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# NUMERICAL STUDY ON ECOLOGICAL COMMENSALISM BETWEEN TWO SPECIES WITH HARVESTED COMMENSAL

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**ABSTRACT**

The present paper deals with the numerical study on the ecological model comprising the commensal species and the host species with a constant harvesting of the commensal species. Further, both the species are considered with the limited resources. The corresponding trajectories of the commensal species (growing, balanced and mortal commensal species) and the host species have been illustrated for wide range of the values of the parameters in the model. The dominance reversal time of the host species over the commensal species or vice versa and the sustainability of the commensalism interaction between the species is also discussed.

**Keywords:** commensalism interaction, commensal species, host species, numerical solutions, trajectories, dominance reversal time.

## 1. INTRODUCTION

There are different kinds of interactions between species like Mutualism, Neutralism, Ammensalism, Commensalism, Prey-Predators and Competition etc. In mathematical ecology the literature including continuous and discrete models plays very important role and then several works have been devoted to investigate these models regarding periodicity, global stability boundedness and other features. Mathematical modeling of ecosystems was initiated by Lotka [9] and Volterra [15]. The general concepts of modeling have been presented in the treatises of Meyer [10], Cushing [6], Paul Colinvaux [11], Freedman [7], Kapur [8] and several others. Phanikumar, Seshagiri Rao and Pattabhi Ramacharyulu [12] studied on the stability of a Host- A flourishing Commensal species pair with limited resources. Later Seshagiri Rao, Phanikumar and Pattabhi Ramacharyulu [13] investigated on the stability of a Host- A declining Commensal species pair with limited resources. The stability analysis of host-Mortal commensal ecological model with host harvesting at a constant rate is carried out by Seshagiri Rao, Kalyani and Pattabhi Ramacharyulu [14]. Acharyulu[1-5] invented various Ammensal Ecological Models.

This paper presents the numerical solutions of the ecological commensalism between two species with the help of classical Runge-Kutta method. The model comprises a Host-Mortal Commensal species pair with limited resources in nature where as the commensal species is harvested at constant rate. This model is characterized by a couple of first order non linear ordinary differential equations. The trajectories of this model by changing the growth, balanced and mortal coefficients of the commensal species over the host species fixing the remaining parameters constants are drawn and the conclusions are given here under. The dominance reversal time is found in all possible cases.

**Nomenclature**

$N_1(t)$ = The population of the commensal $(S_1)$ at time t.

$N_2(t)$ = The population of the host $(S_2)$ at time t.

$d_1$ = The mortal rate of the commensal $(S_1)$.

$a_2$ = The rate of natural growth of the host $(S_2)$.

$a_{11}$ = The rate of decrease of the commensal $(S_1)$ due to the limitations of its natural resources.

$a_{22}$ = The rate of decrease of the host $(S_2)$ due to the limitations of its natural resources.

$a_{12}$ = The rate of increase of the commensal $(S_1)$ due to the support given by the host $(S_2)$.

$k_2 (= a_2 / a_{22})$ = The carrying capacity of $S_2$.

$c (= a_{12} / a_{11})$ = The coefficient of the commensal.

$e_1 (= d_1 / a_{11})$ = The mortality coefficient of $S_1$.

$h_1 (= a_{11} H_1)$ = The coefficient of harvesting of the $S_1$.

$H_1$ = The harvesting/migration of $S_1$ per unit time.

$t^*$ = The dominance reversal time.

$t_{g_1}^*$ = The dominance reversal time of the host over the growing commensal.

$t_0^*$ = The dominance reversal time of the host over the balanced commensal.

$t_{e_1}^*$ = The dominance reversal time of the host over the mortal commensal.

The state variables $N_1(t)$ and $N_2(t)$ as well as all the model parameters $d_1$, $a_2$, $a_{11}$, $a_{12}$, $a_{22}$, $e_1$, c, $k_2$, $h_1$ are assumed to be non-negative constants.

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## 2. BASIC MODEL EQUATIONS

The model equations for a two species commensal-host ecosystem with commensal harvesting at a constant rate employing the above notations are given by the following system of non-linear coupled ordinary differential equations.

(i). Growth rate equation for the Mortal-Commensal species $(S_1)$

$$\frac{dN_1(t)}{dt} = a_{11}\left[-e_1 N_1(t) - N_1^{\,2}(t) + cN_1(t)N_2(t) - H_1\right] \quad (1)$$

(ii). Growth rate equation for the Host species $(S_2)$

$$\frac{dN_2(t)}{dt} = a_{22} N_2(t)\left[k_2 - N_2(t)\right] \quad (2)$$

with the initial conditions

$$N_i(0) = N_{i0} \geq 0, \ \ (i = 1,2) \quad (3)$$

The growth rate equations for **Balanced (i**.e. birth rate of the commensal is equal to its death rate) and **Growing** (i.e. birth rate of the commensal is greater than its death rate) species can be obtained by taking $e_1 = 0$ and $e_1 = -g_1$ in equation (1).

