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A NUMERICAL STUDY OF SCF CONVERGENCE USING ANSYS

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ABSTRACT

In this paper a metallic plate made of steel with an elliptical hole, having fixed long radius and variable short radius is pressure loaded. A comparison is made between the results obtained from analytical equations (from reference 16) for a plate with an elliptical hole and the results obtained from FEA. To show that increasing the order of the element can be one way to improve the results obtained from FEA model two different finite elements (4 node and 8 node plane element) were used on the model. This was shown by measuring the length of the element at the tip of the ellipse. It is possible to produce a more accurate FEA model by increasing the number of elements in a mesh. To show this, the number of elements used to mesh the model were recorded and compared for each ellipse size.

Keywords: ANSYS, FEM, convergence, SCF, elliptical hole.

INTRODUCTION

Machine parts are only as strong as its weakest point. In strictly engineering terms, the design of machine elements is focused reasonably on the regions where stress concentration occurs. Plates having holes are a practical requirement in various aeronautical, marine, naval, mechanical and civil arrangements. In many aerospace and naval machine components these kinds of holes are provided to reduce the overall weight of the assembly and to lay electrical and fuel lines. Many a times such cut-outs are provided for fulfilling some maintenance function like entry and examination of internal sections in aircrafts and in bridges and aero structures having plated structures such as box girders, bridge deck. These cut-outs reduce the strength of the structure and make the structure prone to failure. The stress distribution around an elliptical cut-out in an infinite plate under axial UDL was calculated theoretically by Inglis [1] and Kolosoff [2]. Neuber [3] developed an approximate theoretical method which permits the determination of the value of the maximum stress in a finite plate having a central elliptical cut-out subjected to the same loading. Durelli and Murray [4] worked out the stress distribution around the boundary of an elliptical cut-out in an infinite plate under biaxial loading. Petersons [5] studied the unexpected variations in component geometry of isotropic material subject to static loading and reported its effect on design of machine component. The SCF for different composite materials are also presented by Shiau and Lee [6]. Simha and Mahapatra analytically studied the stress concentration caused by irregular holes [7]. Gao [8] in his pioneering work analytically found the SCF of infinite plate with circular/elliptical cut out for biaxial loading. The effect of dynamic and static loading on stress concentration for a rectangular plate was studied by Zirka *et al*. [9]. Both orthotropic and isotropic plates were studied by them using photo elastic method. Tafreshi [10] used FEM (Finite Element Method) and BEM (Boundary Element Method) for stress analysis of thick flat plates having oblique cut-outs under uniaxial tension and out-of-plane bending. Kumar *et al*. [11] did a parametric study on several plate slenderness ratios and by changing the area ratio of cut out to plate to study the influence of ultimate strength on the size of cut-out. It was found that when the area ratio along the loading direction is increased the ultimate strength decreases. Hasan [12] conducted static analysis on metallic plate for several types of holes shapes, sizes and orientations using finite element method by the commercial software COMSOL. Kalita *et al*. [13-15] has studied the variation of deflection and induced stresses due to presence of central cut-outs under transverse loading.

MATHEMATICAL FORMULATION

The problem of stress analysis is typically a three dimensional problem. However, it is often seen that most practical problems are plane stress or plane strain problems. By considering the case of plane stress or plane strain a three-dimensional problem can be analyzed similarly like a two-dimensional problems. The integration of equilibrium equations together with the compatibility equations and boundary conditions are needed to find the solution of the 2D problem. If the body forces are neglected, the following equations are needed to be satisfied

$$\partial\sigma x / \partial x + \partial\sigma xy / \partial x = 0 \quad (1)$$

$$\partial\sigma y / \partial y + \partial\sigma xy / \partial x = 0 \quad (2)$$

$$(\partial 2 / \partial x2 + \partial 2 / \partial y2))(\sigma x + \sigma y) = 0 \quad (3)$$

By substituting the stress components using displacement components u and v into Eq. (1) to (3), the Eq. (3) is made redundant and Eq. (1) and (2) gets transforms into

$$\partial 2u/\partial x2 + (1\text{-}v)/2(\partial 2u/\partial y2) + (1+v)/2\ (\partial 2v/\partial x\partial y) = 0 \quad (4)$$

$$\partial 2v/\partial y2 + (1\text{-}v)/2(\partial 2v/\partial x2) + (1+v)/2(\partial 2u/\partial x\partial y) = 0 \quad (5)$$

