x} - \\ - m \cdot g + \frac{\partial w_0}{\partial z} w_0 + \frac{\mu \cdot k}{\rho} w_0^2 = 0 \end{cases} \quad (9)$$

Functions  $u_0(z), w_0(z), w_1(z)$  will be determined from the conditions that every factor at for  $l=0,1,2$  in the equations of the system (9) was either zero, or a value close to zero, respectively, the two equations of the system (9) will be approximately satisfied.

Equate to zero first expression:

$$\frac{\partial w_0}{\partial z} w_0 + \frac{\mu \cdot k}{\rho} w_0^2 - m \cdot g = 0 \quad (10)$$

Equation (10) is an ordinary differential equation of the first order. In equation (10) according to [4], we make the change:

$$\xi(z) = w_0^2(z). \quad (11)$$

After the substitution (11), we obtain:

$$\frac{\partial \xi(z)}{\partial z} - \frac{2\mu \cdot k}{\rho} \xi(z) - 2m \cdot g = 0 \quad (12)$$

Equation (12) is a linear first order, the general solution is given by [3]:

$$\xi(z) = e^{\int \frac{2\mu k}{\rho} dz} \left( \int 2m g e^{-\int \frac{2\mu k}{\rho} dz} dz + c_1 \right). \quad (13)$$

We find the integrals on the right side of (13) and by substituting (11) will go back to the function  $w_0(z)$ , we obtain after simplifications:

$$w_0(z) = \sqrt{\frac{mg\rho}{\mu k} + c_1 e^{-\frac{2\mu k z}{\rho}}}. \quad (14)$$

Next, equating to zero the coefficient of  $e^{-x}$  in the second equation of system (9), we obtain:

$$\begin{aligned} & \frac{\partial w_1}{\partial z} w_0(z) + \\ & + \left( \frac{\partial w_0(z)}{\partial z} w_1 + \frac{2\mu \cdot k}{\rho} w_0(z) \right) w_1(z) = 0 \end{aligned} \quad (15)$$

Since the function  $w_0(z)$  is now known, equation (15) is an equation with separable variables for the unknown function  $w_1(z)$ . The general solution of equation (15) has the form:

$$w_1(z) = c_2 e^{-\int \left( \frac{dw_0(z)}{w_0(z) dz} + \frac{2\mu k}{\rho} \right) dz}. \quad (16)$$



Simplifying the function (16) and substituting known from (14) the expression for  $w_0(z)$ , we find the general solution for  $w_1(z)$ :

$$w_1(z) = \frac{c_2 e^{\frac{2\mu kz}{\rho}}}{\sqrt{mg\rho + c_1 \mu k e^{\frac{2\mu kz}{\rho}}}} \quad (17)$$

We now take up find the function  $u_0(z)$ . If we equate to zero the coefficient of  $ex$  in the first equation of system (9), we obtain the equation:

$$\frac{du_0(z)}{dz} w_0(z) = 0, \quad (18)$$

which has an overall solution in the form:

$$u_0(z) = c_3 \quad (19)$$

However, solutions of the form (19) are from outside your problem free discharge of cold air flow as the horizontal projection of the velocity vector will decrease with increasing values as  $x > 0$ , and with increasing values of  $z > 0$ . Thus, we define a function  $u_0(z)$  from the condition that the coefficient of  $e^{2x}$  in the first equation of (9):

$$\frac{\partial u_0(z)}{\partial z} w_1(z) + \left(\frac{\mu \cdot k}{\rho} - 1\right) \cdot u_0^2(z) = 0 \quad (20)$$

Equation (20) is an equation with separable variables for the unknown function  $u_0(z)$ , its general solution has the form:

$$u_0(z) = \frac{1}{c_3 + \int_0^z \frac{\mu k - \rho}{w_1(t)} dt} \quad (21)$$

Substituting the right-hand side of (21) is known from (17) the expression for the function  $w_1(z)$ , we obtain:

$$u_0(z) = \frac{1}{c_3 + \frac{\mu k - \rho}{2\rho c_2} \int_0^z \sqrt{mg\rho + c_1 \mu k e^{\frac{2\mu kt}{\rho}}} e^{\frac{2\mu kt}{\rho}} dt} \quad (22)$$