## 3. NUMERICAL SOLUTIONS OF THE GROWTH RATE EQUATIONS OF THE ECOLOGICAL MODEL

Numerical solutions of the non-linear basic differential equations (1) and (2) with the balanced and growing commensal species have been computed in the time interval [0, 10] in steps of one each employing Runge-Kutta system for a wide range of the model characterizing parameter: $H_1$(harvesting rate) for the commensal, keeping the mortal commensal coefficient $e_1 = 0.35$, commensal coefficient $c = 1$, the self inhibition coefficients $a_{11} = 0.1$, $a_{22} = 0.15$ and the host carrying capacity $k_2 = 4$ constants. The software DEDiscover has been used for this purpose and the graphical illustrations of the results obtained are shown in Figures -1 to 6.

**Case-1:**

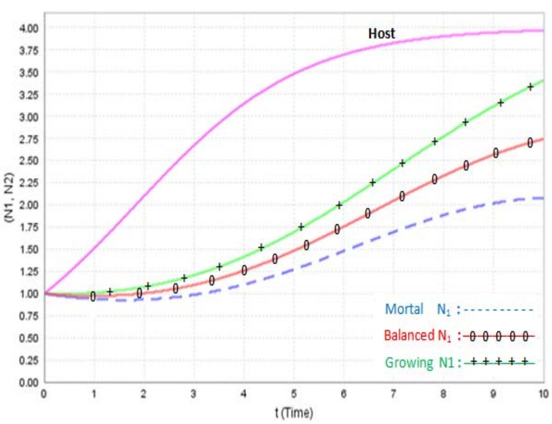


**Figure-1**

Variation of $N_1, N_2$ vs. $t$ for $a_{11} = 0.1, e_1 = 0.35,$
$g_1 = 0.35,\ c = 1,\ H_1 = 0.5,\ a_{22} = 0.15,\ k_2 = 4,$
$N_{10} = 1,\ N_{20} = 1$

In this case the host species always out-numbers the commensal irrespective of the nature of the commensal species. (i.e., growing, balanced and mortal). Also we notice that initially the commensal species have very low growth rate for some time after that they are gradually increasing at slower rate whereas the host species rises and then approaches its asymptotic value. (Vide Figure-1)

**Case-2:**

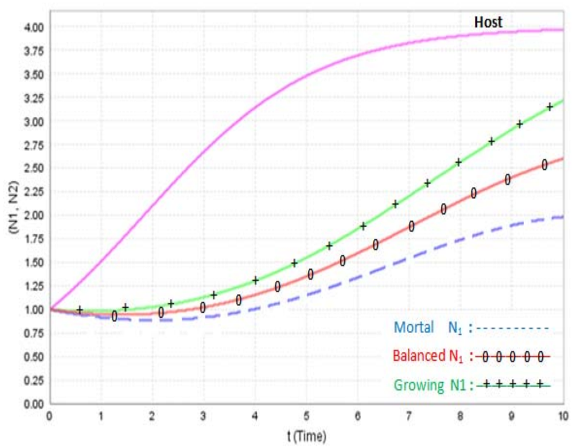


**Figure-2**

Variation of $N_1, N_2$ vs. $t$ for $a_{11} = 0.1, e_1 = 0.35,$
$g_1 = 0.35,\ c = 1,\ H_1 = 0.75,\ a_{22} = 0.15,\ k_2 = 4,$
$N_{10} = 1,\ N_{20} = 1$

**Case-3:**

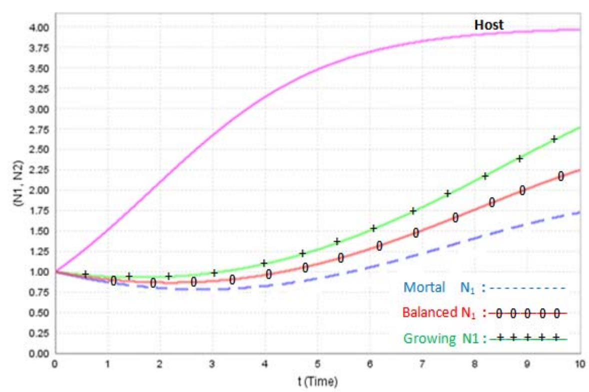


**Figure-3**

Variation of $N_1, N_2$ vs. $t$ for $a_{11} = 0.1, e_1 = 0.35,$
$g_1 = 0.35,\ c = 1, H_1 = 1.2,\ a_{22} = 0.15,\ k_2 = 4,$
$N_{10} = 1,\ N_{20} = 1$

Figures-2 and 3 are the trajectories correspond to small values of the harvesting rates for the commensal species.

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In these two cases 2 and 3, the host dominates the commensal in the following order of dominance, Host > growing commensal > balanced commensal > mortal commensal. (Vide Figures-2 and 3).