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Now by finding u and v from a two dimensional field satisfying the two partial differential Eq. (4) and (5). Instead of determining the two functions u and v the problem can be reduced to solving a single function ψ(x,y), which can be determined by satisfying Eq. (4) and (5). The displacement potential function ψ(x, y) can be defined as

$$u = \partial^2 \psi/\partial x \partial y \quad (6.1)$$

$$v = - [(1-v)\, \partial^2 \psi/\partial y^2 + 2\partial^2 \psi/\partial x^2]/ (1-v) \quad (6.2)$$

By the above definitions the displacement components u and v satisfies Eq. (4) and the only condition reduced from Eq. (5) that the function ψ(x, y) has to satisfy is

$$\partial^4 \psi/\partial x^4 + 2\, \partial^4 \psi/\partial x^2\partial y^2 + \partial^4 \psi/\partial y^4 = 0 \quad (7)$$

So, now the problem is to evaluate a single function Ψ (x, y) from the bi-harmonic Eq. (7), satisfying the boundary conditions specified at the boundary.

This paper is a study of SCF under elastic loading conditions only i.e. the modulus of elasticity does not affect the results since the stress does not exceed the yield point of steel (250 MPa). The maximum stress will be
σ_{max} = SCF x $\sigma_{nom.}$
Here, σ_{nom} is 1000 MPa for all the cases.

Equation (8) is the stress concentration factor for an elliptical hole in a flat plate [16]. Equation (8) is only valid if a/b ratio is between 0.5 and 10. In this paper the ratio of a/b varies from 1 to 10.

$$K = C1 + C2.\left(2.\frac{a}{D}\right) + C3.\left(2.\frac{a}{D}\right)^2 + C4.\left(2.\frac{a}{D}\right)^3 \quad (8)$$

$$C1 = 1.00 + 2.00.\left(\frac{a}{b}\right)$$

$$C2 = -0.351 - 0.021.\sqrt{\left(\frac{a}{b}\right)} - 2.488.\left(\frac{a}{b}\right)$$

$$C3 = 3.621 - 5.183.\sqrt{\left(\frac{a}{b}\right)} + 4.494.\left(\frac{a}{b}\right)$$

$$C4 = -2.270 + 5.204.\sqrt{\left(\frac{a}{b}\right)} - 4.011.\left(\frac{a}{b}\right)$$

Where,
a = the long radius of ellipse (100 mm)
b = the short radius of ellipse (will be varied from 10 mm to 100 mm in 10 mm increments)
D = width of the flat plate (1000 mm)
K = stress concentration factor for an elliptical hole in a flat plate.
When a = b for the last case the elliptical hole becomes a circular hole and equation (8) yields a SCF of 2.538. Reference 16 also contains specific analytical equations for a circular hole.

$$K = 3.00 - 3.13\, (2r/D) + 3.66\, (2r/D)^2 - 1.53\, (2r/D)^3 \quad (9)$$

Where,
r = radius of the circular hole
D = width of the plate

Equation (9) yields a SCF of 2.508 which is only 1.16% less as compared to SCF obtained by equation (8).

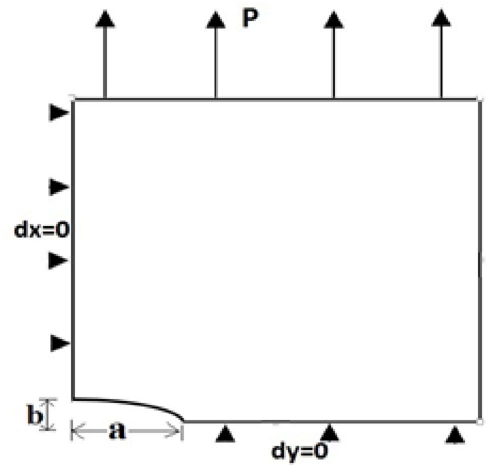

Figure-1. Schematic diagram of the constrained quarter plate and applied load.

