



NOISE REDUCTION AND CONTROL IN DCFS FBW WITH HARDWARE AND DIGITAL FUZZY FILTERS

Luca Piancastelli and Raimondo Alberto Bernabeo

ALMA MATER STUDIORUM University of Bologna, Department of Industrial Engineering, viale Risorgimento, Bologna, Italy

E-Mail: luca.piancastelli@unibo.it

ABSTRACT

This paper introduces the implementation of a few algorithms based on fuzzy logic to improve the performance of a "Fly-by-wire" (FBW) "Digital Flight Control System" (DCFS). These algorithms have been tested on a flight simulator type "FNPT II". This simulator was entirely developed at the Laboratory of Aerospace Engineering of the University of Bologna (Forli site). The algorithms should be simple (reliable) and quick in order to avoid response delay. They should also bring a true advantage in the FBW system. The proposed solutions are on the active filtering of the inputs an adaptive tolerance implementation for the identification of faulty sensors and their deactivation. The field in which this study has demonstrated greater effectiveness is in SW filtering of input signals, where a simple and effective algorithm was implemented. Finally, extremely simple hardware techniques to reduce input noise are also described.

Keywords: noise reduction, Fly-by-wire (FBW), Digital Flight Control System (DCFS), flight simulator.

INTRODUCTION

The fly-by-wire (FBW)-type Digital Flight Control System (DFCS) is a well-established control system. It is widely used both on military and civilian aircraft. Although well proven, there is still considerable room for improvement with regard to some problems of identification and correction of faults.

The FBW as used currently has the characteristic of being very tolerant in terms of noise. For example, tolerances of the order of 15% are accepted on the control parameters calculated by the computer that will act on the PID (Proportional Integrative Derivative) controller control of pitch, roll and yaw [1]. In other words, redundant output channels that differ by 15% are considered equal. This process takes place despite the fact that DFCS already contains filters and sophisticated noise reduction algorithms [2] [3]. The presence of such large tolerances poses serious problems of finding the best channel; namely the one that, within the tolerance of 15%, provides the most reliable value. In any case, there is the problem of identifying the faulty channel(s) and the nature of the fault: temporary, permanent, solvable by the pilot, due to pilot error (e.g. no ignition of the heater anemometer). The differences in the response of the various DFCS control channels are largely due to the noise present in the data of the sensors [4] which may, in some cases, lead to the exclusion of channels that do not have any hardware or software problem.

In this paper, we introduce some examples of systems based on the logic of uncertainty (fuzzy systems) designed to improve the efficiency of FBW DFCS. These controllers are here conceived as mere aids to traditional FBW. The criteria used were those not alter the time-lag of the fly-by-wire, which is itself a critical factor [1] and to maintain the fuzzy systems as simple as possible to increase reliability. Previous works from the authors has shown how the uses of complicated fuzzy systems increase the unpredictability without improving efficiency [5].

A fuzzy controller to evaluate the tolerances of two DFCS-output-values is described. These tolerances may both as a function of the flight conditions and of the input noise of the sensors.

For example, in the flight simulator developed at the Laboratory of Aerospace Engineering of Forli, the authors had problem of drift of analog sensors. Noise-increase with wear and aging is not constant in time. We used the solution to strengthen the signal to cover the entire field of the DA converter (put more commercial potentiometers in series).

A final application of fuzzy systems is to develop a simple fuzzy controller that optimizes the function "direct" for the various flight conditions.

SIMULATORS AS A TOOL FOR PROVING THE FBW

A flight simulator of the FNPT II type was developed since 1996. It enables the simulation of systems operating directly on the aircraft such as the FBW systems. It is characterized by an open multi-computer multi-tasking architecture. A server is responsible equations of motion integrations, data acquisition and TCP/IP protocol handling. Three other PCs display the instruments to the PF (Pilot Flying) and to PNF (Pilot not flying) (Fig. 1). Finally, a fourth computer system is dedicated to the visual. There is also a supervisor station with its own computer for the instructor.

