ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

STRIP DEFLECTION COMPATIBILITY METHOD (SDCM) FOR RECTANGULAR PLATE ANALYSIS WITH CORNERS HELD DOWN

Otoyo D. J., Johnarry T. N. and Ephraim M. E.
Department of Civil Engineering, Faculty of Engineering, University of Science and Technology, Nkpolu-Oroworukwo, Port Harcourt, Nigeria
E-Mail: djotoyo@yahoo.com

ABSTRACT

The subject of plate shall continue to gain application in virtually all fields because of its applicability in numerous engineering projects such as in aerospace, building, naval architecture, offshore engineering, and roads. To ensure safe, fast delivery and economy of projects, engineers and researchers require a simplified and faster method of analysis. Strip Deflection Compatibility Method (SDCM) which utilizes basic equation of elastic curves in x, y, and xy directions to derive a Load Distribution Equation readily comes to rescue. With the new but simple model, the problem is reduced to calculating only the load factors in x, y, and xy directions. The factors are then multiplied by equivalent beam moments and deflection at point of interest to obtain plate values. Twist, deflections and Poisson's ratio have all been used in the derivation of the model, thus guaranteeing a more reliable result and may be applied in any material. Results from the model show great promise as they compare favourably to existing classical solutions.

Keywords: plate, strip, deflection, moment, compatibility, load distribution factors, load distribution equation, corners held down.

1. INTRODUCTION

The complexity of plate solutions has made engineers to devise alternative approaches to simplify the problem. Most of the approaches adopted focused on deriving alternative moment fields (M_x and M_y in x and y directions) for designs without considering the effects of twisting moment (M_{xy}) which is a very important characteristics of plates, Li (2004). Although there exists extensive literature on such approaches, most of them go into lengthy and complicated derivations instead of simplifying the problem. Some of these methods pose difficulties in the distribution of load toward corners of plates where twist is high, thereby failing to give a reliable variation in moments over plate area. This situation leads to uneconomical designs.

Pioneering works in the area of plate includes; Lagrange's biharmonic equation, Johansen (1962, 1972) Yied-line analysis, Hillerborg (1956, 1959, 1975, 1982) Strip Method, Kantorovitch and Krylov (1954) approximations and recently Johnarry (1972, 2011, 2013), Ephraim and Orumu (2002, 2013, 2014) among others have also proffer alternative approaches to the solution of plates.

A novel contribution in Strip Deflection Compatibility Method (SDCM) for Plate Analysis is presented here to further enrich existing literatures by providing a new simple analytical model with simplified methods for calculating moments and deflections at any point on the plate. Design charts and Tables for easier and cost effective application are also presented from the research.

1.1 Basic assumptions

- The plate material is linear elastic and obeys Hooke's law.
- The thickness of the plate is small compared to other dimensions.
- Plane-remain -plane and shear deformation excluded.
- Curvature of plate on deformation is approximated by a second degree polynomial.
- Deflections of strips in all directions are equal at their point of intersection.
- Flexural rigidity of strips is EI for x and y directions, GI for xy direction and D for plate.

2. MATERIALS AND METHODS

2.1 Materials

The study relates to rectangular plate made from homogeneous, continuous isotropic and elastic material with constant thickness and yield stress. The material constant include elastic moduli in x, y, and xy directions and Poisson's ratio. The plate is restrained along all edges by hinge connections and loaded with uniformly distributed pressure over the plate area.

2.2 Method

A typical plate is shown below carrying a uniformly distributed load and supported anyhow:

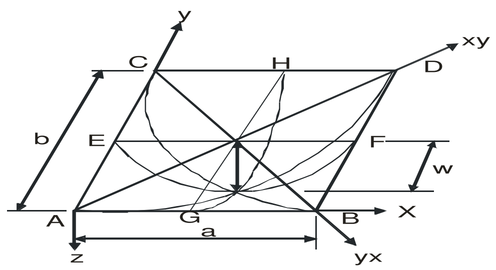


Figure-1. A rectangular plate carrying a uniformly distributed load and supported at edges.

ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

The equation of a plate is given by Lagrange in many texts including Timoshenko and Woinowsky (1959) and several other texts as:

$$\frac{\partial^4 w}{\partial x^4}+\frac{2\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \qquad (1)$$

(***this is sum of loads in x, xy, and y directions***)

Bending and twisting moments from (1) above are expressed as:

$$a)\quad M_{xp} = -D\left[\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right],$$

$$\text{b)}\quad M_{yp} = -D\left[v\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right], \text{ and}$$

$$c)\quad M_{xyp} = -D(1-v)\frac{\partial^2 w}{\partial x\partial y} \qquad (2)$$

Equation (1) can be discretized into load distributions in *x, xy and y* strips respectively as shown below:

a) $\angle^4 w/\angle x^4$; q (x-direction)
b) $2\angle^4 w/\angle x^2\angle y^2$; q (xy direction)
c) $\angle^4 w/\angle y^4$; q (y direction)

