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ON THE METRIC DIMENSION OF SILICATE STARS

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ABSTRACT

A minimum resolving set or a metric basis M for a graph G(V, E) is a small subset of vertices of V such that for every pair of vertices x and y of V \ M, there exist at least one vertex m in M such that the distance between x and m is not equal to the distance between y and m. The number of elements of the metric basis M of G is called metric dimension and the elements of a metric basis are called landmarks. A metric dimension problem for a graph G is to find a metric basis for G. In this paper a new silicate graph called Silicate Stars or Star of Silicate Networks SSL(n) has been derived from Star of David Networks SD(n). The metric dimension problem has been solved for SSL(n) , Single Oxide chain, and single Silicate chain. The problem of finding the metric dimension of a general graph is an NP Complete Problem.

Keywords: star, silicate networks, David networks, oxide chain, Silicate chain, NP-complete, metric dimension.

1. INTRODUCTION AND RELATED WORK

The Silicate network was introduced by Paul *et al*. They studied the topological properties and embedding problem of Silicate networks in [1]. The metric dimension problem for Silicate network is investigated in [2]. In this paper we proposed a interconnection network called Silicate Star and metric dimension problem is investigated for the same. Also the topological properties has been studied as it has been studied for other interconnection networks in [3,4,5,6,7,8,9].

The first metric dimension problem was investigated by Harary and Melter [11]. They gave a characterization for the metric dimension of trees. The metric dimension problem for grid graphs was studied by Melter and Tomescu [12]. Result of Melter and Tomescu have generalized by Khuller *et al*. He proved that the metric dimension of *d* dimensional grid graph is *d* [13]. The metric dimension problem is *NP* complete for any arbitrary graph [14].This problem is also *NP* complete for bipartite graph [15].The metric basis concept has appeared in the literature under different names before 1976. The metric basis was called as reference set by Slater [16] and Later [17]. Slater called the number of elements in a reference set of the graph as location number of the graph. He described the application of metric basis in sonar and loran stations. Chartand have metric basis as minimum resolving set [18]. The metric dimension problem has been studied for trees and grid graphs [13], Petersen graphs [19], honeycomb networks, Hexagonal networks [4], Torus networks [20], Enhanced Hyper cubes [7],Silicate network [2], Regular Trianguline Oxide networks [8] and Star of David networks [9]. We use the proof of correctness method to solve the metric dimension problem for Star of Silicate networks. The above introduction on metric dimension problem is partially referred from [2]. The application of metric dimension to problems of Robot navigation, pattern recognition [13], Network discovery and verification [21], geometrical routing protocols [26], Joins in graphs [23], and coin weighing problems [24].

In section *A*, an algorithm is given to construction a Star of silicate networks from Star of David networks. In Section *B*, construction algorithm for silicate star is explained through an example. In Section *C*, a coordinate system is proposed for Silicate Stars. In section *D*, we have investigated the metric dimension of Silicate Stars, Single Oxide chains and single Silicate chains.

In SiO_4, the corner vertices are called oxygen vertices and the centre vertex is called silicon vertex.

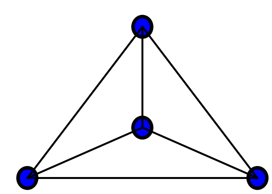

Figure-1. SiO_4 tetrahedron.

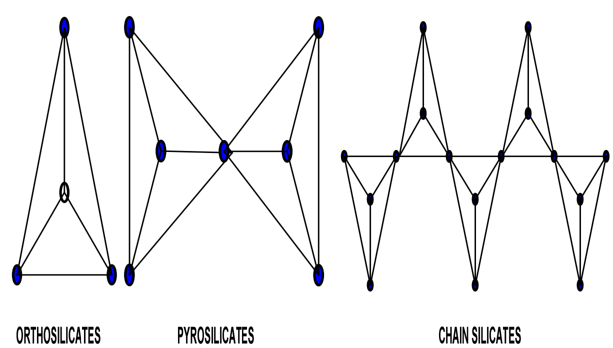

Figure-2. Different kind of silicates.

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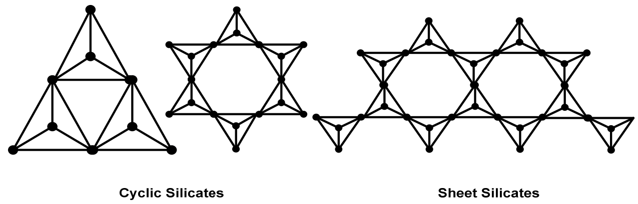

Figure-3. Different cyclic Silicates.

