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MULTIPLE CRACK DETECTION IN BEAMS FROM THE DIFFERENCES IN CURVATURE MODE SHAPES

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ABSTRACT

The presence of crack in a structure tends to modify its modal parameters (natural frequencies and mode shapes). The fact can be used inversely to predict the crack parameters (crack depth and its location) from measurement of the changes in the modal parameters, once a functional relationship between them has been established. The machine components like turbine blade can be treated as a cantilever beam and a shaft as a simply supported beam. Vibration analysis of cantilever beam and simply supported beam can be extended successfully to develop online crack detection methodology in turbine blades and shafts. In the present work, finite element analysis of a cantilever and simply supported beams for flexural vibrations has been considered by including two transverse open U-notches. The modal analysis has been carried out on cantilever and simply supported beams with two U-notches and observed the influence of one U-notch on the other for natural frequencies and mode shapes. This has been done by carrying out parametric studies using ANSYS software to evaluate the natural frequencies and their corresponding mode shapes for different notch parameters (depths and locations) of the cantilever and simply supported beams FEM model. Later, by using a central difference approximation, curvature mode shapes were then calculated from the displacement mode shapes. The location and depth corresponding to any peak on this curve becomes a possible notch location and depth. The identification procedure presented in this study is believed to provide a useful tool for detection of medium size crack in a cantilever and simply supported beam applications.

KEYWORDS: crack detection, vibration, FEM, displacement mode shapes, curvature mode shapes.

INTRODUCTION

Any localized crack in a structure reduces the stiffness in that area. These features are related to variation in the dynamic properties, such as, decreases in natural frequencies and variation of the modes of vibration of the structure. One or more of above characteristics can be used to detect and locate cracks. This property may be used to detect existence of crack or faults together with location and its severity in a structural member. Rizos [1] measured the amplitude at two points and proposed an algorithm to identify the location of crack. Pandey [2] suggested a parameter, namely curvature of the deflected shape of beam instead of change in frequencies to identify the location of crack. Ostachowicz [3] proposed a procedure for identification of a crack based on the measurement of the deflection shape of the beam. Ratcliffe [4] also developed a technique for identifying the location of structural damage in a beam using a 1D FEA. A finite difference approximation called Laplace's differential operator was applied to the mode shapes to identify the location of the damage. Wahab [5] investigated the application of the change in modal curvatures to detect damage in a prestressed concrete bridge. Lakshminarayana [6] carried out analytical work to study the effect of crack at different location and depth on mode shape behaviour. Chandra Kishen [7] developed a technique for damage detection using static test data. Nahvi [8] established analytical as well as experimental approach to the crack detection in cantilever beams. Ravi Prakash Babu [9] used differences in curvature mode shapes to detect a crack in beams.

This paper deals with the technique and its application of mode shapes to a cantilever and a simply supported beam. The paper of Pandey *et al*. [2] shows a quite interesting phenomenon: that is, the modal curvatures are highly sensitive to crack and can be used to localize it. They used simulated data for a cantilever and a simply supported beam model with single damage to demonstrate the applicability of the method. The cracked beam was modeled by reducing the E-modulus of a certain element. By plotting the difference in modal curvature between the intact and the cracked beam, a peak appears at the cracked element indicating the presence of a fault. They used a central difference approximation to derive the curvature mode shapes from the displacement mode shapes. An important remark could be observed from the results of Pandey *et al*. [2]: that is, the difference in modal curvature between the intact and the damaged beam showed not only a high peak at the fault position but also some small peaks at different undamaged locations for the higher modes. To avoid this, a U-notch was modelled at the location of faults instead of reducing the E-modulus of a certain element. Also, analysis was carried out for beams with two cracks instead of one.

So, this paper is done to study about the changes in mode shapes because of the presence of U-notches in the cantilever and simply supported beams and is concerned with investigation of the accuracy when using the central difference approximation to compute the modal curvature and determine the location of the U-notches and to find out the reason of the presence of the misleading small peaks. The application of this technique to

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constructions in which more than one fault positions exist is investigated using a continuous beam with simulated data. The results of this scenario will be analyzed in this paper and U-notches will be detected and localized by using the measured change in modal curvatures.

So, as a summary, in the present work, a methodology for predicting crack parameters (crack depth and its location) in a cantilever and simply supported beam from changes in curvature mode shapes has been developed. Parametric studies have been carried out using ANSYS Software to evaluate mode shapes for different crack parameters (depth and its location). Curvature mode shapes were then calculated from the displacement mode shapes to identify crack location and its severity in the cantilever and simply supported beam.

PROBLEM FORMULATION

Figures-1(a) & 1(b) shows a cantilever and a simply supported beam of rectangular cross section, made of mild steel with two U- notches. To find out mode shapes associated with each natural frequency, FE analysis has been carried out using ANSYS Software for un-notched and U-notched beam.

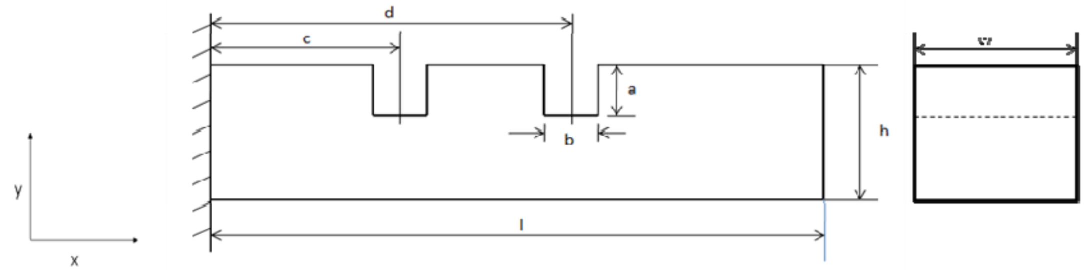

Figure-1(a). Cantilever beam with two U-notch cracks.