And may be rewritten as:

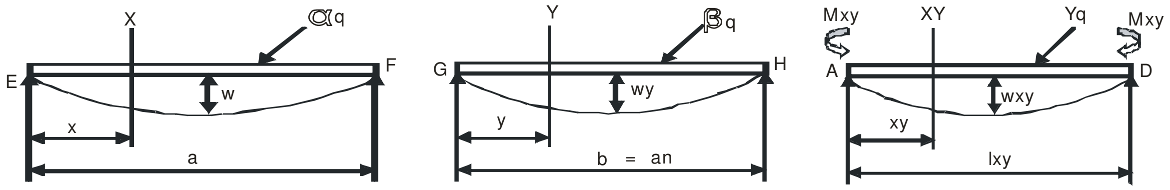
a) $\angle^4 w/\angle x^4 = \boldsymbol{\alpha_p} q/D$,
b) $\angle^4 w/\angle y^4 = \boldsymbol{\beta_p} q/D$ and
c) $2\angle^4 w/\angle x^2\angle y^2 = \boldsymbol{\Upsilon_p} q/D$.
(3)

Where $\boldsymbol{\alpha_p}$, $\boldsymbol{\Upsilon_p}$, and $\boldsymbol{\beta_p}$ are load factors in plate in the x, y, and xy directions respectively.
Substituting (3) in (1) we have;

$\boldsymbol{\alpha_p} + \boldsymbol{\Upsilon_p} + \boldsymbol{\beta_p} = \mathbf{1}$ (The Model or Plate Load Distribution Equation) (4)

The model includes the effects of twist, deflections and Poisson's ratio, thus increasing its applicability in any material. With this simple derivation, the designer is left to calculate only the load factors $\boldsymbol{\alpha_p}$, $\boldsymbol{\Upsilon_p}$ **and** $\boldsymbol{\beta_p}$ and then multiply by the Primitive or beam moments and deflection of equivalent end conditions to achieve plate values.

2.3 Estimation of deflections in x, y, and xy



i ii iii

Figure-2. Strips in x, y, and xy directions, respectively.

From the above, strip moments in x, y, and xy-directions are:

$$m_{xs} = \alpha_s \boldsymbol{M_x} \qquad (5)$$

$$m_{ys} = \beta_s \boldsymbol{M_y} \qquad (6)$$

and Likewise

$$m_{xys} = \Upsilon_s \boldsymbol{M_{xy}} \qquad (7)$$

where: $\mathbf{M_x}$, $\mathbf{M_y}$, and $\mathbf{M_{xy}}$ are equivalent beam moment, $\boldsymbol{\alpha_s}$, $\boldsymbol{\Upsilon_s}$, and $\boldsymbol{\beta_s}$ are load factors in x, y, and xy strips, respectively.

Integrating equation 3(*a*) and (b) and substituting x = a/2 to obtain maximum deflections at the centre (intersection point), we have;

$$w_x = \frac{5\alpha_s q a^4}{384EI} \qquad (8)$$

$$w_y = \frac{5\beta_s q a^4 n^4}{384EI} \qquad (9)$$

Also from Figure-2(iii), the diagonal strip (xy direction); $l_{xy} = a\sqrt{1+n^2}$, $q_{xy} = \Upsilon_s q$ and twisting moment is zero at the centre of strip.

$$\sum M_{centre} = 0$$

and

$$\frac{\Upsilon_s q a^2 \sqrt{1+n^2}^2}{4} - M_{xy} - \frac{\Upsilon_s q a^2 \sqrt{1+n^2}^2}{8} = 0$$

$$\boldsymbol{M_{xy}} = \frac{\Upsilon_s q a^2 (1+n^2)^2}{8} \qquad (10)$$

Plotting this equation against the diagonal we have the graph as in Figure-3.

Using moment area to determine the deflection due to twisting moment we have; $A = \frac{\Upsilon_s q a^2 \ (1+n^2)\ a\sqrt{1+n^2}}{3x8x2}$

$$A = \frac{\Upsilon_s q a^3 \ (1+n^2)\sqrt{1+n^2}}{48} \qquad (11)$$

VOL. 10, NO. 8, MAY 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

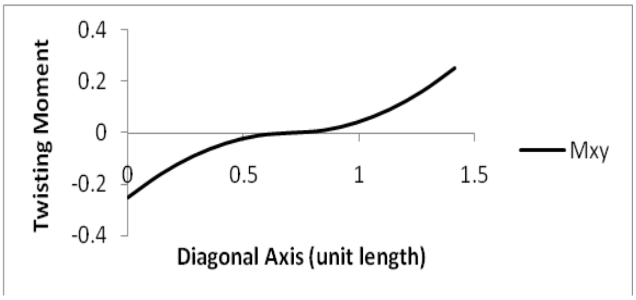


Figure-3. Twisting moment against diagonal axis.