Given the formula (7) and (22), we obtain the expression of the model function  $u(x; z)$ :

$$u_0(z) = \frac{e^{-x}}{c_3 + \frac{\mu k - \rho}{2\rho c_2} \int_0^z \sqrt{mg\rho + c_1 \mu k e^{\frac{2\mu kt}{\rho}}} e^{\frac{2\mu kt}{\rho}} dt} \quad (23)$$

The expression for the model function  $w(x; z)$ : accordingly follows from (10), (14), (17):

$$w(x; z) = \sqrt{\frac{mg\rho}{\mu k}} + c_1 e^{\frac{2\mu kz}{\rho}} + \frac{c_1 e^{\frac{2\mu kz}{\rho}} e^{-x}}{\sqrt{mg\rho + c_1 \mu k e^{\frac{2\mu kt}{\rho}}}} \quad (24)$$

Elucidate the behavior of function  $u(x; z)$ : and  $w(x; z)$  when  $x > 0$  and  $z > 0$ . It is not difficult to see that

$$\frac{\partial u(x; z)}{\partial x} < 0,$$

therefore  $u(x; z)$  decreases over  $x$ . We single out the function  $\varphi(z)$ :

$$\varphi(z) = c_3 + \frac{\mu k - \rho}{2\rho c_2} \int_0^z \sqrt{mg\rho + c_1 \mu k e^{\frac{2\mu kt}{\rho}}} e^{\frac{2\mu kt}{\rho}} dt \quad (25)$$

This function is located in the denominator of the formula (23) for  $u(x; z)$ , and is monotonically increasing in  $z$  (since the integrand in the representation  $\varphi(z)$  is positive), respectively, the function  $u(x; z)$ , decreases monotonically with respect to  $z$ .

It is also easy to see that the derivative of the function  $w(x; z)$  with respect to  $x$  is negative, that is,  $w(x; z)$  is monotonically decreasing in  $x$ .

#### 4. FINDING THE ARBITRARY CONSTANTS

To obtain the final solution of the problem we find the values of the arbitrary constants  $c_1, c_2, c_3$ . To determine the constant  $c_3$  use the equation (23), in which we substitute in both sides of  $x = 0$  and  $z = 0$ . We obtain the equation:

$$u(0, 0) = u_0 = \frac{1}{c_3 + 0},$$

hence:

$$c_3 = \frac{1}{u_0} \quad (25)$$

Assuming in equation (24) the values of variables  $x=0$  and  $z=0$  obtain:

$$0 = \sqrt{\frac{mg\rho}{\mu k}} + c_1 + \frac{c_2}{\sqrt{mg\rho + c_1 \mu k}} \quad (26)$$

From equation (26) we express the constant  $c_2$  by  $c_1$ :

$$c_2 = -\frac{mg\rho + c_1 \mu k}{\sqrt{\mu k}} \quad (27)$$

To find the arbitrary constant  $c_1$  differentiate the function (24) in the variable  $z$ . We first find the derivative of the function  $w_0(z)$ :



$$\begin{aligned} \frac{dw_0(z)}{dz} &= \\ &= \frac{d}{dz} \left( \sqrt{\frac{mg\rho}{\mu k} + c_1 e^{\frac{2\mu kz}{\rho}}} \right) = \quad (28) \\ &= \frac{-c_1 \mu^2 k^2 e^{\frac{2\mu kz}{\rho}}}{\rho \sqrt{\mu k} \sqrt{mg\rho + c_1 \mu k e^{\frac{2\mu kz}{\rho}}}} \end{aligned}$$

