INTELLIGENT TUNING OF PI CONTROLLER IN AN AEROBIC BIOREACTOR

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ABSTRACT

This paper proposes an intelligent controller tuning technique for a PI controller using a nature inspired metaheuristic algorithm, Firefly algorithm (FA). The PI controller is designed to maintain Dissolved Oxygen (DO) concentration in an aerobic biological reactor of a waste water treatment plant (WWTP). The proposed technique is compared with IMC based PID tuning method and PI parameters of Benchmark Simulation Model1 (BSM1). The PI controller tuned using the proposed technique produces satisfactory response and gives better results when subjected to set-point tracking and disturbance rejection test respectively.

Keywords: firefly, PID, DO control, WWTP, ASM1, BSM1.

1. INTRODUCTION

Wastewater treatment plants (WWTPs) are large non-linear systems, highly interactive and are uncertain due to the composition of the incoming wastewater. Activated Sludge in WWTPs is a biological process which uses bacteria and other micro organisms to remove contaminants from the wastewater. Some of these microbes grow only under aerobic conditions (dissolved oxygen present). It is very important to supply enough oxygen in the aeration tanks for the effective metabolism of microorganisms. Therefore, maintaining dissolved oxygen concentration (typically 1.0 - 3.0 mg/L) in the aerobic reactor is an important control problem [4]. Modeling and control of an activated sludge process is always a challenging task. Activated Sludge Models (ASMs) developed by International Water Association (IWA) task group includes ASM1, ASM2, ASM2d and ASM3 [2]. ASM1 is used most commonly to represent carbon oxidation, nitrification and de-nitrification processes [3].

In process industries, most of the controllers are PI/PID (Proportional plus Integral / Proportional plus Integral plus Derivative) type. The most important step in the application of PI/PID controller is tuning of its parameters. There are many procedures available for tuning the PI/PID controllers (for WWTPs) in the literature [5, 6]. It is necessary to find an advanced technique to tune the PI/PID controller which makes tuning procedure easier. The recent advancements in biologically inspired optimization algorithms optimize the controller parameter(s) based on the objective function(s) [10-12]. The firefly algorithm (FA) is a naturally inspired metaheuristic algorithm developed by Xin-She in 2007. The FA is based on the idealized behavior of the flashing characteristics of fireflies [7]. Being an effective optimizing algorithm, FA is used in many engineering applications [8, 9]. In control problems, FA and its variants are used to optimally tune the PI/PID controller [10, 11].
The aerobic biological reactor is modeled based on the model equations described in ASM1 [3]. The reactor is characterized as thirteen state variables and their corresponding equations are given below. The below reactor model shows that the thirteen state variables have a strong coupling with each other which makes the system non-linear and difficult to control. The suffix ‘i’ in the mathematical equation represent the inlet to the reactor. The mathematical model of the process is given below.

\[
\frac{dS_i}{dt} = \frac{1}{V} \left( Q_i S_{in} - Q_o S_x - V \left[ \frac{\rho_1}{Y_H} + \frac{\rho_2}{Y_H} - \rho_7 \right] \right) \quad (1)
\]

\[
\frac{dX_i}{dt} = \frac{1}{V} \left( Q_i X_{in} - Q_o X_x + V \left[ (1 - f_p) \rho_4 + \rho_5 \right] - \rho_7 \right) \quad (2)
\]

\[
\frac{dX_{BH}}{dt} = \frac{1}{V} \left( Q_i X_{BH} - Q_o X_{BH} + V \left[ \rho_1 + \rho_2 - \rho_4 \right] \right) \quad (3)
\]

\[
\frac{dX_{BA}}{dt} = \frac{1}{V} \left( Q_i X_{BA} - Q_o X_{BA} + V \left[ \rho_3 - \rho_5 \right] \right) \quad (4)
\]

\[
\frac{dX_p}{dt} = \frac{1}{V} \left( Q_i X_{p} - Q_o X_{p} + V \left[ f_p \rho_4 + f_p \rho_5 \right] \right) \quad (5)
\]

\[
\frac{dS_O}{dt} = \frac{1}{V} \left[ \rho_1 \left( 1 - \frac{Y_H}{Y_A} \right) + \frac{(4.57 - Y_A) \rho_4}{Y_A} \right] \quad (6)
\]

\[
\frac{dS_{NO}}{dt} = \frac{1}{V} \left[ Q_i S_{NO} - Q_o S_{NO} + V \left[ \rho_1 \left( 1 - \frac{Y_H}{Y_A} \right) + \frac{(4.57 - Y_A) \rho_4}{Y_A} \right] \right] \quad (7)
\]