$$\bar{x} = \frac{a\sqrt{1+\mathrm{n}^2}}{2}\frac{1}{4} = \frac{a\sqrt{1+\mathrm{n}^2}}{8}$$

$$\boldsymbol{w_{xy}} = \frac{A\bar{x}}{GI} \tag{12}$$

$$\boldsymbol{w_{xy}} = \frac{\Upsilon_s q a^4 (1+n^2)^2}{384GI} = \frac{\Upsilon_s q a^2(1+v)(1+n^2)^2}{192EI} \tag{13}$$

Moments from strips are as follows:

From 4i

$$\boldsymbol{m_{xs}} = \boldsymbol{EI}\frac{\partial^2 w}{\partial x^2} = \alpha_s \boldsymbol{M_x} \tag{14}$$

From 4ii

$$\boldsymbol{m_{ys}} = \boldsymbol{EI}\frac{\partial^2 w}{\partial y^2} = \beta_s \boldsymbol{M_y} = \beta_s \boldsymbol{M_x} n^2 \tag{15}$$

And from 4iii

$$\boldsymbol{m_{xys}} = \boldsymbol{GI}\frac{\partial^2 w}{\partial x \partial y} = \Upsilon_s \boldsymbol{M_{xy}} \tag{16}$$

The motive here is to determine α_p, β_p, and Υ_s such that they are multiplied by primitive moments to obtain plate values:

$$M_{xp} = \alpha_p M_x,\ M_{yp} = \beta_p M_y \text{ and } M_{xyp} = \Upsilon_p \boldsymbol{M_{xy}} \tag{17}$$

Where M_{xp}, M_{yp}, and M_{xyp} are plate moment in x, y, and xy, respectively.

2.4 Algorithm for application of the Model

- Determination of load factors α_p, β_p and Υ_p;

a) equate strip deflections at point of intersection P(x,y); w_x, = w_y, = w_{xy} and compute β_s, and Υ_s in terms of *n and v*

b) Substitute equations (14), (15), (16), (17) and the ratios from *i* above into equation (2) a, b and c to obtain relationships for α_p, Υ_p and β_p in terms of α_s, *n and v.*

c) Substitute the relations in *ii* above into equation (4) to obtain α_s, α_p, Υ_p and β_p.

- Calculate plate moments from; $M_{xp} = \alpha_p \mathbf{M_x}$, $M_{yp} = \beta_p \mathbf{M_y}$ and $M_{xyp} = \Upsilon_p \mathbf{M_{xy}}$
- Calculate deflections from: $w = 5\alpha_s q a^4/(384D(1-v^2))$
- Repeat steps i-iii for any point on the plate
- Computer simulation of plates with various aspect ratios.

3. TESTING / APPLICATION OF THE MODEL

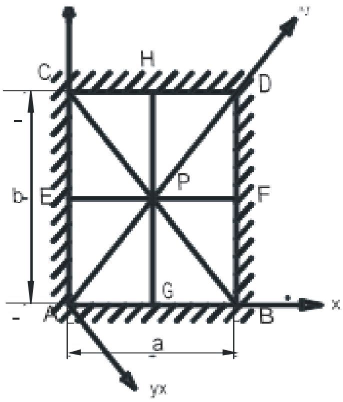


Figure-4. Simply supported rectangular plate carrying a uniformly distributed load over its area.

3.1 Displacement compatibility

- Determination of load factors α_p, β_p and Υ_p;

a. Compute and equate strip deflections in x, y, and xy: $w_x = w_y = w_{xy} = w$

$$\boldsymbol{w} = \frac{5\alpha_s q a^4}{384EI} = \frac{5\beta_s q a^4}{384EI} = \frac{\Upsilon_s q a^4 (1+n^2)^2}{384GI}$$

$$\beta_s = \frac{\alpha_s}{n^4},\ \Upsilon_s = \frac{5\alpha_s}{2(1+v)(1+n^2)^2} \tag{18}$$

Substitute (18) into plate equations (2) a, b, and c;

$$a)\quad M_{xp} = \alpha_p \boldsymbol{M_x} = -D\left[\frac{\boldsymbol{m_{xs}}}{EI} + v\frac{\boldsymbol{m_{ys}}}{EI}\right]$$

Remember

$$\frac{\partial^2 w}{\partial x^2} = \frac{\alpha_s \boldsymbol{M_x}}{EI} = \frac{\boldsymbol{m_{xs}}}{\mathbf{EI}},\quad \frac{\partial^2 w}{\partial y^2} = \frac{\beta_s \boldsymbol{My}}{\boldsymbol{EI}} = \frac{\boldsymbol{m_{ys}}}{EI},\quad \frac{\partial^2 w}{\partial x \partial y} = \frac{\Upsilon_s \boldsymbol{M_{xy}}}{GI} = \frac{\boldsymbol{m_{xys}}}{GI},$$