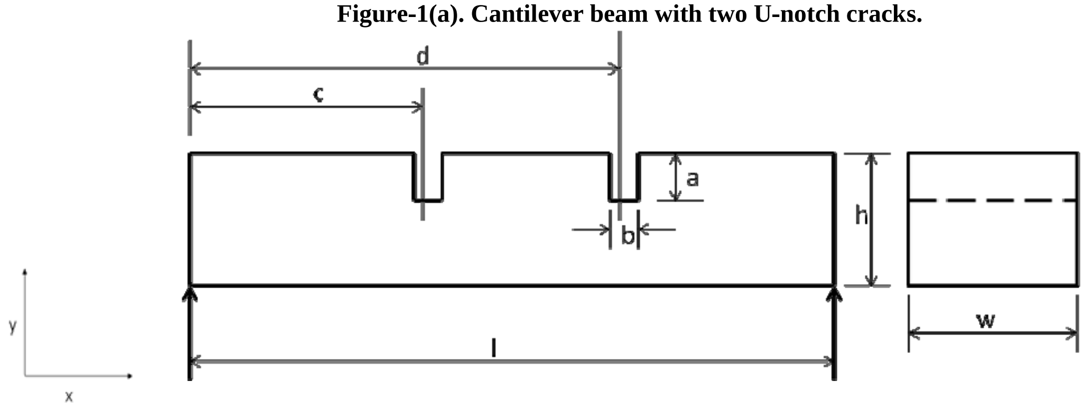

Figure-1(b). Simply supported beam with two U-notch cracks.

The mode shapes of the multiple U-notched cantilever and simply supported beams are obtained for U-notches located at normalized distances (c/l & d/l) from the fixed end of a cantilever beam and from left end support of a simply supported beam with a normalized depth (a/h).

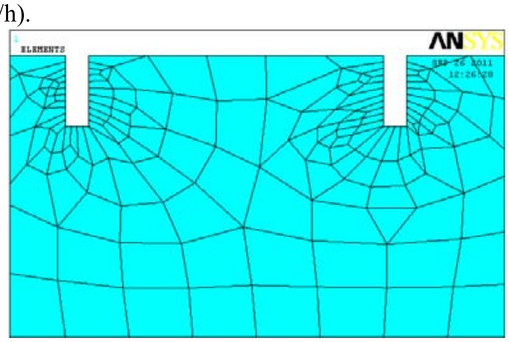

Figure-2. Discretised model of beam with two U-notches.

Figure-2 shows the discretised model (zoomed near the position of U-notches) of a beam with two U-notches. Parametric studies have been carried out for thin beam having length (l) = 260 mm, width (w) = 25 mm and thickness (h) = 4.4 mm. The breadth (b) of each U-notch has been kept as 0.32 mm. The U-notch locations from the fixed end (c & d) of the cantilever beam have been taken in different combinations near the fixed end, free end and middle of the beam. Similarly, U-notch locations from the left end support (c & d) of the simply supported beam have been taken in different combinations near the supports and middle of the beam. The intensity of U-notch (a/h) was varied by increasing its depth over the range of 0.25 to 0.75 in the steps of 0.25. This represented the case of a varying degree of crack at particular location. For each model of the two U-notch locations, the first three natural frequencies and corresponding mode shapes were calculated using ANSYS software.

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Governing equation

Determination of modal parameters (natural frequency & mode shape) in a beam is an Eigen value problem. ANSYS Software is used for theoretical modal analysis of the beam and the governing equation for general Eigen value problem is:

$$[M]\{\ddot{X}\}+[K]\{X\}=0 \tag{1}$$

Disposing the brackets without ambiguity equation (1) is rewritten as follows:

$$M\ddot{X}+KX=0 \tag{2}$$

Pre-multiplying both sides of equation (2) by M^{-1}:

$$M^{-1}M\ddot{X}+M^{-1}KX=M^{-1}0$$

Now, $M^{-1}M\ddot{X}=I\ddot{X}$ and

$M^{-1}KX=AX$ (say)

$$I\ddot{X}+AX=0 \tag{3}$$

Where, $A=M^{-1}K$ = system matrix.

Assuming harmonic motion $\ddot{X}=-\lambda X$; where, $\lambda=\omega^2$ equation (3) becomes

$$[A-\lambda I]X=0 \tag{4}$$

The characteristic of motion is then:

$$\Delta=|A-\lambda I|=0 \tag{5}$$

The n-roots λ_i, where i=1, 2, 3…n of the characteristic equation (5) are called Eigen values.

The natural frequencies are found as:

$$\omega_i=\sqrt{\lambda_i}\text{, i = 1,2,3………n.} \tag{6}$$

Substitution of the Eigen values (λ_i s) in (4) gives the mode shapes X_i corresponding to λ_i. These are Eigen vectors.

RESULTS AND DISCUSSION

In this analysis, it is assumed that crack is of U-notched shape. The depth (a) and locations (c & d) of these notches are normalized to the height and length of the cantilever and simply supported beams respectively. The first three mode shapes for the beam were calculated using ANSYS software and were shown below for different crack depths and crack location ratios.