Derivative of the function  $w_1(z)$  is found by using logarithmic differentiation:

$$\begin{aligned} \ln \frac{c_2 e^{\frac{2\mu kz}{\rho}}}{\sqrt{mg\rho + c_1 \mu k e^{\frac{2\mu kz}{\rho}}}} &= \\ &= \ln c_2 - \frac{2\mu kz}{\rho} - \quad (29) \\ & - \frac{1}{2} \ln \left( mg\rho + c_1 \mu k e^{\frac{2\mu kz}{\rho}} \right) \end{aligned}$$

Then the derivative of the function  $w_1(z)$  is:

$$\begin{aligned} \frac{dw_1(z)}{dz} &= \\ & \left( -\frac{2\mu k}{\rho} + c_1 \frac{\mu^2 k^2}{\rho \left( mg\rho + c_1 \mu k e^{\frac{2\mu kz}{\rho}} \right)} \right) \times \quad (30) \\ & \times \frac{c_2 e^{\frac{2\mu kz}{\rho}}}{\sqrt{mg\rho + c_1 \mu k e^{\frac{2\mu kz}{\rho}}}} \end{aligned}$$

Considering the formula (28) and (30) we find the derivative of  $w(x; z)$ :

$$\begin{aligned} \frac{dw}{dz} &= \frac{-c_2 e^{\frac{2\mu kz}{\rho}}}{\rho \sqrt{\mu k} \sqrt{mg\rho + c_1 \mu k e^{\frac{2\mu kz}{\rho}}}} + \\ & \left( -2 + \frac{c_2 e^{\frac{2\mu kz}{\rho}}}{mg\rho + c_1 \mu k e^{\frac{2\mu kz}{\rho}}} \right) \times \quad (31) \\ & \times \left( \frac{\mu k c_2 e^{\frac{2\mu kz}{\rho}} e^{-x}}{\rho \sqrt{mg\rho + c_1 \mu k e^{\frac{2\mu kz}{\rho}}}} \right) \end{aligned}$$

Assuming in equation (31) the values of variables  $x = 0$  and  $z = 0$ , we obtain:

$$\begin{aligned} g &= \frac{-c_1 \mu^2 k^2}{\rho \sqrt{\mu k} \sqrt{mg\rho + c_1 \mu k}} + \\ & \left( -2 + \frac{c_1 \mu k}{mg\rho + c_1 \mu k} \right) \times \quad (32) \\ & \times \left( \frac{\mu k c_2}{\rho \sqrt{mg\rho + c_1 \mu k}} \right) \end{aligned}$$

We make in the equation (32) the change:

$$\sqrt{mg\rho + c_1 \mu k} = \theta. \quad (33)$$

In the context of the change (33), taking into account formula (27), expressed in terms of constant  $\theta$ ,  $c_1 \mu k$  and  $c_2$ :

$$c_1 \mu k = \theta^2 - mg\rho; \quad c_2 = \frac{\theta^2}{\sqrt{\mu k}}. \quad (34)$$

Substituting the representation (34) into equation (32), we obtain:

$$\begin{aligned} g &= \frac{-\mu k (\theta^2 - mg\rho)}{\rho \theta \sqrt{\mu k}} - \\ & - \left( -2 + \frac{\theta^2 - mg\rho}{\theta^2} \right) \frac{\mu k \theta^2}{\rho \theta \sqrt{\mu k}} \quad (35) \end{aligned}$$

It is not difficult to see that, after simplifications, equation (35) takes the form:

$$\frac{\rho g \theta}{\sqrt{\mu k}} = 2mg\rho.$$

hence

$$\theta = 2m \sqrt{\mu k}. \quad (36)$$

In view of (36) from (34) we find:

$$c_1 = 4m^2 - \frac{mg\rho}{\mu k}; \quad c_2 = -4m^2 \sqrt{\mu k}. \quad (37)$$

Thus, the projection of the velocity vector are given by (23), (24) where the constants  $c_1, c_2, c_3$  are given by (25) and (37).

The problem is solved. The resulting equations allow us to calculate the projection of the velocity vector of the flow of cooled air exiting the cooling chamber of the refrigerator when the door is open.