But $EI=D(1-v^2)$ and $GI = \frac{EI}{2(1+v)}$ and so;

$$\alpha_p \boldsymbol{M_x} = \frac{\alpha_s \boldsymbol{M_x}}{(1-v^2)} + \frac{v\alpha_s \boldsymbol{M_x}}{n^2(1-v^2)} \tag{19}$$

$$\alpha_p = \frac{\alpha_s(n^2+v)}{n^2(1-v^2)} \tag{20}$$

and from 2(b):

$$\beta_p = \frac{\alpha_s(n^2 v+1)}{n^4(1-v^2)} \tag{21}$$

Likewise from 2(c);

$$\Upsilon_p = 2\Upsilon_s \tag{22}$$

And so;

$$\Upsilon_p = \frac{5\alpha_s}{(1+v)(1+n^2)^2} \quad (23)$$

Load distribution

Substitute equation (20), (21), and (23) into (4);

$$\boldsymbol{\alpha_p + \Upsilon_p + \beta_p = 1}.$$

We have;

$$\frac{\alpha_s(n^2+v)}{n^2(1-v^2)} + \frac{5\alpha_s}{(1+v)(1+n^2)^2} + \frac{\alpha_s(n^2v+1)}{n^4(1-v^2)} = 1$$

$$\frac{n^4(1-v^2)(1+n^2)^2}{n^2(n^2+v)(1+n^2)^2+5n^4(1-v)+(n^2v+1)(1+n^2)^2} \quad (24)$$

For $n = 1$, $v = 0.3$, $\alpha_s = 0.26187$, $\alpha_p = 0.3741 = \beta_p$, $\Upsilon_p = 0.2517$

From equation (17)

$M_{xp} = \alpha_p M_x = \alpha_p qa^2/8 = 0.04676qa^2$,

$M_{yp} = \beta_p M_y = 0.04676qa^2$

Deflection

Using our initial assumption;

$w = 5\,\alpha_s qa^4/(384D(1-v^2))$ we can estimate the deflection

$w = 5\text{x}0.26187qa^4/(384D(1-0.3^2)) = 0.003747qa^4/D$.

4. RESULTS AND DISCUSSIONS

4.1 Results

The Strip Deflection Compatibility Method (SDCM) has been developed and used to solve a Simply Supported Plate carrying a UDL over its area with corners held down, giving reasonable results as follows:

Table-1. Load, moment and deflection coefficients for all-round simply supported rectangular plates carrying a uniformly distributed load over its area with corners held down for v = 0.3.

n	α_s	α_p	β_p	Υ_p	$\Upsilon_p M_x$ (SDCM	M_y (SDCM)	w (SDCM)
					.qa^2	.qa^2	.qa^4/D
1.0	0.2619	0.3741	0.3741	0.2518	0.0468	0.0468	0.003747
1.2	0.3659	0.4859	0.2777	0.2364	0.0607	0.0500	0.005236
1.4	0.4629	0.5865	0.2103	0.2032	0.0733	0.0515	0.006623
1.6	0.5472	0.6717	0.1622	0.1661	0.0840	0.0519	0.007829
1.8	0.6169	0.7407	0.1273	0.1320	0.0926	0.0516	0.008827
2.0	0.6728	0.7948	0.1017	0.1035	0.0994	0.0508	0.009627
2.2	0.7169	0.8367	0.0825	0.0809	0.1046	0.0499	0.010258
2.4	0.7515	0.8688	0.0679	0.0633	0.1086	0.0489	0.010753
2.6	0.7786	0.8936	0.0567	0.0497	0.1117	0.0479	0.011141
2.8	0.7999	0.9127	0.0479	0.0394	0.1141	0.0470	0.011446
3.0	0.8169	0.9276	0.0410	0.0314	0.1159	0.0461	0.011688
100	0.9099	1.0000	0.0000	0.0000	0.1250	0.0375	0.013020

ARPN Journal of Engineering and Applied Sciences

Table-2. Load, moment and deflection coefficients for all-round simply supported rectangular plates carrying a uniformly distributed load over its area with corners held down for v=0.2.