Curvature finite difference approximation

Localized changes in stiffness result in a mode shape that has a localized change in slope, therefore, this feature will be studied as a possible parameter for crack detection purposes. For a beam in bending the curvature (k) can be approximated by the second derivate of the deflection:

$$k=\frac{d^2y}{dx^2} \tag{7}$$

In addition, numerical mode shape data is discrete in space, thus the change in slope at each node can be estimated using finite difference approximations. In this work, the central difference equation was used to approximate the second derivate of the displacements u along the x - direction at node i:

$$k_i=\frac{y_{i-1}-2y_i+y_{i+1}}{h^2} \tag{8}$$

The term $h=x_i-x_{i-1}$ is the element length. In this process meshing and node numbering is very important. Equation (8) require the knowledge of the displacements at node i, node i -1 and node i+1 in order to evaluate the curvature at node i. Thus, the value of the curvature of the mode shapes could be calculated starting from node 2 through node 261 in case of this beam. After obtaining the curvature mode shapes the absolute difference between the uncracked and cracked state is determined to improve crack detection.

$$\Delta(k_i)=|k_i|_{No\ crack}-|k_i|_{With\ crack} \tag{9}$$

As a result of this analysis, a set of curvature vectors for different crack localizations are obtained.

Uncracked case

By using the same finite element model shown in Figure-2, linear mode shapes was performed in ANSYS. The numerical results were exported to MATLAB to be processed.

The associated mode shapes were sketched evaluating the displacements in y direction of the 261 equidistant nodes located at the bottom line of the beam. In order to unify the results from the different cases, mode shapes were normalized by setting the largest grid point displacement equal to 1. It can be noticed from figures that all the mode shapes smoothed functions, what indicate the absence of cracks. Cracked beam mode shapes will be used to compare further results. Since changes in the curvature are local in nature, they can be used to detect and locate cracks in the beam.

Simple cracked case

The different combinations of crack location scenarios are selected for studying the effect of localized cracks in the beam. Although the reduction in natural frequencies is related to the existence of crack and its severity, this feature cannot provide any useful information about the location of the crack. Thus, curvature mode shapes were calculated and compared with the uncracked case. It can be seen that the maximum difference value for each mode shape occurs in the crack locations. In other areas of the beam this characteristic was much smaller. Although the third mode shapes was the most sensitive to the failure it is important that any of the three curvature mode shapes peak at the cracked locations.

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Cantilever beam

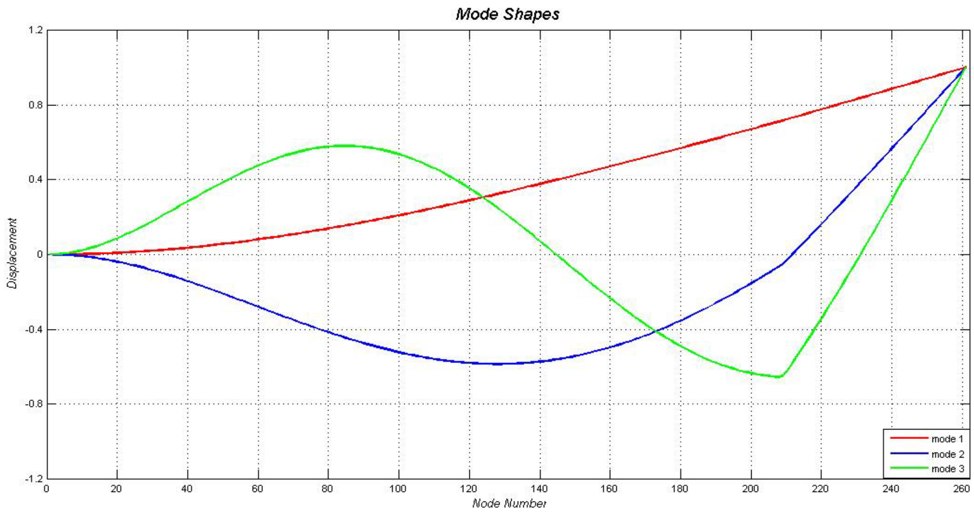

Figure-3. Mode shapes for crack at c/l = 0.15 of depth a/h = 0.25 and crack at d/l = 0.8 of depth a/h = 0.75.

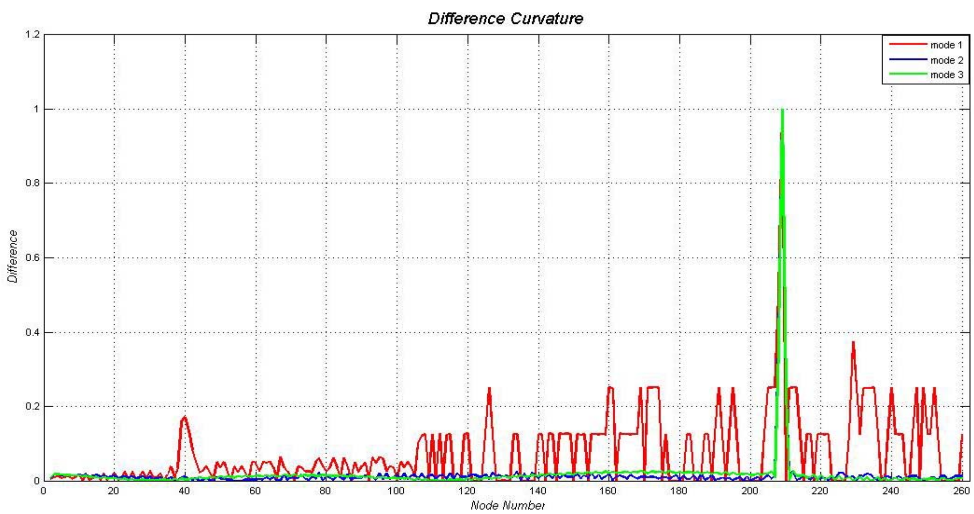

Figure-4. Difference curvature for crack at c/l = 0.15 of depth a/h = 0.25 and crack at d/l = 0.8 of depth a/h = 0.75.