Theorem 1[1]**:** The number of nodes in Silicate network $SL(n)$ is $15n^2 + 3n$ and edges is $36n^2$.

Theorem 2[2]**:** The metric dimension of Silicate network $SL(n)$ is $6n$.

a) Construction of star of silicate networks from SD(n)

A silicate network can be constructed in different ways [1]. We describe the construction of a new Star of silicate network from a Star of David network.

Step-1: Draw a Star of David graph *H*, of dimension one.

Step-2: Divide each edge into 2^n -1 edges by inserting 2^n - 2 vertices at each edge of *H*.

Step-3: Connect any two vertices *u* and *v* by an edge if one is the mirror image of the other and if they are at odd distance 1, 3 , 5, 7, ..., ($6n$-1) from, one corner vertices except the pairs at a distance ($2n$-1).

Step-4: Fix a new vertex at each new edge crossing. The resulting network is called the Star of David network of dimension *n* and is denoted by $SD(n)$ [9].

Step-5: Replace each sub graph K_3 by tetrahedron (Figure 1). This network is called Silicate Star or Star of Silicate network of dimension *n* and is denoted by $SSL(n)$.

Note: Silicate networks $SL(n)$ and Oxide networks $OX(n)$ defined in [1] are proper subgraph of $SSL(n)$.

b) Method of construction of SSL(2) from SD(2)

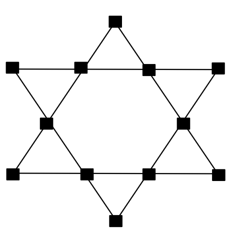

Figure-4. Step-1.

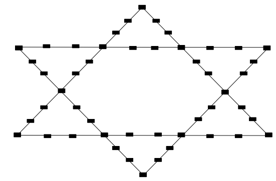

Figure-5. Step-2.

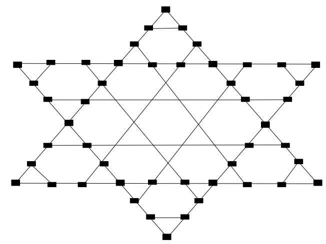

Figure-6. Step-3.

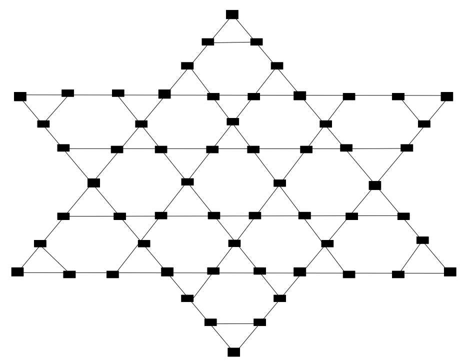

Figure-7. Step-4: Star of David network of dimension 2 $SD(2)$.

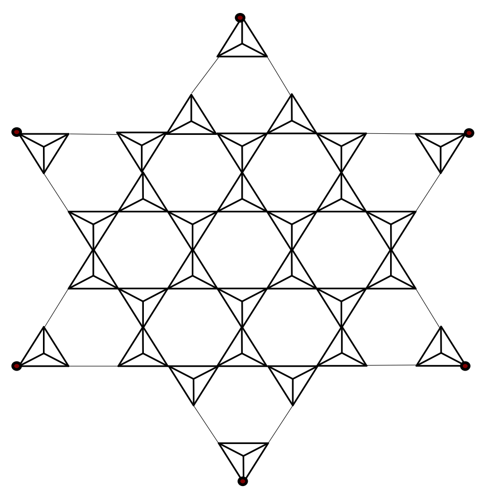

Figure-8. Step-5: Star of Silicate Network $SSL(2)$.

c) Coordinate system for star of silicate networks.

A coordinate system is proposed that assigns an address to each vertex of star of David network. This coordinate system is then extended to Star of Silicate

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network. The basic idea is due to Stojmenovic [5, 6] and to Nocetti *et al.* [10] who proposed a system for a honeycomb network and a hexagonal network, respectively.