n	**α_s**	**α_p**	**β_p**	**Υ_p**	**$\Upsilon_p M_x$** (SDCM	**M_y** (SDCM)	**w** (SDCM)
					$.qa^2$	**$.qa^2$**	**$.qa^4/D$**
1.0	0.2824	0.3529	0.3529	0.2941	0.0441	0.0441	0.003830
1.2	0.3948	0.4683	0.2554	0.2763	0.0585	0.0460	0.005354
1.4	0.4998	0.5737	0.1886	0.2377	0.0717	0.0462	0.006778
1.6	0.5910	0.6637	0.1420	0.1943	0.0830	0.0454	0.008016
1.8	0.6661	0.7367	0.1089	0.1544	0.0921	0.0441	0.009035
2.0	0.7259	0.7940	0.0851	0.1210	0.0992	0.0425	0.009846
2.2	0.7726	0.8380	0.0676	0.0944	0.1048	0.0409	0.010479
2.4	0.8087	0.8716	0.0546	0.0737	0.1090	0.0393	0.010968
2.6	0.8366	0.8973	0.0449	0.0579	0.1122	0.0379	0.011347
2.8	0.8583	0.9169	0.0374	0.0458	0.1146	0.0366	0.011642
3.0	0.8753	0.9320	0.0315	0.0365	0.1165	0.0355	0.011872
100	0.9600	1.0000	0.0000	0.0000	0.1250	0.0250	0.013020

Table-3. Load, moment and deflection coefficients for all-round simply supported rectangular plates carrying a uniformly distributed load over its area with corners held down forv =0.1.

n	**α_s**	**α_p**	**β_p**	**$\Upsilon_p M_x$** (SDCM)	**$\Upsilon_p M_x$** (SDCM	**M_y** (SDCM)	**w** (SDCM)
					$.qa^2$	**$.qa^2$**	**$.qa^4/D$**
1.0	0.2977	0.3308	0.3308	0.3383	0.0414	0.0414	0.003916
1.2	0.4165	0.4499	0.2321	0.3180	0.0562	0.0418	0.005478
1.4	0.5277	0.5603	0.1660	0.2738	0.0700	0.0407	0.006941
1.6	0.6243	0.6552	0.1209	0.2239	0.0819	0.0387	0.008211
1.8	0.7035	0.7325	0.0896	0.1779	0.0916	0.0363	0.009252
2.0	0.7660	0.7930	0.0677	0.1393	0.0991	0.0338	0.010074
2.2	0.8142	0.8394	0.0521	0.1085	0.1049	0.0315	0.010708
2.4	0.8510	0.8745	0.0408	0.0846	0.1093	0.0294	0.011193
2.6	0.8791	0.9011	0.0326	0.0664	0.1126	0.0275	0.011562
2.8	0.9005	0.9212	0.0264	0.0524	0.1152	0.0259	0.011844
3.0	0.9170	0.9366	0.0217	0.0417	0.1171	0.0244	0.012061
100	0.9900	1.0000	0.0000	0.0000	0.1250	0.0125	0.013021

ARPN Journal of Engineering and Applied Sciences

Table-4. Load, moment and deflection coefficients for all-round simply supported rectangular plates carrying a uniformly distributed load over its area with corners held down for v =0.

n	α_s	α_p	β_p	$\Upsilon_p M_x$ (SDCM)	$\Upsilon_p M_x$ (SDCM	M_y (SDCM)	w (SDCM)
					$.qa^2$	$.qa^2$	$.qa^4/D$
1.0	0.3077	0.3077	0.3077	0.3846	0.0385	0.0385	0.004006
1.2	0.4306	0.4306	0.2077	0.3617	0.0538	0.0374	0.005607
1.4	0.5462	0.5462	0.1422	0.3117	0.0683	0.0348	0.007111
1.6	0.6464	0.6464	0.0986	0.2550	0.0808	0.0316	0.008416
1.8	0.7281	0.7281	0.0694	0.2025	0.0910	0.0281	0.009481
2.0	0.7921	0.7921	0.0495	0.1584	0.0990	0.0248	0.010314
2.2	0.8408	0.8408	0.0359	0.1233	0.1051	0.0217	0.010948
2.4	0.8775	0.8775	0.0264	0.0960	0.1097	0.0190	0.011426
2.6	0.9050	0.9050	0.0198	0.0751	0.1131	0.0167	0.011784
2.8	0.9257	0.9257	0.0151	0.0592	0.1157	0.0148	0.012054
3.0	0.9413	0.9413	0.0116	0.0471	0.1177	0.0131	0.012257
100	1.0000	1.0000	0.0000	0.0000	0.1250	0.0000	0.013021

Table-5. Comparison of moments from SDCM and analytical methods for a rectangular plate carrying a uniformly distributed load over its area with corners held down for v =0.3.

n	M_x (SDCM)	M_y (SDCM)	Mx - analy	My-analy
	$.qa^2$	$.qa^2$	$.qa^2$	$.qa^2$
1	0.0468	0.0468	0.0479	0.0479
1.2	0.0607	0.05	0.0627	0.0501
1.4	0.0733	0.0515	0.0755	0.0502
1.6	0.084	0.0519	0.0862	0.0492
1.8	0.0926	0.0516	0.0948	0.0479
2	0.0994	0.0508	0.1017	0.0464

Table-6. Effect of Poisson's Ratio on moment for a simply supported plate carrying uniformly distributed load over its area with corners held down.

