



## DYNAMIC ANALYSIS OF ROTATING COMPOSITE CANTILEVER BLADES WITH PIEZOELECTRIC LAYERS

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### ABSTRACT

Rotating plates in form of turbine blades or machinery parts are often encountered in industrial engineering. The dynamic characteristics of these plates are useful information from design point of view. This paper deals with vibrational analysis of the skew composite plates with piezoelectric layers. In this paper an attempt has been made to study the influence of skew angle and rotational velocity on the free vibration frequencies of a cantilever composite plate with piezoelectric layers. A commercial finite element package ANSYS is used as solver for the problem. The obtained results are compared with existing literature and good convergence in results is seen.

**Keywords:** piezoelectric, composite, natural frequency, FEM, dynamic analysis.

### INTRODUCTION

Plates as well as structures made of plates find a wide variety of application in aerospace, marine, construction and automobile industry. Thus a good knowledge of the static and dynamic behavior is essential to assess and use the full potential of plates. A number of articles are available on static [1-5] and dynamic analysis [6-10] of plates. In many cases we can see that the effect of vibration is very prominent whether it is small in amplitude or large. Blades of any turbine or aeroplane wings the effect of vibration can be severe as those are flexible structures. Due to the effect of vibration, deflection occurs i.e. strain comes into picture. This deflection can cause instability. To make the structure more flexible and economically viable the use of composite materials has been introduced in order to decrease the weight. But still the effect due to vibration could not be minimized to satisfactory level, and if it was done by anyway i.e. compromising the flexibility of the structure. So there lies a keen need for a system which will optimize the flexibility and also the control to the vibration aroused. Piezoelectric materials display mechanical distortions when any electric field or potential is applied. This type of materials has recently received a lot of attention to the control of the flexible structure. These piezoelectric materials, bonded to the surface to the surface of a structural element (as a plate or beam), transfer forces to the structural member according to the magnitude of the excitation voltage applied to them. This type of structures is known as adaptive or smart structures in which the active control to the deflection of the structural element is included. In recent times a remarkable amount of research in the area of adaptive control has been going on. The adaptive control of structures using piezoelectric materials as the satisfactory means of decrement of the vibration in aero structures without compromising the flexibility is of significant interest. The finite element method is one of the most reliable and widely used tools for the numerical analysis of

piezoelectric materials. Plenty of literature is available on the application of FEM for simulating piezoelectric materials.

Kadivar and Samani [11] used layerwise theory to study the free vibration of a rotating thick laminate with simply supported boundary conditions. Koo [12] investigated the effects of layerwise in-plane displacements on fundamental frequencies for laminated composite plates. Tahani and Nosier [13] applied layer wise theory to determine the inter-laminar stresses in a rectangular cross-ply laminated plate under pure bending. The work was limited to simply supported boundary conditions. Tahani and Nosier [14] later used the theory to investigate the inter-laminar stresses near the free edges of a cross-ply simply supported laminated plate due to axial extensional loading. David [15] developed a two dimensional plate theory for the analysis of multilayered piezoelectric plates.

Recently much work has been done by Biswas and Ray [16] on the formulation of finite element of the smart or adaptive structures. Beam theory has played a vital role in study of vibration characteristic of mechanical elements like for turbine blade. Several excellent literatures are available on vibratory behavior of twisted beams [17-20]. Thomas and Soares [21] have analyzed the dynamic response of rotating structures by using a super parametric shell element. The work of Trompette and Lalanne [22] analyzed rotating turbine blades by considering 3D models. A 24 node isoparametric elements was used in their study. Their work is quite significance because they have presented the effect of varying temperature on turbine blade frequencies. However in their study Coriolis acceleration effect was neglected, which resulted in approximate solutions. An adaptive structure was analyzed by Bruke and Hubbard [23] by treating it as one dimensional beam model. Krishna and Mei [24] calculated buckling and post-buckling stresses of thin four-sided laminated plates with piezoelectric materials by means of finite element analysis.



Ramamurthy and Sreenivasamurthy [25] performed a comprehensive dynamic study on rotating pre twisted plates in addition to tapered cantilever plates.

In this paper, a composite cantilever plate is modeled and solved to study its dynamic characteristics. Rectangular plates with skew angles have been analyzed with considering the influence of rotational forces together with the Coriolis Effect on the plate structure. Most of the other literature had neglected the Coriolis Effect due to the complex nature of the problem in hand. Though there are a couple of articles based on analytical and empirical methods on problem, to the best of our knowledge there is no such investigation done using CAE based FEA for vibrational analysis of rotating composite plates with piezoelectric layers. The paper is structured in the following fashion. Section 1 gives a brief idea on the previous research works and motivation for the present work. Section 2 contains the description of the problem and its finite element modeling. Validation as well as results are provided in section 3 followed by summary bulletin of the study under Section 4.