METHODOLOGY

A flat square plate with an elliptical hole made of steel (E=210 GPa and Poisson's ratio 0.3) with dimensions of 1000 mm width with long radius of 100 mm and a short radius that is varied in length from 100 mm to 10 mm in 10 mm increments, was loaded with a 1000 N pressure load. The objective of this work is to study the influence of an elliptical hole on the stress distribution. The sharpness of the ellipse is increased such that it transforms from a circle to a narrow crack (Figure-1).

A quarter plate (Figure-1) is modeled in ANSYS with side 4 constrained in X- direction and side 1 constrained in Y-direction. A pressure load of 1000 N/mm^2 is applied to the top edge. Free mesh on quarter

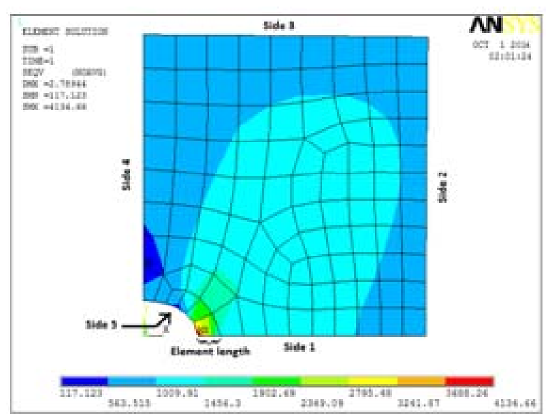

Figure-2. A typical meshed simulation result.

plate is applied by using the element smart size option available in ANSYS and the SCF so obtained is recorded in Table-1. This value is compared with the analytical SCF obtained from equation (8) and deviations are also recorded in Table-1.

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The next step is to determine the SCF within 1% accuracy of that obtained from the analytical equations. This is done by applying manual controls to mesh the geometry.

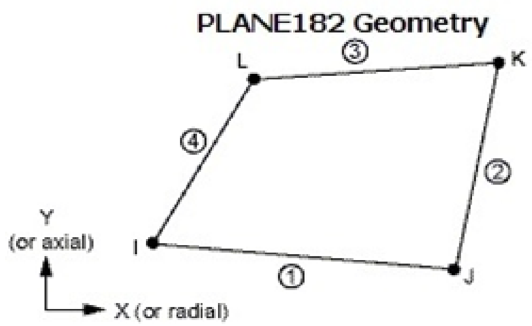

Figure-3. A typical 4 node element used in the analysis.

Suitable line divisions for element creation is selected and the values are recorded in Table-2 along with the new found SCF which is in better agreement with the analytical results. The following command is used to easily set the desired number of elements per line in the model.

LESIZE, _Y1, , ,10, 3, , , ,1

This command will generate 10 elements on the assigned line with a scaling factor of 3, i.e. the elements at the near end of ellipse will be 3 times as small as the element at the far end on the same line. This procedure is needed so that the FEA analysis is able to capture the SCF at the uniform boundary easily. To establish that increasing the order of element would improve our results, two element types are compared- a 4 node plane element and an 8 node plane element (specified as PLANE182 and PLANE183 in the ANSYS library). PLANE182 is generally used for modeling 2-D solid structures. It has four nodes each having two degrees of freedom: Ux and Uy i.e. translational DOFs in x and y directions. PLANE183 is a higher order 2-D, eight node or six node element. PLANE183 takes quadratic displacement behaviour and is convenient for modeling irregular meshes. PLANE183 has 8 nodes or 6 nodes each with 2 DOF: Ux and Uy i.e. translational DOFs in x and y directions. Both the elements may be used as a plane element (plane stress, plane strain and generalized plane strain) or as an axisymmetric element. Both this elements have plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. They also have mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyperelastic materials.