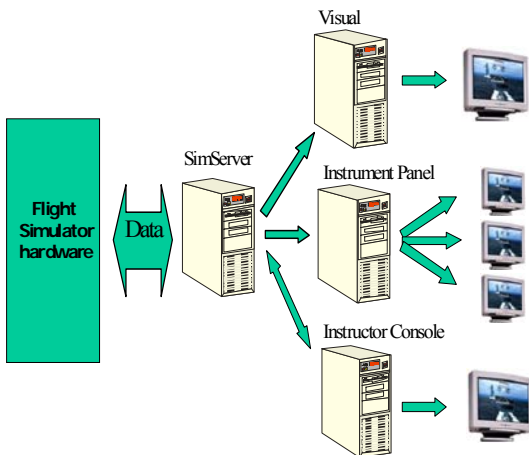


Figure-1. Flight simulator HW architecture.

The visual system and the equation integrator are implemented in C++ with direct tcp/ip connection between the two computer systems.

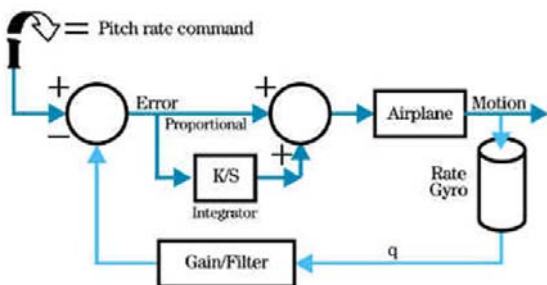


Figure-2. The FBW system for controlling the pitch [9] in an airplane.

National Instruments LabView is used for data acquisition, instrument visualization and the communication via LAN (Local Area Network).

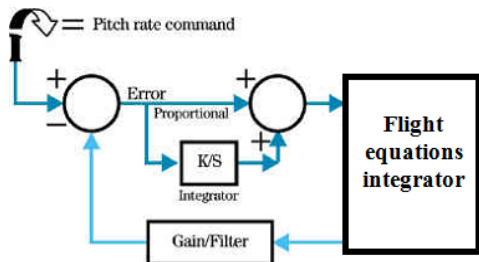


Figure-3. FBW system for pitch control in flight simulator.

The hardware controls (pedals, throttles, levers, switches and buttons) are detected by the acquisition software on the computer of the PF that acts as a master (for the instruments and the controls) and runs to the various programs that handle the low speed functions of the simulator. This system is fully open and it is possible

to easily insert a control module FBW (Figure-2) that acquires as input data from the throttle, stick and pedals. The other input data come from the simulation of the sensors and gyroscopic instruments. The module FBW (Figure-2) processes the commands given by the pilot, filters the noise, and sends signals to the subroutines that simulate the actuators of the control surfaces. The integration software (integrator) can then proceed to a further step of the calculation.

The simulation of a FBW system is commonly used for the design and tuning on the "iron machine" [1], but it requires some caution. First of all, the integrator must operate at a frequency at least an order of magnitude higher than the frequency of operation of the FBW. In our case, the integrator operates at about 1000 Hz, while the FBW operates at 100 Hz. Moreover it is necessary to simulate the response of the actuators with the associated delays and the noise, and inaccuracies of the sensors. The latter argument is particularly delicate since there are noisy sensors and sensor almost free from noise. In the case of controlling the pitch (v. Figure-4) the feedback coming from the integration of the vertical load factor (N_z or g), the speed of pitch (\dot{q}), the angle of pitch (q) and the angle of attack (α). This approach is common to control the pitch in the FBW [9], and goes back to the first tests on the FBW [6].

In our case the angle of attack, the pitch angle and the vertical load are obtained from the software that simulates the inertial platform. It is then necessary that the signals actually have the "noise" generated by the "inertial platform". In our case, the input data "stick angle" is very "clean", since a noise suppression system is easy to implement on this sensor.

In systems with multiple redundancies, it is also advisable that the redundancy is not restricted to just control channels, but also, when possible, in the number of sensors used. It is good that the inputs for each channel are separated, or that come from different sensors, different inertial platforms, in some cases even different technology with different noise. In this case the simulator is required to simulate the type of sensor.

In addition to these noises, there are other kinds of "errors". A very important one is a complete absence of signal due to a brief interruption in communications between the sensor and the computer of flight in the case of sensors with DA converter (Digital / Analog) resident, due error flag on, false contacts, overflow sensor fault or failure. There are also disorders EM (electromagnetic), the level of which varies according to many factors internal and external to the airplane and its systems. It is easily possible to represent them in the simulator [4].