#### PROBLEM DEFINITION AND FEA MODELLING

As already pointed out the speed of rotation ( $\Omega$ ) and pre twist angles have significant influence on natural frequencies. Hence natural dynamic response of the blades have been calculated for various parameters and the value of natural frequency is presented for several non-dimensional speed of rotation and of skew angle ( $\alpha = 0^\circ, 45^\circ, 90^\circ$ ). The non-dimensional speed of rotation is considered between 0 and 1 and pre-twist of  $0^\circ, 45^\circ, 90^\circ$  are considered. Figure-1 shows a plate with skew angle  $45^\circ$  fixed to a hub. The rotation of the hub causes the plate to rotate similar to the blades of a turbine. The mesh sizes were chosen coarse at the fixed part of the structure (the hub) and fine at the rotatory part of the simplified structure. Throughout the study the material of the turbine blade (which is basically a thin plate) is expected to be homogeneous and isotropic. The Poisson's ratio is taken as 0.3. Structural steel has been used for the material of the isotropic core plate. For piezoelectric material simulation purposes PZT5 is used. It is assumed that the material is polarized perpendicularly to Y axis, the stiffness matrix  $c$  [ $\text{N m}^{-2}$ ], piezoelectric matrix  $e$  [ $\text{C m}^{-2}$ ] and relative permittivity matrix  $g$  [n/n] are given respectively as:

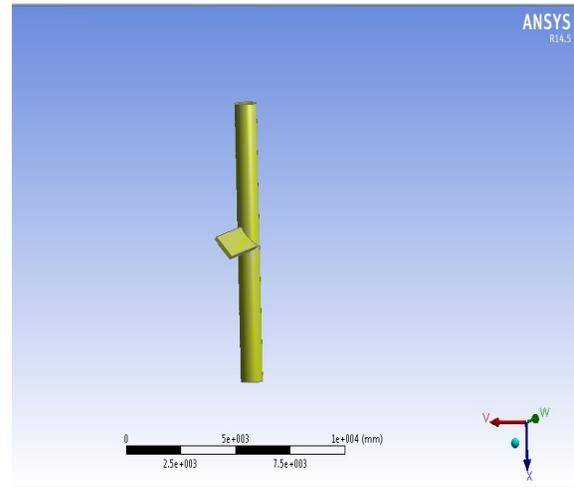
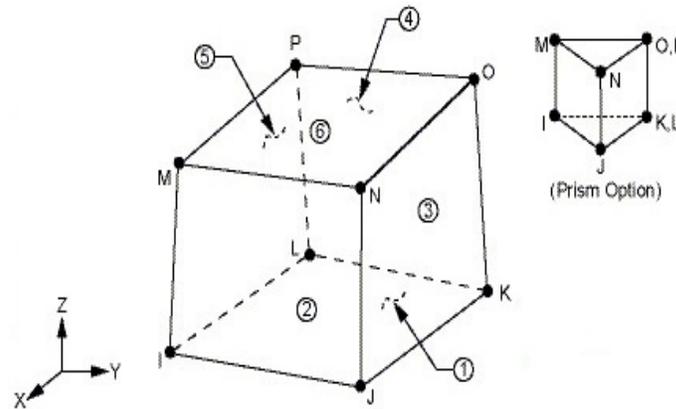


Figure-1.  $45^\circ$  skew angle of the plate with hub.

Composite plates with piezoelectric material are analyzed for different aspect ratio keeping the thickness 1 mm for the upper and bottom layers of piezoelectric material and 10 mm for the isotropic plate sandwiched between the two layers. The fiber orientation of the composite is  $0/90/0$ . SOLID5 element is used to mesh the isotropic core plate and SOLSH190 (SOLID SHELL-3D FINITE STRAIN) element is used to mesh the two piezoelectric layers on the top and bottom of the isotropic core plate. SOLID5 (Figure-2) has a 3-D magnetic, thermal, electric, piezoelectric and structural field capability with limited coupling between the fields. The element has eight nodes with up to six degrees of freedom at each node. Scalar potential formulations (reduced RSP, difference DSP, or general GSP) are available for modeling magneto static fields in a static analysis. When used in structural and piezoelectric analyses, SOLID5 has large deflection and stress stiffening capabilities.