n	M_x (SDCM)v= 0.3	M_y (SDCM)v=0.3	M_x (SDCM)v=0.2	M_y (SDCM)v=0.2	M_x (SDCM)v=0.1	M_y (SDCM)v=0.1	M_x (SDCM)v=0	M_y (SDCM)v=0
	$.qa^2$	$.qa^2$	$.qa^2$	$.qa^2$	$.qa^2$	$.qa^2$	$.qa^2$	$.qa^2$
1	0.0468	0.0468	0.0441	0.0441	0.0414	0.0414	0.0385	0.0385
1.2	0.0607	0.05	0.0585	0.046	0.0562	0.0418	0.0538	0.0374
1.4	0.0733	0.0515	0.0717	0.0462	0.07	0.0407	0.0683	0.0348
1.6	0.0840	0.0519	0.083	0.0454	0.0819	0.0387	0.0808	0.0316
1.8	0.0926	0.0516	0.0921	0.0441	0.0916	0.0363	0.091	0.0281
2	0.0994	0.0508	0.0992	0.0425	0.0991	0.0338	0.099	0.0248
2.2	0.1046	0.0499	0.1048	0.0409	0.1049	0.0315	0.1051	0.0217
2.4	0.1086	0.0489	0.109	0.0393	0.1093	0.0294	0.1097	0.019
2.6	0.1117	0.0479	0.1122	0.0379	0.1126	0.0275	0.1131	0.0167
2.8	0.1141	0.047	0.1146	0.0366	0.1152	0.0259	0.1157	0.0148
3	0.1159	0.0461	0.1165	0.0355	0.1171	0.0244	0.1177	0.0131

100	0.125	0.0375	0.125	0.025	0.125	0.0125	0.125	0

Table-7. Comparison of SDCM and analytical deflections for a rectangular plate carrying a uniformly distributed load over its area with corners held down for v =0.3.

n	**w** (SDCM)	**w-Analy**
	.qa^4/D	.qa^4/D
1.0	0.003747	0.00406
1.2	0.005236	0.00564
1.4	0.006623	0.00705
1.6	0.007829	0.00830
1.8	0.008827	0.00931
2.0	0.009627	.01010

4.2 Discussions

The load, moment, and deflection factors (α_s, α_p, β_p, Υ_p, Mxp, M_{yp} and w) against aspect ratio n for a simply supported rectangular plate carrying a uniformly distributed load over its area with corners held down are shown in Tables 1 to 4. Increase in aspect ratio n increases load factors and moments in the shorter span (x-direction). From a minimum value at n = 1, the moment increases as n becomes very large and converges to 0.125qa^2. This characteristic is observable irrespective of Poisson's ratio v. The shorter span moments does not show much difference with Poisson's ratio as the graph of moments for various Poisson's ratios are close/cluster and actually intersect between n = 2.0 and 2.2 (see Table-5 and Figure-A.7). On the contrary the moments M_{yp} in the y-strip converges from maximum values to 0.0375qa^2, 0.0250qa^2, 0.0125qa^2 and 0 for v = 0.3, 0.2, 0.1 and 0, respectively as n becomes very large (see Table-5 and Figure-A.7). There is a great show of disparity among longer span moments with Poisson's ratio (see also Table-5 and Figure-A.7). Similarly when aspect ratio is unity, twisting moment is maximum and reduces to zero as n become very large. This scenario is applicable to all materials (as observable in Tables 1 to 4).

The implication of this to engineers is that at large aspect ratios, simply supported plate behaves similar to simply supported beam and can be designed with shorter span moments only as a wide beam of 1m width. Moment in y-direction can be taken care of using distribution steel if plate is a concrete slab. The effect of twist can be ignored without affecting the strength of the plate. The deflection of the plate calculated when v = 0 is 0.004006qa^4 at n = 1 and increases with aspect ratio to converge at 0.013021qa^4. Poisson's ratio does affect deflection.

Tables 6 compare SDCM moments with analytical solutions with reasonable confidence. Also Table-7 compare SDCM deflections with analytical solutions and the deviation from exact solutions lie between 0.5 to 3 percent thereby showing a reliable and dependable approximation for plate solutions.

Graphs for moments and deflections factors for various conditions are also provided in the appendix for references.

5. CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The Strip Deflection Compatibility Model for Plate Analysis (SDCM) has been developed and used to solve a simply supported plate. Results compare with Analytical solutions and have shown good correlation and reliability. The new model:

- Enhances the serviceability of the strip method of plate analysis.
- Incorporates twisting effects from the onset by including deflection in the xy direction in its formulation.
- Guarantees easier application in design offices without references and modification of results
- Guarantees variation of moments and deflections over plate surface with ease.
- Provides various tables and graphs for use in design offices to fast track designs.

Because of its simplicity the method can be applied to solve other plates e.g. simply supported plate with corners free to lift, fixed and mixed boundary conditions.