The software implementation of a FBW is easy with the "PID module" of LabView, The Flight Computer, is described by [3], and was also implemented with LabView.



HARDWARE METHODS FOR NOISE REDUCTION

Multiple transducers

The reduction of some of the noise issues of DFCS can be done by working on the hardware. This type of testing is especially easy on flight simulators static type FTD (Flight Training Device) or FNPT (Flight Navigation Training Program) where low-cost components are used with a consequent increase of the noise. In particular, in this type of simulators is particularly convenient to use resistive potentiometers to detect the position of the throttle and thick. The resistive potentiometers, however, have problems of zero drift, noise and wear. While the problems of zero can be partially controlling by manually resetting the zero each time you restart the simulator. The noise problems on both potentiometers on the DA converters (Digital / Analog) must be solved with different strategies.

First, for the smooth operation of the converter is necessary that this work in its optimal range. If, for example, the range is [-5, +5] V, it is necessary that the handwheel full stroke in one direction the voltage is -5 V and in the opposite end of stroke is equal to +5V.

A better solution is to use the potentiometers in serial arrangement, in our case the four knobs (360/84 = 4). In this manner, it is possible to reduce the noise using as a measuring voltage to the formula (1):

with $n = 4$ (1)

$$V_{output} = \frac{\sum_{i=1}^n V_i}{n} \text{ with } n = 4 \quad (1)$$

It is possible to distribute the potentiometers at different points of the kinematic to compensate play and stretch of parts. It is the case of complex transmission system with elastic parts as, for example, the chain connecting the PF&PNF yokes.

Oversampling

Oversampling is a classic technique to reduce the measurement noise, in fact, at least in theory, it is possible to increase the number of measurements performed for a same size and perform the reduction of the measurement noise with (1). If, for example, you must sample a magnitude at 20 Hz, it is possible to sample it at 100 Hz, with an oversampling of 5 times, applying (2) with $n = 5$, a considerable noise reduction is obtained.

In some particularly noisy sensors this technique appears to be particularly advantageous. Even here, however, there are problems, in fact, if, as in our example, the sampling frequency is equal to 20 Hz with an interval of time equal to 0.05 if you make an to 100 Hz, the measurement that is obtained from (2) is related to 0.025 s before the current sample. Thus there is a phenomenon of slow response of the system or delayed unsuitable for a system (DFCS) "Fly by Wire". To overcome this

phenomenon, there are various techniques, the simplest is to add to (1) of the weights. This makes the last data more important (2):

$$V_{output} = \frac{\sum_{i=1}^n w_i V_i}{\sum_{i=1}^n w_i} \quad (2)$$

However, as it adds "weight" to the last data, it tends to nullify the effect of noise reduction.

A more efficient approach is to fit into a cubic polynomial with the "least squares" method. The new data can be extrapolated at the right time and inputted to the DFCS. In the bibliography [3], there are numerous other algorithms of "filter" the noise, some of which are based on neural networks. However, the oversampling with the least squares proved to be an extremely effective method, at least in our simulator.

On airplanes, the noise problem is most felt, because you have reliability problems of the sensors, already "sensitive" by definition. In addition the D / A converter is immersed in a hostile environment, with serious problems of EMC (Electro Magnetic Compatibility) and EMI (Electro Magnetic Interference). In the true world an oversampling may not be as free from risks such as in a simulator. Furthermore, in the control loop, the "filter oversampling" constitutes a further delay between measured data and corrective action making the control loop less stable. Ultimately, for aircraft, it is better to have the possibility to use higher frequency for the controller and not only of the acquisition system. Kalman filtering can then be used for noise reduction.

FUZZY LOGIC

Fuzzy logic controllers may not enter into competition with the traditional ones, but they are complementary to and are they are more effective for a few applications. They are suitable to the control systems of highly non-linear and / or difficult analytical representation because they are characterized by marked uncertainties. Typical examples are the control of the operation of traffic lights for access ramps to roads with a high traffic [7], [8], the control of the management of the ovens for the preparation of the cement [9].