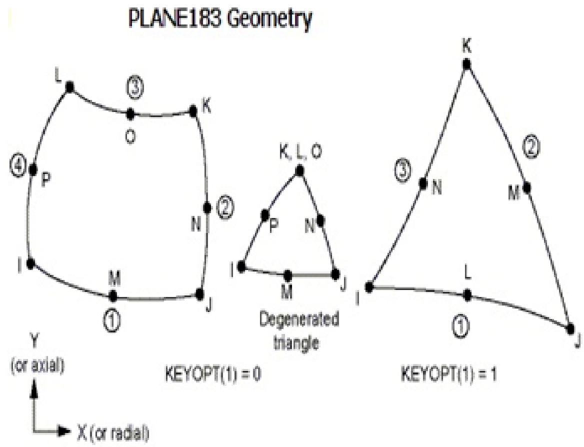

Figure-4. A typical 4 node element used in the analysis.

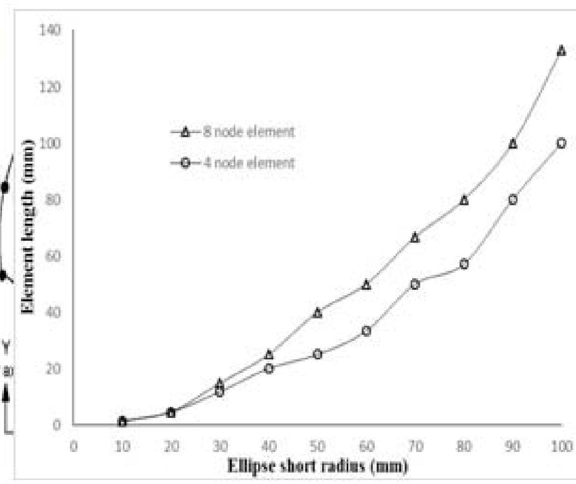

Figure-5. Element length vs. ellipse short radius.

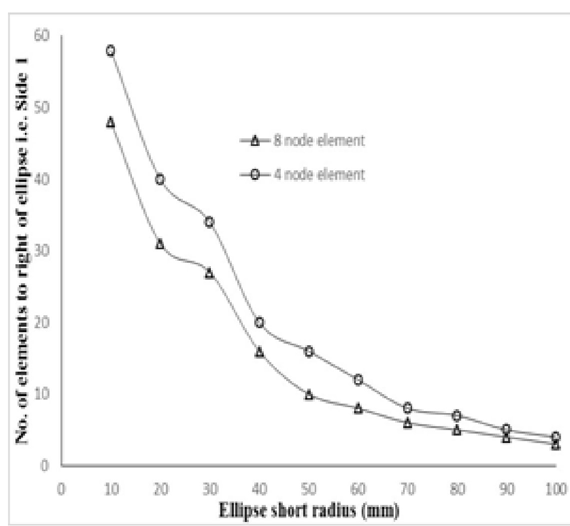

Figure-6. No. of elements used to mesh side 1.

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Table-1. Comparison of SCF obtained analytically and from quarter plate FEA model using smart mesh size 2.

a	**b**	**D** (width)	**a/b**	**SCF** (analytical)	**SCF** (Plane 182)	**% Error**	**SCF** (Plane 183)	**% Error**
100	10	1000	10	17.030	20.360	-19.554	21.054	-23.6291
100	20	1000	5	8.932	11.431	-27.979	11.226	-25.6835
100	30	1000	3.333	6.250	8.130	-30.082	7.976	-27.6184
100	40	1000	2.5	4.916	6.169	-25.500	6.223	-26.5984
100	50	1000	2	4.118	4.952	-20.242	5.229	-26.9678
100	60	1000	1.667	3.589	4.591	-27.921	4.585	-27.7535
100	70	1000	1.429	3.212	3.967	-23.502	4.100	-27.6424
100	80	1000	1.250	2.930	3.622	-23.602	3.733	-27.3896
100	90	1000	1.111	2.712	3.425	-26.294	3.519	-29.7601
100	100	1000	1	2.538	3.238	-27.598	3.321	-30.8684

Table-2. Comparison of SCF obtained analytically and from quarter plate FEA model using manual element size selection procedure with 4 node plane element (PLANE 182).