5.2 Recommendations

The model should be applied to ascertain its efficacy on mixed boundary conditions and point loads. Stress and strain conditions of plates should be investigated using the new model.

REFERENCES

Ephraim M. E and Orumu S. T. 2014. Analysis of Uniformly Loaded Simply Supported Rectangular Plates with 7-1 Lifting Corner Using Strip Moment Ratio

(SMR) Method. Journal of Civil and Environmental Research. 6(1): 31-38.

Hillerborg A. 1956. Equilibrium theory for Reinforced concrete slabs. Betong. 41(4): 171-182.

Hillerborg A. 1959. Strip Method for slabs on columns, L-shaped plates etc. Sevenska Riksbyggen, Stockholm.
Hillerborg A. 1975. Strip Method of Design, viewpoint publications, cement and concrete Association. Wexham Springs, Slough, England.

Hillerborg A. 1982. The Advanced Strip Method - a simple Design Tool. (1982) Mag. Conc. Res. 34(121): 175-181.

Johansen K. W. 1962. Yield-Line Theory. London, Cement and Concrete Association. p. 181 (English translation).

Johansen K. W. 1972. Yield-Line formulae for Slabs. London, Cement and Concrete Association. p. 106

Johnarry T. N. 1984. Simpler Nodal Forces for Yield-line Analysis of Reinforced Concrete slab. Magazine of Concrete Research. 36, (127).

Johnarry T. N. 2011. Acceleration-Displacement Ratio Transform of Differentials for Transverse plates and constant Buckling solution. Innovative System Design and Engineering. 2(4): 250-263.

Johnarry T. N. 2013. Buckling by Constant Stiffness and curvature-Deflection Resonance. Journal of Civil Engineering Research, Scientific and Academic Publishing USA. doi:10.5923/j.jce .2013031.01.

Kantorovich L. V. and Krylov V. I. 1954. Approximate method of Higher Analysis, John Wiley and Sons, New York.

Orumu S. T. 2002. Serviceability Solution of Rectangular Plates from Isolated Strip Moment Ratio (SMR); Doctoral Thesis (Structural Engineering), Rivers State University of Science and Technology, Portharcourt, Nigeria, 2002.

Orumu S. T. and Ephraim M. E. (2013). Strip Moment Ratio (SMR) Theory of plate. Analysis for Uniformly Loaded simply Supported Rectangular Plates with Corners Held Down. IOSR Journal of Engineering (ISORJEN) e-ISSN: 2250-3021, P-ISSN: 2278-8719 3, ISSNE: 1/pp. 28-45.

Timoshenko S. and Woinowsky - Krieger S. 1959. Theory of Plates and Shells, Mc.Graw Hill, New York, USA.

APPENDIX

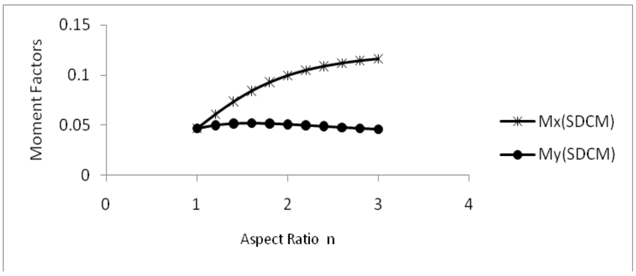


Figure-A.1. Moment Factors against Aspect Ratios for a Simply Supported Plate carrying a Udl over its area (v=0.3).

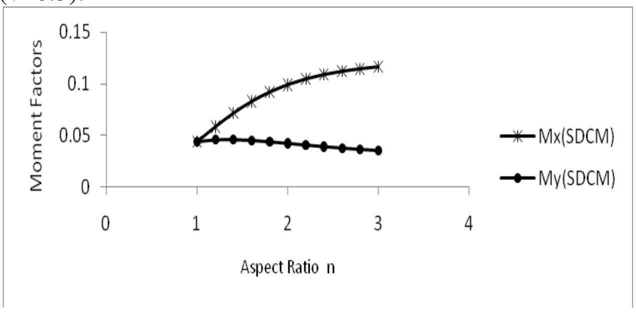


Figure-A.2. Moment Factors against Aspect Ratios for a Simply Supported Plate carrying aUdl over its area (v=0.2).

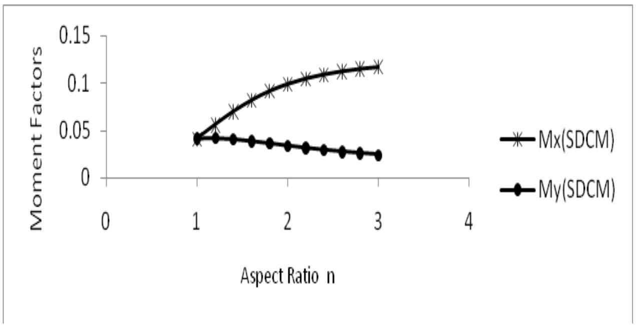


Figure-A.3. Moment Factors against Aspect Ratios for a Simply Supported Plate carrying aUdl. Over its area (v=0.1).