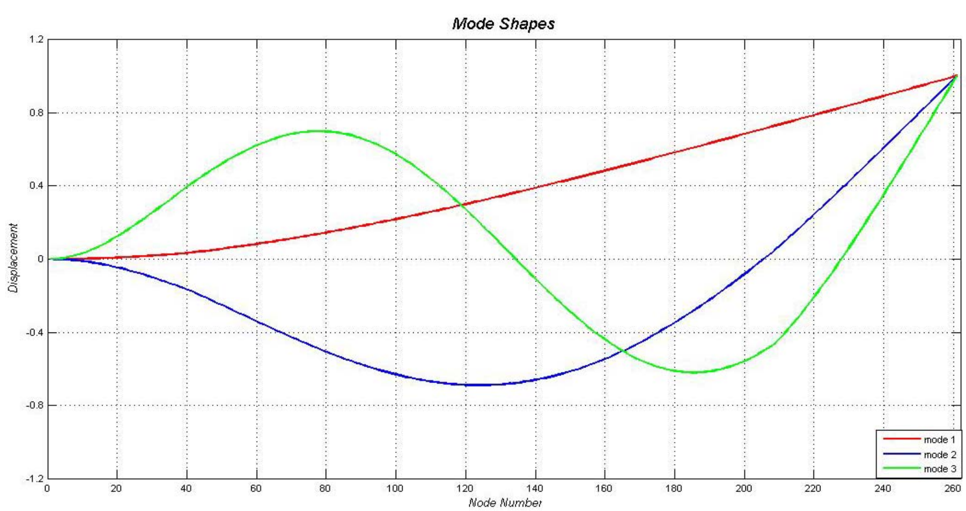

Figure-5. Mode shapes for crack at c/l = 0.15 of depth a/h = 0.5 and crack at d/l = 0.8 of depth a/h = 0.5.

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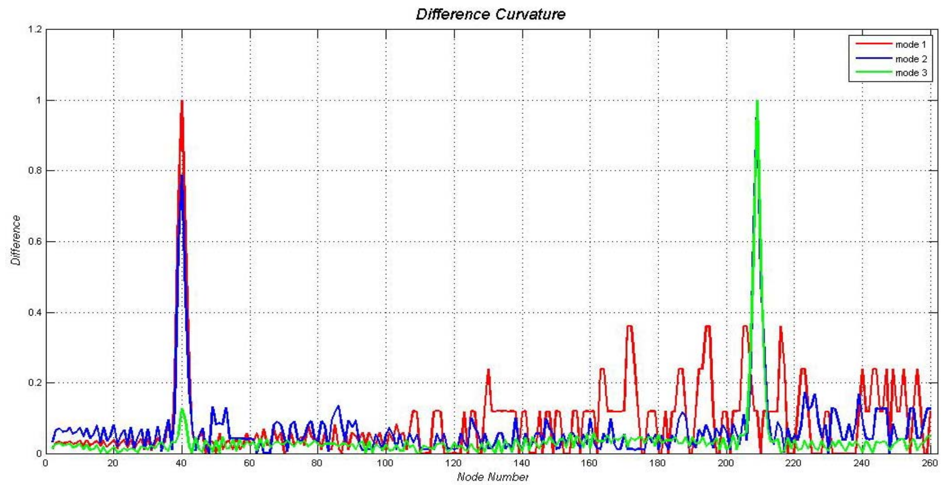

Figure-6. Difference curvature for crack at c/l = 0.15 of depth a/h = 0.5 and crack at d/l = 0.8 of depth a/h = 0.5.

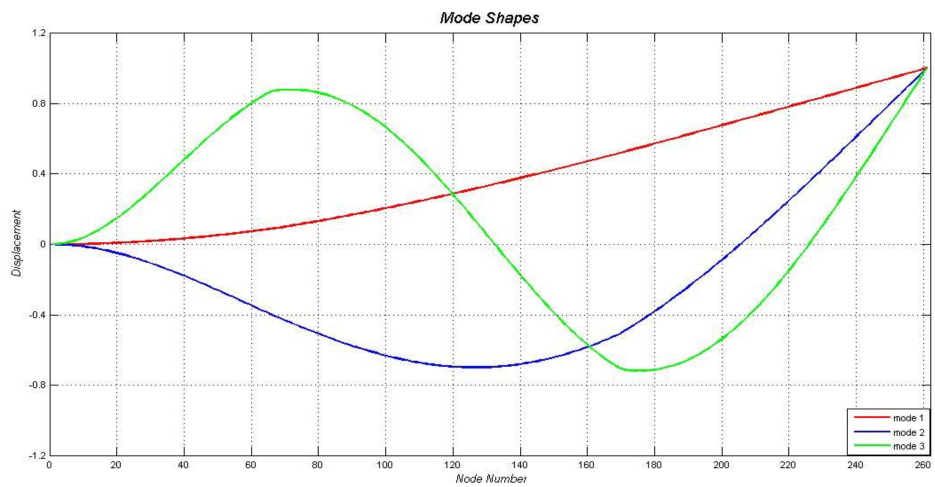

Figure-7. Mode shapes for crack at c/l = 0.25 of depth a/h = 0.5 and crack at d/l = 0.65 of depth a/h = 0.5.