\[
\frac{dS_{NH}}{dt} = \frac{1}{V} \left[ Q_i S_{NH} - Q_o S_{NH} + V \left[ \left( 1 - \frac{Y_H}{Y_A} \right) \rho_2 \right] \right] \quad (8)
\]

\[
\frac{dS_{ND}}{dt} = \frac{1}{V} \left[ Q_i S_{ND} - Q_o S_{ND} + V \left[ (1 - f_p) \rho_4 + f_p \rho_5 \right] \right] \quad (9)
\]

\[
\frac{dX_{ND}}{dt} = \frac{1}{V} \left[ Q_i X_{ND} - Q_o X_{ND} + V \left[ (1 - f_p) \rho_4 + f_p \rho_5 \right] \right] \quad (10)
\]

\[
\frac{dS_{ALK}}{dt} = \frac{Q_i S_{ALK} - Q_o S_{ALK} + V \left[ \left( 1 - \frac{Y_H}{Y_A} \right) \rho_2 \right]}{V} \quad (11)
\]

The terms \( Q_i \) & \( Q_o \) represents the inlet and outlet flowrate respectively; \( Z_i \) & \( Z \) are the component concentration of inlet and outlet respectively. The symbol \( \rho_1 \ldots \rho_8 \) represents the eight processes of ASM1 [3] that take place inside the bioreactor. The biological parameter values required to model the reactor correspond to the temperature of 15°C [1].

### 3. CONTROLLER DESIGN

#### PI Control algorithm

The PI controller is widely used in many industrial applications due to its simplicity and effective control action. The continuous control law of PI controller is

\[
u(t) = K_p e(t) + K_I \int_0^t e(t) dt + u_0 \quad (12)
\]
where $e(t)$ is the error signal between the set-point and actual process output; $u(t)$ is the controller output; $u_0$ is the controller output at time $t=0$. The parameters of PI controller are $K_p$, $K_i$. The PI controller directly operates on the error signal and takes the appropriate control action.

**Step Response Characteristics**

The PI controller is designed by developing a linear model of the bioreactor. To obtain the step response, a simple technique is followed. One such response was obtained in [5]. To the aerobic reactor operating at the steady state condition ($S_o=1.491mg/L$), a positive step change (36%) is given to the manipulated variable (oxygen transfer coefficient, $K_{La}$). The dissolved oxygen concentration in the reactor increases and reaches another steady state value. The resulting data is the step response of the reactor. Aerobic biological process is reduced to the First-Order-Plus-time-Delay (FOPTD) model as given in Eq.13. Figure-3 shows the step response of the aerobic reactor and the derived transfer function model respectively.

$$G(s) = \frac{0.028e^{-0.0027s}}{0.0064s + 1}$$  \hspace{1cm} (13)

**FA – PI method**

Metaheuristic algorithms like FA are usually nature-inspired, used to achieve global optimization. The pseudo code of the Firefly Algorithm is given in Table I [7].

The attractiveness function of the firefly is given by the following equation

$$\beta = (\beta_0 - \beta_{min})\exp(-r^2) + \beta_{min}$$  \hspace{1cm} (16)

where $r$ is the distance between the two fireflies. $\beta_0$ is the attractiveness at $r = 0$. $\beta_{min}$ is the minimum attractiveness between any two fireflies. $\gamma$ is the light absorption coefficient.

The distance ($r_{ij}$) between any two fireflies $i$ and $j$ at $x_i$ and $x_j$ respectively in the solution space is given in Eq.17 and $d$ is the dimension number. The movement of a firefly $i$ towards $j$ in the solution space, is given in Eq.18. Here $\alpha$ is the randomization parameter whose value ranges from 0 to 1. As $\alpha$ is a useful parameter for algorithm convergence, based on the application, it can be written as a function of iteration. In Eq.18, $\text{rand}$ returns pseudorandom value drawn from the standard uniform distribution on the open interval (0,1).

$$r_{ij} = \sqrt{\sum_{k=1}^{d} (x_{ik} - x_{jk})^2}$$  \hspace{1cm} (17)

$$x_i = x_i + \beta (x_j - x_i)^2 + \alpha(\text{rand} - 0.5)$$  \hspace{1cm} (18)