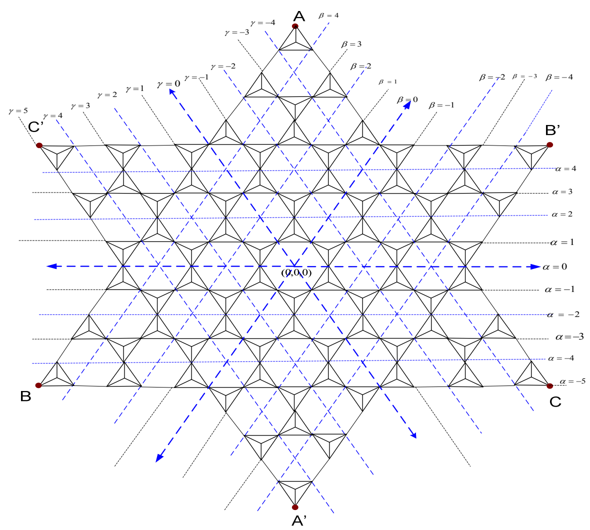

Figure-9. Coordinate system in star of silicate network of dimension 3.

Three axes α, β and γ are introduced to three edge directions of silicate star. The angle between any two axis is 120 degree. The equation of three coordinate axes are $\alpha = 0$, $\beta = 0$, and $\gamma = 0$. Any line parallel to α- axis, or β-axis, or γ-axis is called α-line,or β-line,or γ-line respectively . Any vertex of Star of David $SD(n)$ is called (i,j,k) if the vertex is the point of intersection of lines $\alpha = i$, $\beta = j$, and $\gamma = k$. Address for a Silicon vertex is the centroid of Oxygen vertices of a tetrahedral SiO_4.

Theorem 3: The number of vertices in Star of David $SD(n)$ is $18n^2 - 6n$ and edges is $36n^2 - 24n + 6$.

Theorem 4: The number of vertices in Silicate Star $SSL(n)$ is $30n^2 - 18n + 6$ and edges is $42n^2 - 10n + 4$.

d) The metric dimension of silicate stars

Theorem 5: The metric dimension of Star of Silicate network $SSL(n)$ is 6.

Proof: Let us find the lower bound for the metric dimension of Silicate star. Let u be a boundary vertex of $SSL(n)$ with degree 3. Let v be the silicon node adjacent to u. Then for any node w in $SL(n)$, we have $d(u, w) = d(v, w)$. Thus any metric basis will contain either u or v. There are 6 boundary nodes with degree 3 in $SSL(n)$. Hence any metric basis of $SSL(n)$ should contain at least 6 nodes of $SSL(n)$. Therefore the metric dimension of silicate star is at least 6.We claim that the set of boundary vertices with degree 3, $\{A, A', B, B', C, C'\}$ is the metric basis (Figure 9). Since the network is symmetrical with respect to any α or β or γ axis , we begin our discussion with respect to α-lines. A line $\alpha = k$ is called as odd or even whenever k is odd or even. The region between two consecutive α-lines is called as an α-channel. It is noted that only odd α-lines contain the boundary vertices of $SSL(n)$ with degree 3. And Centroid vertices (silicon vertices) lies on channels (Figure-9).

Let $u(x_1,y_1,z_1)$ and $v(x_2,y_2,z_2)$ be any two distinct vertices of $G = SD(n)$. Clearly G is subgraph of $SSL(n)$. Suppose u and v lies in a same α line, then $x_1 = x_2$, and hence either

$$d(u, A) \neq d(v, A) \text{ or } d(u, B) \neq d(v, B) \tag{1}$$

Similarly, if u and v lies in a same β or γ lines, then $\{B,C\}$ or $\{A,C\}$ resolves u and v respectively. Therefore $\{A,B,C\}$ is a resolving set for G. Let $T_1(G)$ be the subgraph of G enclosed by the lines $\alpha = -(2n-1)$, $\beta = (2n-1)$ and $\gamma = -(2n-1)$. And let $T_2(G)$ be the subgraph of G enclosed by the lines $\alpha = (2n-1)$, $\beta = -(2n-1)$ and $\gamma = (2n-1)$. Clearly $G = T_1(G) \cup T_2(G)$ and $T_1(G) \cap T_2(G)$ is a subgraph of Hexagonal network $HX(2n)$ which is called Hex Oxide windows $HOW(n)$.