In case of complex systems the fuzzy controllers shows their limit [5]. In fact, for these cases, it becomes difficult to implement the tables of the rules. Therefore, it is the belief of the authors that a single complicated fuzzy controller is not a good option, while a set of simpler controllers, each performing a specific task, is more functional. For this reason, in this paper, fuzzy controllers are always used with a few variables in both the input and the output. Great advantage of a simple fuzzy controller, compared to traditional methods, is its robustness.



APPLICATIONS OF FUZZY CONTROLLERS IN THE FBW

Fuzzy controller for "reference"

FBW DFCS have normally quadruple redundancy [2] [3]. In practice, four distinct controllers analyze the flight data, the input data of the PF and output the correct control parameters. The control parameters are compared and if out of tolerance they are excluded. The admissible tolerance is around 15%. The inputs that give the flight parameters are usually very noisy. Others as the PF commands are quite clean. For the latter it is possible to further reduce the noise by using techniques such as those described in section 3.1. It is also possible to use the only little input noisy and use the mathematical model of the aircraft to evaluate an additional reference value. This procedure, however, in addition to being cumbersome, is likely to be unreliable: In fact the actual flight conditions of the aircraft are not known in advance. There are "external" data difficult to assess such as, for example, the presence of ice, turbulence, the true thrust of the engines... that make this approach unfeasible. Instead, fuzzy controller can be implemented with very simple basic rules such as:

- By increasing the thrust of the engines and leaving other controls the aircraft climbs.
- Pulling the stick of the aircraft climbs.
- Pull the stick with leveled trust the IAS (Indicated Air Speed) decreases.

This rules are not true anywhere, for example in case of winds shear they are not true. However, in most conditions they are true. A fuzzy simulator can then be implemented. It can use only the "clean" inputs and the previous readings for the noisy inputs. It is then obtained a variation of the tolerance band as a function of the flight conditions.

You can also identify the ranges of meaningful input, knowing that some inputs need to be "growing", "leveling" or "stationary", so discarding the source of errors numeric conversion converters [4]. It is thus to replace a tolerance of acceptance fixed with a variable (adaptive) depending on the flight conditions of the aircraft.

Another use of fuzzy controllers could be to identify the conditions of the atmosphere around the aircraft. It is known that in summer during the hottest hours the air is turbulent and during the winter the turbulence decreases. Using environmental sensors available on all IFR (Instrument Flight Rules) aircrafts and the data from the inertial platform autopilot it is possible to adapt the "acceptance tolerance" by means of a fuzzy controller.

Fuzzy controller for signal conditioning

In FBW, the drastic nature of analog filters must be controlled. The higher the level of filtering the less is the information. In fact, a low bypass filter with a narrow band will not be able to detect sudden maneuvers. Consequently the filter, inevitably, in addition to reducing the peaks due to disturbances also reduce the peaks due to rapid maneuvers imposed by the pilot. The airplane appears to be sluggish in some conditions. This is normal for "old time" pilots. They were used to anticipate the aircraft, especially airliners to obtain the desired effect. For younger pilot, who come directly from lighter aircraft, this is unusual.

The overall system performance can be improved if you increase the analog filter band and you add a digital filter immediately after the AD converter. The filters more suitable for this purpose are those suffice on Fuzzy Logic, since these are based on the rule based. It consists of a memory cell that stores the last digital input data assumed to be correct. The fuzzy controller determines whether the new particular sample is valid or not. The test program of the Fuzzy filter is shown in Figure-4.

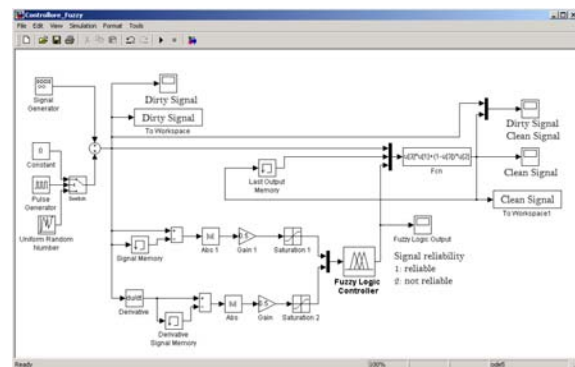


Figure-4. LabView program for the testing of a Fuzzy filter.