**Case-4:**

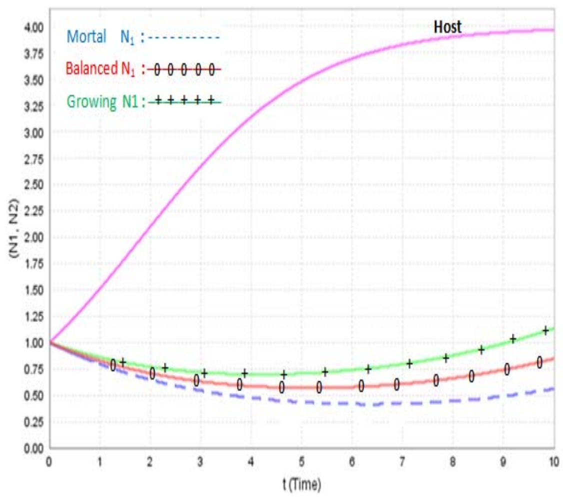


**Figure-4**

Variation of $N_1, N_2$ vs. $t$ for $a_{11} = 0.1, e_1 = 0.35$, $g_1 = 0.35$, $c = 1$, $H_1 = 2$, $a_{22} = 0.15$, $k_2 = 4$, $N_{10} = 1$, $N_{20} = 1$

**Case-5:**

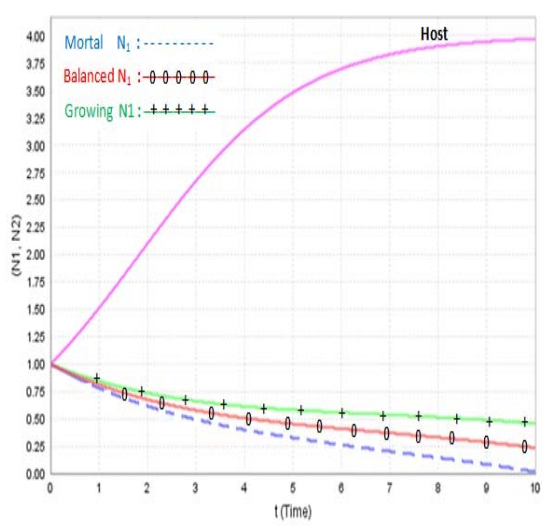


**Figure-5**

Variation of $N_1, N_2$ vs. $t$ for $a_{11} = 0.1, e_1 = 0.35$, $g_1 = 0.35$, $c = 1$, $H_1 = 2.16$, $a_{22} = 0.15$, $k_2 = 4$, $N_{10} = 1$, $N_{20} = 1$

The situation for higher values of the harvesting rates for the commensal species presented in Figures-5 and 6 of cases: 4 and 5. In these cases the host out-numbers the commensal throughout interval irrespective of the nature of the commensal species. Further the host species rises steeply and then reaches its asymptotic value where as the commensal species decreases irrespective of the nature of the commensal species.

**Case-6:**

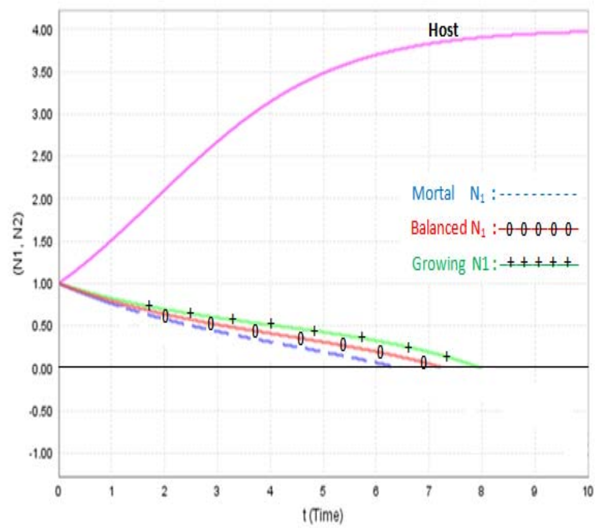


**Figure-6**

Variation of $N_1, N_2$ vs. $t$ for $a_{11} = 0.1, e_1 = 0.35$, $g_1 = 0.35$, $c = 1$, $H_1 = 2.35$, $a_{22} = 0.15$, $k_2 = 4$, $N_{10} = 1$, $N_{20} = 1$

In this case the host species always dominate over the commensal species irrespective of the nature of the commensal. Further we see that the host species rises initially and later maintains a steady variation with no appreciable growth rate where as the commensal decrease and become extinct at times t = 6.46 (mortal commensal), t = 7.302 (balanced commensal) and t = 7.991 (growing commensal) respectively as seen in Figure-6.

## CONCLUSIONS

(i). For higher harvesting rates $H_1$ of the commensal species, the host dominates over the commensal irrespective of the nature of the commensal and in the course of time the commensal decreases gradually and would become extinct.

(ii).There is no dominance reversal time between the host and the commensal species for the higher values $H_1$ of the commensal species for fixing all other parameter values in the basic model equations.

(iii).The commensal species extinct in the order, first the mortal commensal, next the balanced commensal and then the growing commensal for higher harvesting values $H_1$ where as the host species converges to its asymptotic value.

**Open problems**

The following problems are proposed for further investigation in these basic model equations of the ecological commensalism interaction between the host and commensal species.

(i). Situations involving delayed commensalism as they occur are of interest in nature at times.
(ii). Immigration at variable rate of the commensal.
(iii). Immigration of the commensal and migration of the host at (a). constant and (b). variable rates.
(iv). Constant migration and immigration of both the species in the model equations.

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