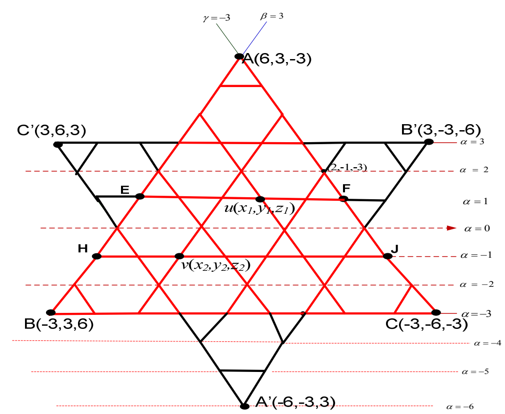

Figure-10. Edges of equilateral triangle graph $T_1(G)$ is highlighted with red color.

Case-1: If u and v belongs to $T_1(G)$ and $x_1 = x_2$, then

$$d(u, B) \neq d(v, B) \text{ and } d(u, C) \neq d(v, C) \tag{2}$$

If u and v belongs to $T_1(G)$ and $y_1 = y_2$, then

$$d(u, A) \neq d(v, A) \text{ and } d(u, B) \neq d(v, B) \tag{3}$$

If u and v belongs to $T_1(G)$ and $z_1 = z_2$, then

$$d(u, A) \neq d(v, A) \text{ and } d(u, C) \neq d(v, C) \tag{4}$$

Case-2: Similarly the equations (2), (3), and (4) are true in $T_2(G)$.

Case-3: If u and v belongs to $T_1(G) \cap T_2(G)$, then the equations (2), (3), and (4) are also true.

Case-4: If u belongs to $T_1(G)$ and v belongs to $G - T_1(G)$ then $d(u, A) \neq d(v, A)$ if $x_1 = x_2$, $d(u, B) \neq d(v, B)$ if $y_1 = y_2$ and $d(u, C) \neq d(v, C)$ if $z_1 = z_2$.

Case-5: If u and v are vertices in $T_1(G)$ with $x_1 \neq x_2$, $y_1 \neq y_2$ and $z_1 \neq z_2$ then $d(A, u) \neq d(A, v)$.

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Let us prove the case 5. If u and v are vertices in $T_1(G)$ with $x_1 \neq x_2$, $y_1 \neq y_2$ and $z_1 \neq z_2$, then there exist two equilateral triangle cycle path t_1 (AEF) sub graph and t_2 (AHJ) subgraph as in Figure-10, clearly $d(u, A) \neq d(v, A)$

Similarly we can prove the following case-6.

Case-6: If u and v are vertices in $T_2(G)$ with $x_1 \neq x_2$, $y_1 \neq y_2$ and $z_1 \neq z_2$ then $d(A', u) \neq d(A', v)$. From Case 5 and 6, we get case 7.

Case-7: If $u(x_1,y_1,z_1)$ and $v(x_2,y_2,z_2)$ are vertices in $SD(n)$ then $d(u, A) = d(v, A)$ if and only if $d(u, A') = d(v, A')$.

By the above discussion we get case-8.

Case-8: If $u(x_1,y_1,z_1)$ and $v(x_2,y_2,z_2)$ are vertices in $T_2(G)$ with $x_1 \neq x_2$, $y_1 \neq y_2$ and $z_1 \neq z_2$, then $d(u, A') \neq d(v, A')$ if and only if $d(u, A) \neq d(v, A)$.

Case-9: If $u(x_1,y_1,z_1)$ and $v(x_2,y_2,z_2)$ are vertices on a same α channel then all the vertices in $\{B, B',C, C'\}$ will satisfy the required condition.

Case-10: Similarly , If $u(x_1,y_1,z_1)$ and $v(x_2,y_2,z_2)$ are vertices on a same β , γ channels then all the vertices in $\{A,A', B,B'\}$,$\{A,A',C,C'\}$ will satisfy the required condition, respectively.

Case-11: If $u(x_1,y_1,z_1)$ and $v(x_2,y_2,z_2)$ are vertices on any two different channels then all vertices in $\{A,A',B,B', C,C'\}$ will satisfy the required condition.

Case-12: If $u(x_1,y_1,z_1)$ is a vertex on any α or β *or* γ channel and $v(x_2,y_2,z_2)$ is a vertex on any α or β *or* γ lines , then

(i) $d(u, A) = d(v, A)$ if and only if $d(u, A') \neq d(v, A')$ if and only if u and v are adjacent and lying on a same tetrahedron.

(ii) $d(u, B) = d(v, B)$ if and only if $d(u, B') \neq d(v, B')$ if and only if u and v are adjacent and lying on a same tetrahedron.

(iii) $d(u, C) = d(v, C)$ if and only if $d(u, C') \neq d(v, C')$ if and only if u and v are adjacent and lying on a same tetrahedron.