Flowing air mass  $m$  is proportional to the density of the air temperature  $p$ . when the air temperature  $T=5^\circ$ , the density  $\rho = 1,27 \frac{\text{kg}}{\text{m}^3}$ , when  $T=25^\circ$ , the density

$$\rho = 1,185 \frac{\text{kg}}{\text{m}^3} \text{ therefore proportional to the mass } m$$

$$\Delta\rho = 0,085 \frac{\text{kg}}{\text{m}^3}$$

The value of  $U_0$  (initial flow rate), and the coefficient  $k$  (coefficient characterizing the specific



consumption in the section of the flow at the initial time) are determined on the basis of experimental data.

### DISCUSSIONS AND CONCLUSIONS

The investigations have allowed the authors to substantiate and develop recommendations for the design of household refrigerators, in terms of reduction of heat influxes on doors of the cooler:

1. Necessary to ensure a decrease in the time spent in the refrigerator door open.
2. Necessary to achieve a reduction in the flow of cold air, replaced by warm with the door open the refrigerator chamber.

The first area provides, for example, constructive implementation of the refrigerator shelves round and rotated [5]. This allows relatively quick to sort and extract the product from the refrigerator. Also invited to the doorway of the cooler after the main door of the refrigerator is covered with a screen, with a window to allow access only to the desired shelf, it contributes to our range of products to their extraction process, without loss of cooled air [6]. To speed up the search for the right product, and thereby to reduce the time spent by the door in the open position, developed Chest freezers, with round-robin mechanism for moving the food baskets [7]. For refrigerators with a large amount of their partial filling of the products it is advisable to apply the technology changes in the volume of cameras [8].

Styling or product recovery in one of the offices (on a separate shelf, other compartments or racks separated flaps that prevent the movement of air [9] of the inter-shelf space.

Reducing the amount of substitution of cooled air is also achieved by the use of containers for filling it with cold air [10].

In such a refrigerator, on the rear side of the cooler installed flexible container for storage of cold air.

3. Obtained according to the definition of the projections of the velocity vector of the flow of cool air coming out of the refrigerator compartment of the refrigerator when the door is open.

4. Results of the study to determine the characteristics of the flow of cooling air and to evaluate the thermal losses associated with the replacement of the cooled air is warmer when you open the doors of the chambers of a domestic refrigerator.

5. Influence on the design features, the dependencies of the different models of domestic refrigerators accounted for on the basis of simple experiments, and can be further described models.

### REFERENCES

[1] Mathematical Model of Refrigerated Air from the Fridge Speed Calculation. 2014. M.A. Lemeshko,

E.V.Duvanskaya, S.P. Petrosov, V.N.Kohanenko. World Appl.Sci., 30 (9): 1145-1151.

- [2] Lemeshko M.A., Laletin V.I., Mitsik M.F. 2011. Definition speeds cool air when you open the cabinet doors household refrigeration appliance. Engineering Gazette Don, №4. <http://www.ivdon.ru/magazine/archive/n4y2011/604/>.
- [3] Lemeshko M.A., Mitsik M.F. 2013. A mathematical model of free discharge cooled air from the chamber of domestic refrigeration appliance. News of higher educational institutions. North-Caucasian region. Series: Engineering. №4. 16-19.
- [4] Korn, G. 1970. Mathematical Handbook for Scientists and Engineers / G. Korn, T. Korn. – M.: Science. 720 pp.
- [5] Household refrigerator (options). Patent RU № 2418248, 10.05.2011.
- [6] Household refrigerator. Patent RU № 2402727, 27.10.2010.
- [7] Chest Freezer. Patent RU № 2402721, 27.10.2010.
- [8] Household refrigerator. Patent RU № 2234036, 10.08.2004.
- [9] Household electric refrigerator (options). Patent RU №2152569, 10.07.2000.
- [10] Household refrigerator. Patent RU № 2416060, 10.04.2011.