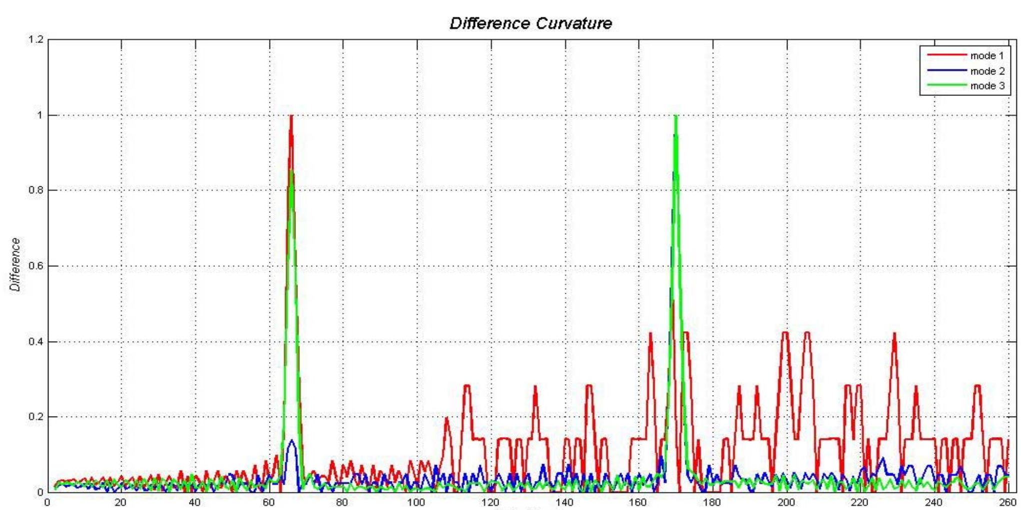

Figure-8. Difference curvature for crack at c/l = 0.25 of depth a/h = 0.5 and crack at d/l = 0.65 of depth a/h = 0.5.

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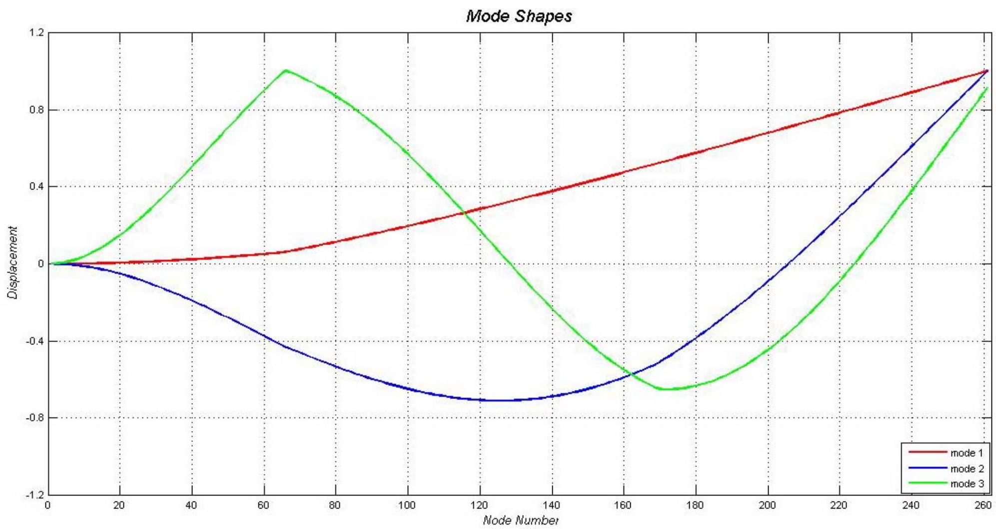

Figure-9. Mode shapes for crack at c/l = 0.25 of depth a/h = 0.75 and crack at d/l = 0.65 of depth a/h = 0.5.

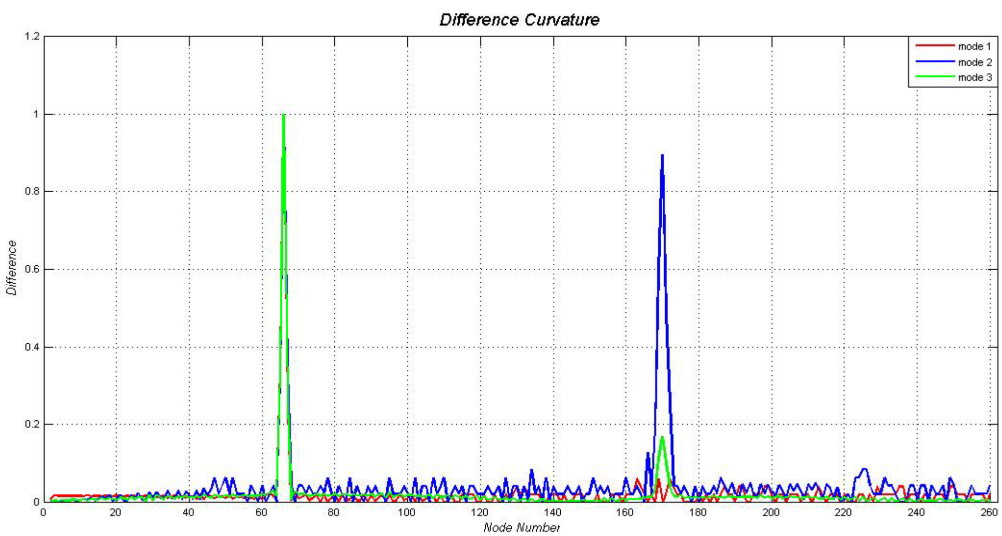

Figure-10. Difference curvature for crack at c/l = 0.25 of depth a/h = 0.75 and crack at d/l = 0.65 of depth a/h = 0.5.

To examine the curvature mode shape technique for cantilever beam having several crack locations, the span of the beam is discretized by 260 elements. Two crack locations are assumed at a time. Firstly, two cracks at c/l = 0.15 of depth a/h = 0.25 and at c/l = 0.8 of depth a/h = 0.75 were considered. The first three displacement mode shapes are shown in Figure 3. The difference in modal curvature between the uncracked and the cracked beam is plotted in Figure 4 for the first three modes. For mode 1 in Figure 4, it can be observed that the peak at c/l = 0.15 is very small comparing to that at d/l = 0.8. And also, for mode 1 the modal curvature at c/l = 0.15 is less and has very small values at the nodes next to it. In contrast, at d/l = 0.8 high modal curvature takes place. This indicates the severity of the crack depth ratio a/h = 0.75 at d/l = 0.8 compared to crack depth ratio a/h = 0.25 at c/l = 0.15. And also, first mode shape modal curvature is more sensitive near the fixed end compared to second and third mode shape modal curvatures. Again second and third mode shape modal curvatures are more sensitive near the free end compared to first mode shape modal curvature. The same can be observed for some other crack scenarios shown in Figures-5 & 6, 7 & 8, 9 & 10. So, depending on the absolute ratio between the modal curvature values for a particular mode at two different locations, one peak can dominate the other. Therefore, one can conclude that in case of several crack locations in a structure, all modes should be carefully examined.