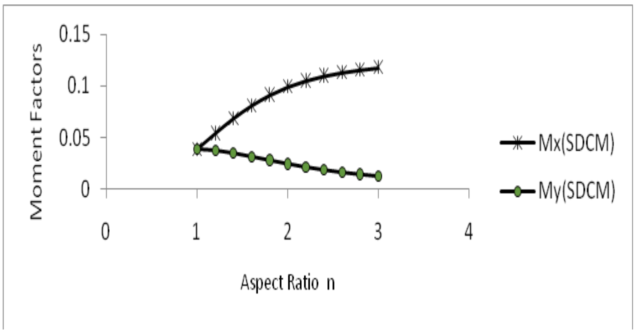


Figure-A.4. Moment Factors against Aspect Ratios for a Simply Supported Plate carrying aUdl over its area (v=0).

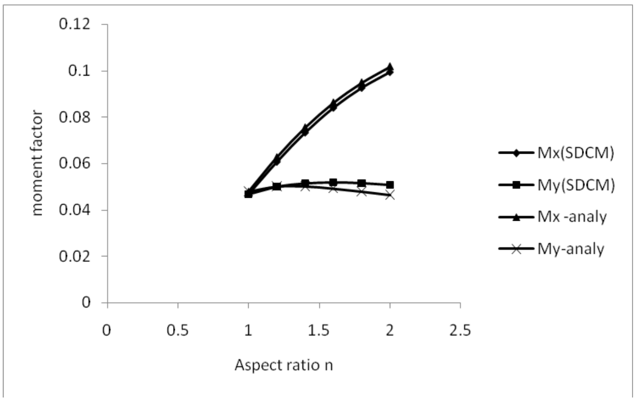


Figure-A.5. Comparison of SDCM and Analytical moments.

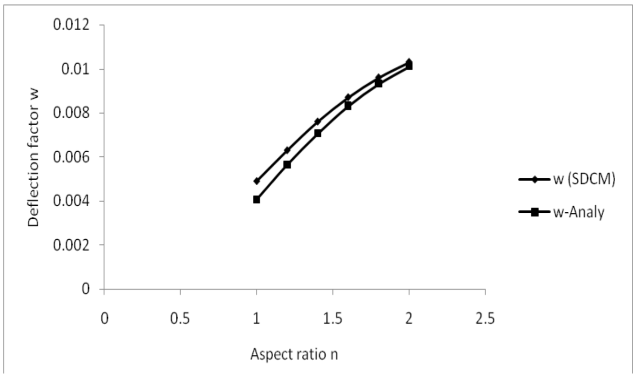


Figure-A.6. Comparison of SDCM and Analytical Deflections for v = 0.3.

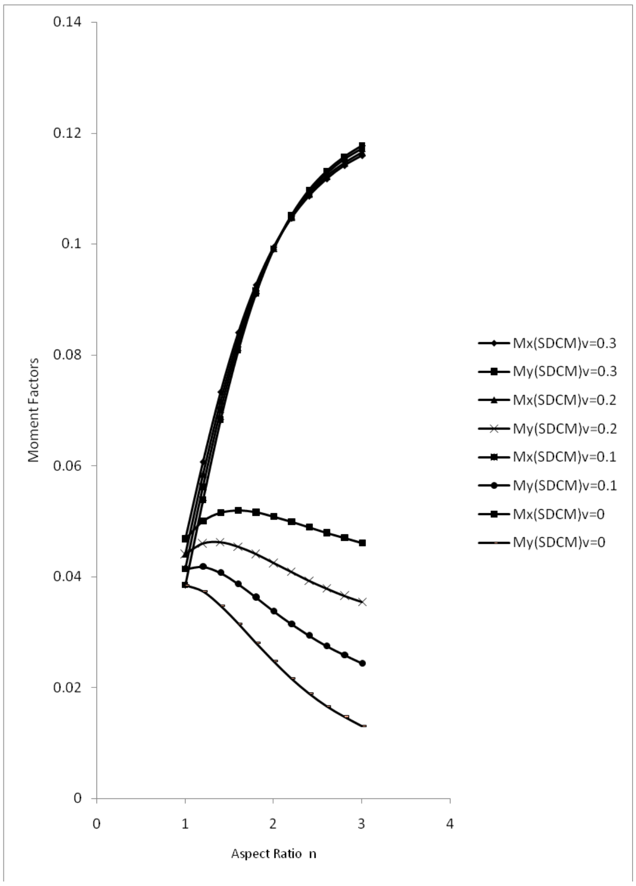


Figure-A.7. Effect of Poisson's Ratio on Deflection for a Simply Supported Plate carrying Udl Over its area with corners held down.