The geometry, node locations, and the element coordinate system for SOLSH190 element are shown in Figure-3. The element is defined by eight nodes. The element coordinate system follows the shell convention where the z axis is normal to the surface of the shell. The node ordering must follow the convention that the I-J-K-L and M-N-O-P element faces represent the bottom and top shell surfaces, respectively. SOLSH190 employs incompatible modes to enhance the accuracy in in-plane bending situations. The satisfaction of the in-plane patch test is ensured. A separate set of incompatible modes is adopted to overcome the thickness locking in bending dominant problems. The incompatible modes introduce seven internal DOFs that are inaccessible to users and condensed out at the element level. SOLSH190 utilizes a suite of special kinematic formulations to avoid locking when the shell thickness becomes extremely small.



SOLID5 Geometry

Figure-2. SOLID 5 Element geometry.

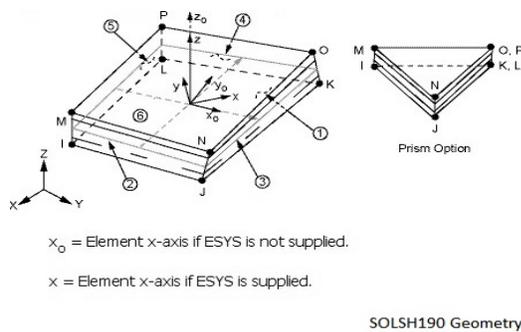


Figure-3. SOLSH190 element geometry.

RESULTS AND DISCUSSIONS

To demonstrate the influence of the rotational velocity on the natural frequencies of the vibration of the structure only the results in the case of aspect ratio 4 are presented in tabular form. However all the results can be obtained to within 5-10% accuracy of the published

literature? It is also worth mentioning here that the slight variation with the published results is also may be due to the fact that Coriolis effect is considered in this study unlike in reference [20]. The edge-wise bending, torsional and flap-wise bending modes as a function of the normalized angular speed is shown in Tables 1-3, respectively. The normalized angular speed  $\Omega$  is defined as speed of rotation per unit frequency at rest.

i.e.  $\Omega = \frac{\text{speed of rotation}}{\text{frequency at rest}}$

The normalized frequency parameter is defined as the non-dimensional frequency which is scaled under the values where the required changes have been found. A non-dimensional frequency parameter,  $\beta$  is introduced

which is defined as  $\beta = \omega b^2 \sqrt{\rho h / D}$ . The plates assumed to be of uniform thickness with  $h/l = 0.12$ , where “h” is the thickness of the plate and “l” is the length.

Table-1. Influence of skew angle on 1<sup>st</sup>edge-wise bending mode frequency.

Skew angle	Present		Ramamurthy[20]	
	Rotational velocity ( $\Omega$ )	Frequency ( $\beta$ )	Rotational velocity ( $\Omega$ )	Frequency ( $\beta$ )
0°	0	13.11	0	13.07
	0.13	12.72	0.14	13.05
	0.67	12.6	0.69	12.84
45°	0	13	0	13.07
	0.13	12.94	0.14	13.06
	0.67	12.77	0.68	12.95
90°	0	13.07	0	13.08
	0.13	13.01	0.13	13.07
	0.67	12.7	0.66	13.03

**Table-2.** Influence of skew angle on 1<sup>st</sup>torsional mode frequency.

Skew angle	Present		Ramamurthy[20]	
	Rotational velocity ( $\Omega$ )	Frequency ( $f$ )	Rotational velocity ( $\Omega$ )	Frequency ( $f$ )
0°	0	15.05	0	14.99
	0.13	15.31	0.14	15.02
	0.67	15.52	0.67	15.35
45°	0	15.39	0	14.87
	0.13	15.45	0.13	14.99
	0.67	15.5	0.67	15.1
90°	0	15.05	0	15.01
	0.13	15.37	0.13	15.01
	0.67	15.47	0.67	15.02

**Table-3.** Influence of skew angle on 1<sup>st</sup>flap-wise bending mode frequency.

Skew angle	Present		Ramamurthy[20]	
	Rotational velocity ( $\Omega$ )	Frequency ( $f$ )	Rotational velocity ( $\Omega$ )	Frequency ( $f$ )
0	0	3.68	0	3.44
	0.13	4.71	0.13	3.5
	0.67	4.74	0.67	4.43
45	0	3.63	0	3.43
	0.13	4.69	0.13	3.43
	0.67	4.8	67	3.98
90	0	3.65	0	3.43
	0.13	4.71	0.13	3.44
	0.67	4.72	0.66	3.56

Change in skew angle has insignificant effect on the 1<sup>st</sup> edge-wise bending mode frequency. With rise in the normalized rotational velocity a small decrease in the 1<sup>st</sup> edge-wise bending mode frequency is observed. The 1<sup>st</sup> torsional mode frequency rises with increase in rotational speed. It also increases slightly with increase in skew angle. The 1<sup>st</sup> flap-wise bending mode frequency rises monotonously with rise in rotational speed for all the skew angles. However the change in skew angle of the rotating blade has minor influence on the 1<sup>st</sup> flap-wise bending mode frequency.