Simply supported beam

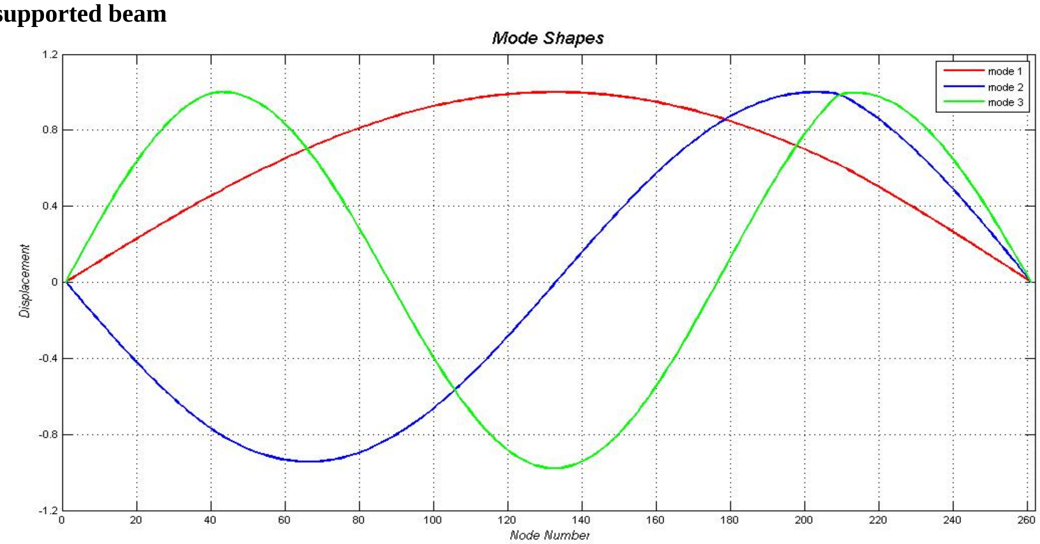

Figure-11. Mode shapes for crack at c/l = 0.15 of depth a/h = 0.25 and crack at d/l = 0.8 of depth a/h = 0.5.

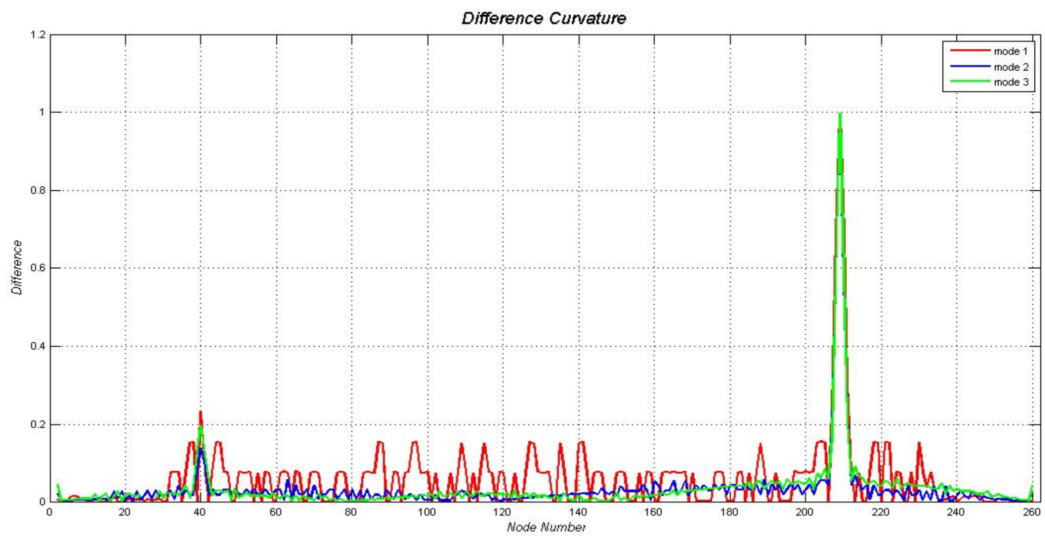

Figure-12. Difference curvature for crack at c/l = 0.15 of depth a/h = 0.25 and crack at d/l = 0.8 of depth a/h = 0.5.

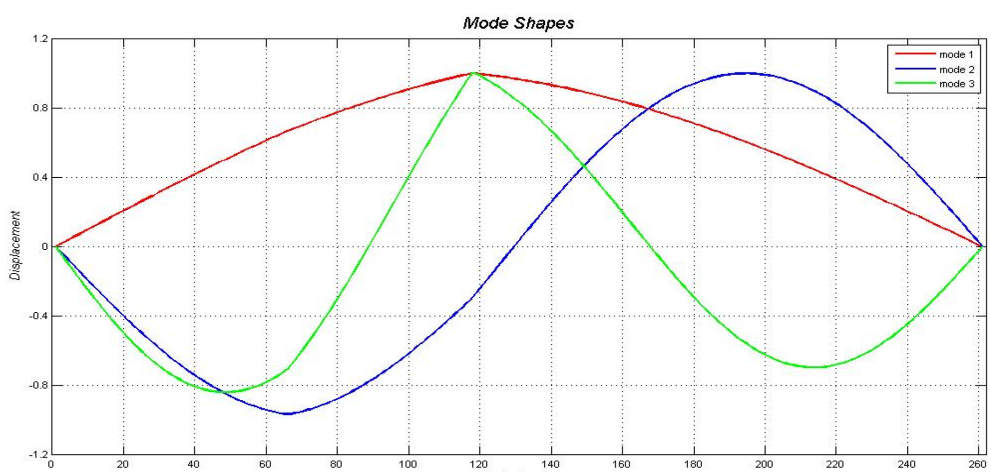

Figure-13. Mode shapes for crack at c/l = 0.25 of depth a/h = 0.5 and crack at d/l = 0.45 of depth a/h = 0.75.