Let us prove the case-12. Let $u(x_1,y_1,z_1)$ is a vertex on any α or β *or* γ channel, $v(x_2,y_2,z_2)$ is a vertex on any α or β or

γ lines. If $d(u, A) = d(v, A)$ (5)

then $d(u, A') \neq d(v, A')$ (6)

Suppose $d(u, A') = d(v, A')$, then there exist a cycle of even length $2d_1+2d_2$ with a path such that $d(u, A) = d(v, A) = d_1$ and $d(u, A') = d(v, A') = d_2$ or cycle of length 3 with tails. See Figure-11.

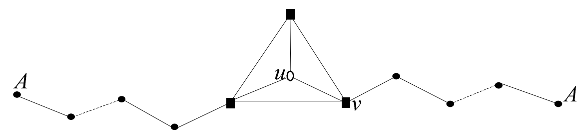

Figure-11. Cycle with tails.

But this is true if and only if u and v lies on a same (α or β or γ) channel or lines respectively. This is contradicts our equation (5). Hence $d(u, A) = d(v, A)$ if and only if $d(u, A') \neq d(v, A')$. Similarly we can prove (ii) and (iii).

Thus a set of vertices $\{A,A',B,B',C,C'\}$ is a metric basis for $SSL(n)$. Hence the metric dimension of *Star of Silicate network* of dimension n is 6.

Single Oxide and silicate Chains

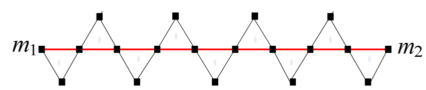

Figure-12. Single Oxide chain of length 9.

Theorem 6: The metric dimension of Single Oxide chain of length l is 2.

Proof: It is easy to see that this graph does not contain an even cycle. Let m_1 and m_2 are initial and final node of the Oxide chain of length l respectively. Any two nodes u and v not lie in red line are resolved by both m_1 and m_2. If u not lies in red line , v lies in red line with $d(u , m_1)= d(v, m_1)$ then we must have $d(u , m_2) \neq d(v, m_2)$. Suppose that $d(u , m_1)= d(v, m_1)$ and $d(u , m_2) = d(v, m_2)$ then there exist cycle of even length , which is a contradiction. Therefore $d(u , m_2) \neq d(v, m_2)$, and $\{ m_1, m_2\}$ resolves the chain. Hence the result.

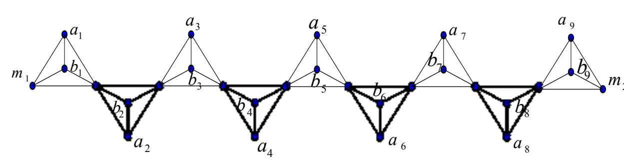

Figure-13. Single Silicate chain of length 9.

Theorem 7: The metric dimension of Single Silicate chain of length l is at least l +2.

Proof: Let m_1 and m_2 be initial and final node of the chain of length l respectively. For $i=j$, each pair (a_i ,b_j) are equal distance from all other nodes, therefore , one of each pair(a_i,b_j) must present in the metric basis(Figure-14).
Let us assume the set $\{ a_i$ / i= 1to l $\}$ is present in the basis, and m_1 or Silicate node adjacent to m_1 must present in the basis , because both are equal distance from all other nodes . Similarly m_2 or silicate node adjacent m_2 must present in the basis because both are equal distance from all other nodes. Hence the cardinality of the metric basis is at least l+2.□

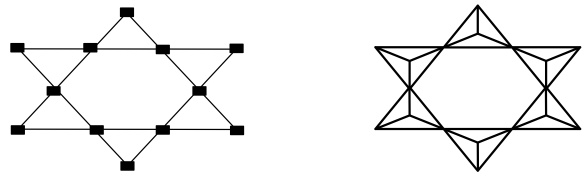

Figure-14. Cyclic oxide and cyclic silicate of length 6.

2. CONCLUSIONS

In this paper we have investigated the metric dimension of Silicate star, single oxide chains, and single Silicate chains. The metric dimension of silicate network is depends up on the dimension of the network. Here we have achieved new silicate structures for which the metric dimension is always constant irrespective of its dimension. Silicate stars is also a better networks than honeycomb networks, because it admits more processor and links than honeycomb network and has constant metric dimension 6.The new silicate windows like hexagonal, triangle and rectangle windows are under investigation.

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