## CONCLUSIONS

The natural frequencies have been calculated for pre twisted turbine blades for various aspect ratios, and for numerous values of non-dimensional speed of rotation ( $\Omega$ ) between 0 to 1 of skew angle ( $\alpha=0^\circ, 45^\circ, 90^\circ$ ). Excellent convergence with the existing literature has been seen. It is seen that as rotational velocity is increased the fundamental edge-wise bending mode frequency of the plate decreases whereas the first 1<sup>st</sup> torsional mode frequency and the fundamental flap-wise bending mode frequency increases.



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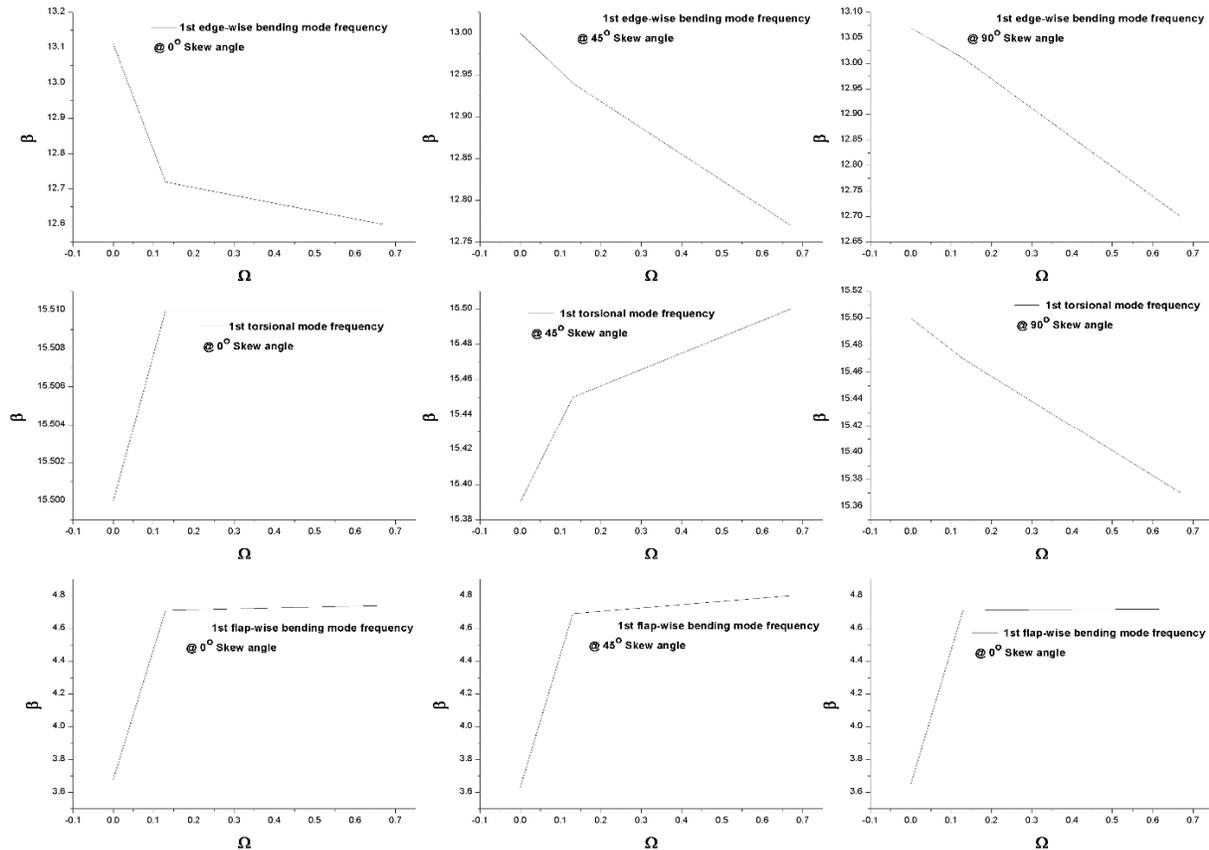


Figure-4. Non-Dimensional frequency parameter vs. rotational velocity.

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