**Table-1. Pseudo-code of FA.**

| **Objective function $f(x)$, $x = (x_1, \ldots, x_d)$** |
| **Generate initial population of fireflies $x_i$ ($i = 1, 2, \ldots, n$)** |
| **Light intensity $I_i$ at $x_i$ is determined by $f(x_i)$** |
| **Define light absorption coefficient $\Upsilon$** |
| while ($t < \text{MaxGeneration}$) |
| for $i = 1 : n$ all n fireflies |
| for $j = 1 : n$ all n fireflies (inner loop) |
| if($I_i < I_j$), Move firefly $i$ towards $j$; end if |
| Vary attractiveness with distance $r$ via $\exp(-r^2)$ |
| Evaluate new solutions and update light intensity |
| for $j$ |
| end for |
| Rank the fireflies and find the current global best $g$ |
| end while |
| Post-process results and visualization |
number of iterations in the solution space. The flashing light intensity (I) of the firefly is formulated in such a way that it is associated with the objective function which in our process is the minimization of IAE Eq.19. The parameters of FA are given in Table-II.

\[ I = \frac{0.1}{IAE + 0.1} \]  

(19)

**Figure-4. FA based PI controller tuning**

**Table-2. Parameters of FA.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of fireflies, n</td>
<td>10</td>
</tr>
<tr>
<td>Randomness, α</td>
<td>0.5</td>
</tr>
<tr>
<td>Initial attractiveness, β₀</td>
<td>1.0</td>
</tr>
<tr>
<td>Minimum attractiveness, βₘᵣₜ</td>
<td>0.2</td>
</tr>
<tr>
<td>Absorption coefficient, γ</td>
<td>1.0</td>
</tr>
<tr>
<td>Generation number, nₒ</td>
<td>10</td>
</tr>
</tbody>
</table>

**4. SIMULATION RESULTS & DISCUSSIONS**

The algorithm is simulated for the controller range of 22 < Kₚ < 43 and 1500 < Kᵢ < 4700. The best controller parameter values obtained from the three methods are given in Table-III. In order to check the effectiveness of the controller, the system is subjected to disturbance. To the reactor that operating at the steady state condition (DO = 2 g/m³), the Sₒᵢ of the influent wastewater is increased from 1.43g/m³ to 1.64 g/m³ for a period of 0.025day. The response of the controller for the three methods is shown in the Figure-5. On applying disturbance, the DO concentration of the process deviates from the set point. The time taken by the closed loop system to overcome the disturbance and settle is called recovery time. Table-IV shows the performance of the three controllers quantitatively for the disturbance rejection test.

**Table-3. Controller Parameters.**

<table>
<thead>
<tr>
<th>Method</th>
<th>KP</th>
<th>Kᵢ</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSM1</td>
<td>25.00</td>
<td>12500.0</td>
</tr>
<tr>
<td>IMC-PI</td>
<td>35.75</td>
<td>4589.22</td>
</tr>
<tr>
<td>FA-PI</td>
<td>28.10</td>
<td>4612.70</td>
</tr>
</tbody>
</table>

The aerobic reactor is perturbed from the steady state operating condition (DO = 1.491mg/L) and the response obtained for the three methods are shown in the Figure-6. The performance of the controller is evaluated using the performance criteria shown in Table V. From the servo response and the performance summary table, it is clear that FA based tuning method outperforms the other two tuning methods with minimum IAE, fast settling time, less overshoot and small undershoot. Figure-7 depicts the overall performance measures of the controller obtained from the servo test for the three methods.
5. CONCLUSIONS

In this paper, an intelligent tuning method for a PI controller was proposed to control the dissolved oxygen concentration in an aerobic biological reactor. In the proposed method, the controller is tuned to minimize IAE. Different performance criteria like minimizing ISE, ITAE, overshoot and its combinations may be used to formulate the objective function to tune the controller. The servo response and the regulatory response for all the three methods are evaluated. The simulation results demonstrate that FA tuned PI controller gives best results and can improve the control system performance in terms of time domain specifications when compared with the other two methods. As a future work, we intend to design an adaptive PI control strategy using FA for the entire operating region. Also stability of the control system will be addressed while tuning the controller.

Table 5. Servo Response – Performance Summary.

<table>
<thead>
<tr>
<th>Set Point</th>
<th>IAE/ITAE</th>
<th>Settling Time (Day)</th>
<th>Overshoot/Undershoot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FA-PI</td>
<td>IMC-PI</td>
<td>BSM1</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0290</td>
<td>0.0290</td>
<td>0.2257</td>
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<td>2.25</td>
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<td>2.50</td>
<td>0.0308</td>
<td>0.0333</td>
<td>0.1350</td>
</tr>
<tr>
<td>2.00</td>
<td>0.0299</td>
<td>0.0304</td>
<td>0.2272</td>
</tr>
</tbody>
</table>

REFERENCES