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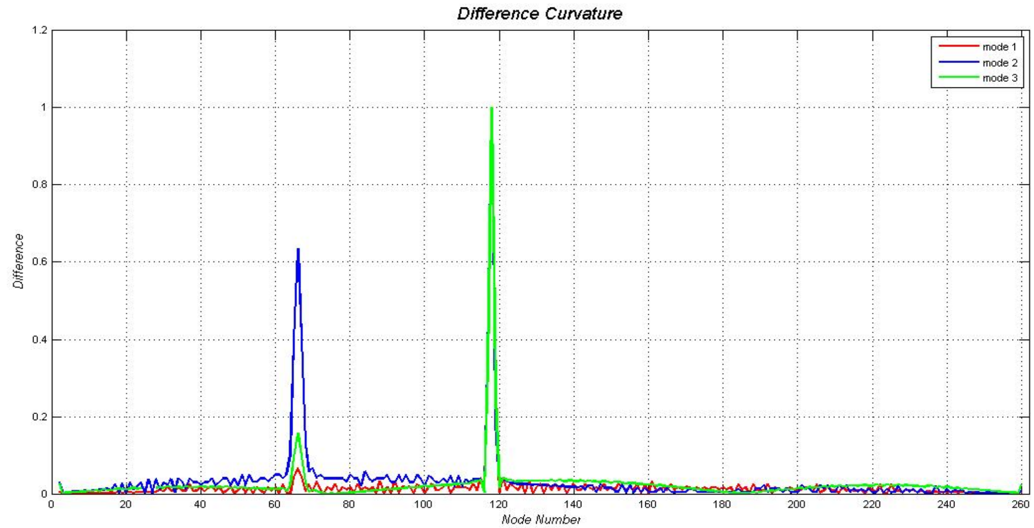

Figure-14. Difference curvature for crack at c/l = 0.25 of depth a/h = 0.5 and crack at d/l = 0.45 of depth a/h = 0.75.

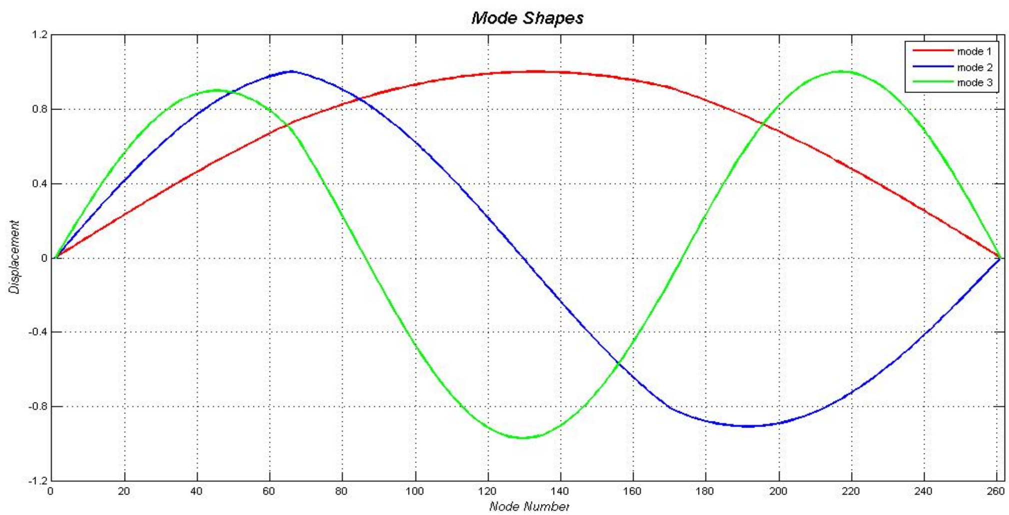

Figure-15. Mode shapes for crack at c/l = 0.25 of depth a/h = 0.5 and crack at d/l = 0.65 of depth a/h = 0.5.

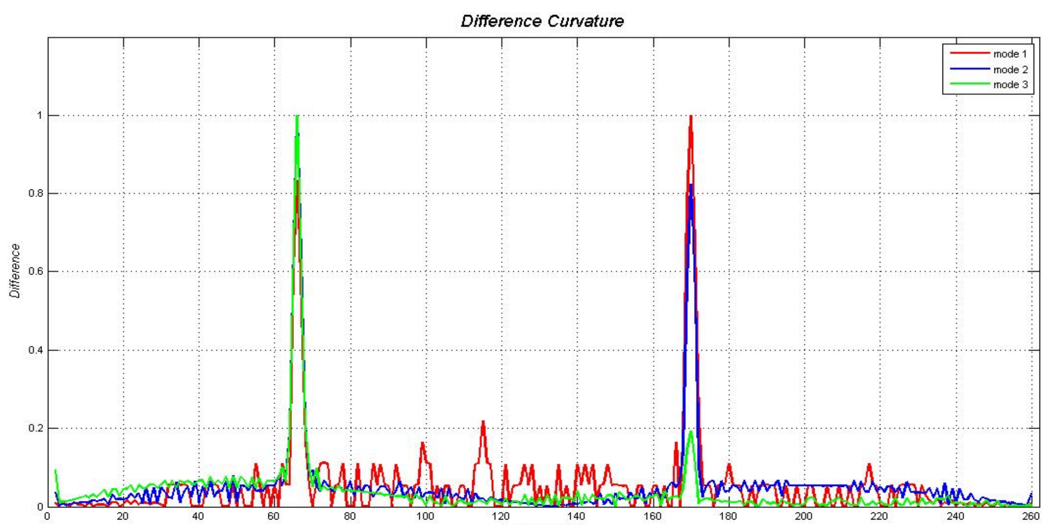

Figure-16. Difference curvature for crack at c/l = 0.25 of depth a/h = 0.5 and crack at d/l = 0.65 of depth a/h = 0.5.