a/b	**SCF** (analytical)	**SCF** (FEA)	**No. of elements/side**					**Element length** (mm)	**Total no. of elements**
			Side 1	**Side 2**	**Side 3**	**Side 4**	**Side 5**		
10	17.030	17.031	58	30	30	50	50	1.262	1742
5	8.932	8.859	40	30	30	40	40	4.603	1471
3.33	6.250	6.243	34	12	12	17	16	11.765	408
2.5	4.916	4.897	20	12	12	17	12	20	283
2	4.118	4.107	16	9	9	12	6	25	141
1.67	3.589	3.624	12	8	8	10	6	33.33	109
1.43	3.212	3.227	8	7	7	9	5	50	76
1.25	2.930	2.954	7	7	7	7	5	57.143	59
1.11	2.712	2.696	5	6	6	6	5	80	44
1	2.538	2.536	4	4	4	4	4	100	20

Table-3. Comparison of SCF obtained analytically and from quarter plate FEA model using manual element size selection procedure with 8 node plane element (PLANE 183).

a/b	**SCF** (analytical)	**SCF** (FEA)	**No. of elements/side**					**Element length** (mm)	**Total no. of elements**
			Side 1	**Side 2**	**Side 3**	**Side 4**	**Side 5**		
10	17.030	16.975	48	20	20	30	36	1.462	688
5	8.932	8.909	31	17	17	25	25	4.654	522
3.33	6.250	6.253	27	12	12	17	16	14.815	308
2.5	4.916	4.947	16	12	12	15	10	25	225
2	4.118	4.135	10	11	9	8	6	40	105
1.67	3.589	3.623	8	8	8	8	6	50	77
1.43	3.212	3.24	6	7	7	7	5	66.67	56
1.25	2.930	2.941	5	7	7	7	5	80	54
1.11	2.712	2.732	4	5	5	6	4	100	31
1	2.538	2.528	3	3	3	3	3	133	12

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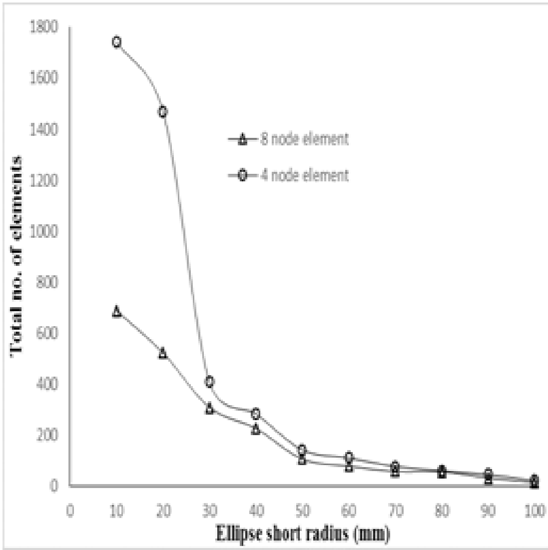

Figure-7. Number of elements used to mesh the quarter plate model.

RESULTS AND DISCUSSIONS

Figure-5 shows that for capturing the same SCF around geometry the FEA analysis using an 8 node element uses a larger element length than a 4 node element. This means that the 4 node element needs more refinement to capture the same SCF which is within +/- 1% of the analytical solution. Also, we see that as the hole becomes narrower an increasingly smaller element is needed. For instance, Plane 182 had an element length of 1.262 mm as compared to 1.462 mm of Plane 183 element in the model with b = 10 mm. Figure-6 and Figure-7 shows that the accuracy of the FEA model would increase by increasing the number of elements used. Both these Figures show that the 8 node element is more efficient at meshing the geometry since it produces the same results as the 4 node element even when using considerably lesser number of elements.

CONCLUSIONS

Any disruption in structure causes change in stress flow patterns and it reduces the strength of the structure. Stress concentration always occurs near the discontinuity in structure. It is seen that the analytical equations for calculating SCF around elliptical holes can be used to calculate SCF around circular holes with sufficient accuracy. It is also seen that the accuracy of an FEA model can be increased by two methods. The first method is to increase the order of the element used in the model. It is seen that the 4-noded plane element (Plane 182) needed smaller elements and more of them in order to capture the stress concentration factor to be within +/- 1% of the closed form solution for the stress concentration factor for this specific geometry, as compared to the 8-noded plane element (Plane 183). The second method is to increase the number of elements in the model. This is done by showing that as the ellipse became narrower; more elements were needed in order to obtain the stress concentration factor within 1% of the closed form solution to equation (8) for the specific geometry.

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