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Figure-17. Mode shapes for crack at c/l = 0.65 of depth a/h = 0.5 and crack at d/l = 0.85 of depth a/h = 0.5.

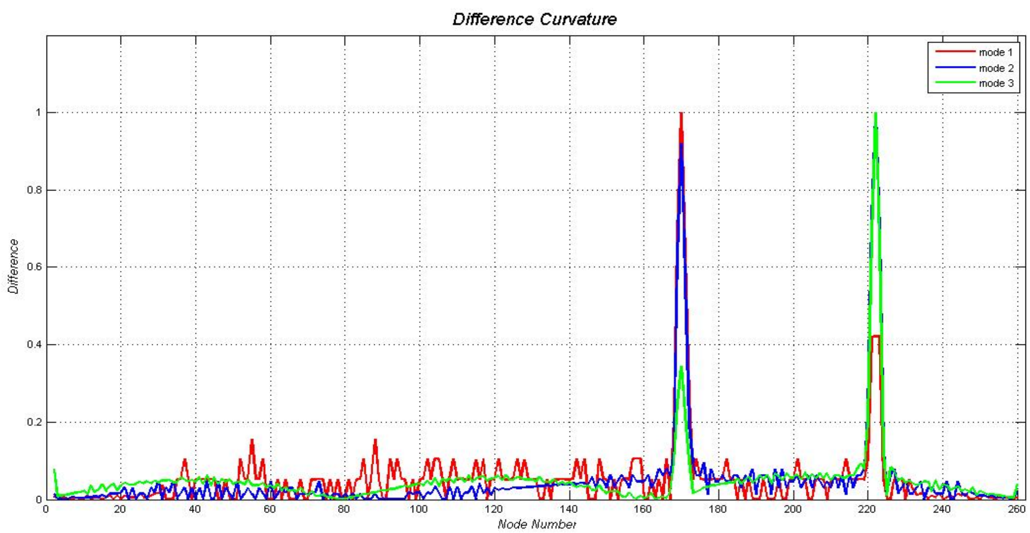

Figure-18. Difference curvature for crack at c/l = 0.65 of depth a/h = 0.5 and crack at d/l = 0.85 of depth a/h = 0.5.

To examine the curvature mode shape technique for simply supported beam having several crack locations, the span of the beam is also discretized by 260 elements. Two crack locations are assumed at a time. Firstly, two cracks at c/l = 0.15 of depth a/h = 0.25 and at c/l = 0.8 of depth a/h = 0.5 were considered. The first three displacement mode shapes are shown in Figure 11. The difference in modal curvature between the uncracked and the cracked beam is plotted in Figure 12 for the first three modes. For all the modes in Figure 12, it can be observed that the peak at c/l = 0.15 is very small comparing to that at d/l = 0.8. In contrast, at d/l = 0.8 high modal curvature takes place. This indicates the severity of the crack depth ratio a/h = 0.5 at d/l = 0.8 compared to crack depth ratio a/h = 0.25 at c/l = 0.15. And also, third mode shape modal curvature is more sensitive near the supports compared to first and second mode shape modal curvatures. The same can be observed in Figure 18 at d/l = 0.85. Again in Figure 14 third mode shape modal curvature is more sensitive near the middle portion of the beam at d/l = 0.45 compared to other mode shape modal curvature and the second mode modal curvature is more sensitive near the one-fourth of the length of the beam at c/l = 0.25. The same can be observed for some other crack scenarios shown in Figure 16 at d/l = 0.65 and in Figure 20 at c/l = 0.65 for second mode shape. So, depending on the absolute ratio between the modal curvature values for a particular mode at two different locations, one peak can dominate the other. Therefore, one can conclude that in case of several crack locations in a structure, all modes should be carefully examined.

And also, the crack is assumed to affect stiffness of the cantilever beam. The stiffness matrix of the cracked element in the FEM model of the beam will replace the stiffness matrix of the same element prior to damaging to result in the global stiffness matrix. Thus frequencies and mode shapes are obtained by solving the eigen value problem $[K] - \omega^2 [M] = 0$. So it can be seen in Figure 3, 9, 13 very clearly the changes in slopes and deviations in mode shape at crack location for crack depth ratio 0.75.

CONCLUSIONS

A method for identifying multiple crack parameters (crack depth and its location) in beams using modal parameters has been attempted in the present paper. Parametric studies have been carried out using ANSYS Software to evaluate modal parameters (natural frequencies and mode shapes) for different crack parameters. A theoretical study using simulated data for a cantilever and simply supported beams has been conducted. When more than one fault exists in the structure, it is not possible to locate crack in all positions from the results of only one mode. All modes should be carefully examined in order to locate all existing faults. Also, due to the irregularities in the measured mode shapes, a curve fitting can be applied by calculating the curvature mode shapes using the central difference approximation. The curvature mode shapes technique for crack localization in structures is investigated in this paper. The results confirm that the application of the curvature mode shape method to detect cracks in engineering structures seems to be promising. Techniques for improving the quality of the measured mode shapes are highly recommended. The identification procedure presented in this paper is believed to provide a useful tool for detection of medium size cracks in beams.

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