As stated before, the "Fuzzy Controller" has two input values from the reading and processing of the sampled signal:

- 1 signal change of the signal with respect to sampling time ΔP
- 2 derivative change of the signal with respect to sampling time ΔV

Three functions fuzzy ("low", "medium", "high") are associated to the numeric variables in input.

The output of the "Fuzzy Controller" has output three fuzzy functions that are:

1. reliable, which generates a numerical value close to 1
- 2 not so reliable (medium reliable), which generates a numerical value close to 0
3. unreliable, which generates a numeric value between 0 and 1

The "heuristic rules" of the fuzzy controller are shown in Table-1:

**Table-1.** Heuristic rules of the fuzzy filter.

If ΔP is low and ΔV is low \rightarrow data is reliable
If ΔP is low and ΔV is medium \rightarrow data is reliable
If ΔP is low and ΔV is high \rightarrow data is medium reliable
If ΔP is medium and ΔV is low \rightarrow data is reliable
If ΔP is medium and ΔV is medium \rightarrow data is medium reliable
If ΔP is medium and ΔV is high \rightarrow data is medium reliable
If ΔP is high and ΔV is low \rightarrow data is medium reliable
If ΔP is high and ΔV is medium \rightarrow data is not reliable
If ΔP is high and ΔV is high \rightarrow data is not reliable

Two examples were considered for the tests

1) Presence of a loud noise, of limited duration, during a routine maneuver.

2) Presence of a loud noise during an emergency maneuver.

In the first case the input signal to the controller is the one shown in Figure-5. Time [s] is in the X axis while yoke angle potentiometer output [V] is on the Y axis.

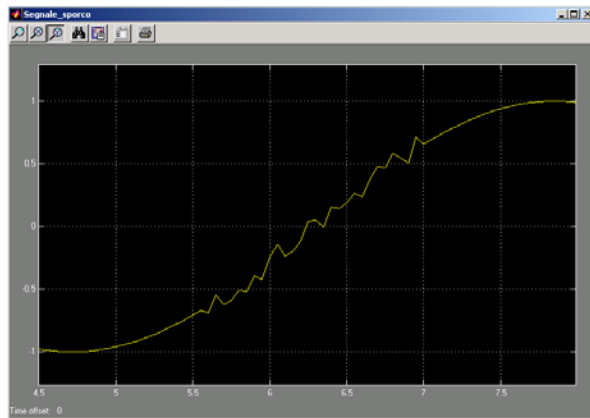


Figure-5. Input signal: strong for limited duration during a normal operation.

This is a normal low frequency control (about 1 Hz), in which it is introduced a random noise for about 1.5s. This noise has width of about 10% of the maximum of the clean signal. The output of the fuzzy filter is shown Figure-6, where the thicker line (yellow) is the sampled signal, while the thinner line (purple) is filtered one:

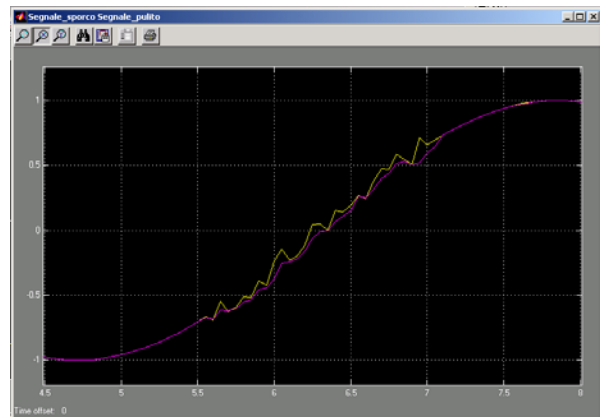


Figure-6. Input signal (yellow) and filtered output (purple) in the presence of a strong noise of limited duration during a normal operation.

In areas in the absence of the disturbance signal is observed that the two signals are perfectly superimposed: Therefore, the filter does not generate a danger "time-lag". In the disturbed zone, however, the output signal is much less noisy. Furthermore it can be observed that, in the noisy area, the response of the filter (purple line) always remains reliable. In fact, if in the absence of reliable input the controller generates a value similar to the previous value available.

In the second example, signal noise is introduced during of the flare maneuver. As on final approach, when the PF has to pull for the presence of an obstacle on the landing strip. Figure-7 shows the signal coming from the stick with noise superimposed.

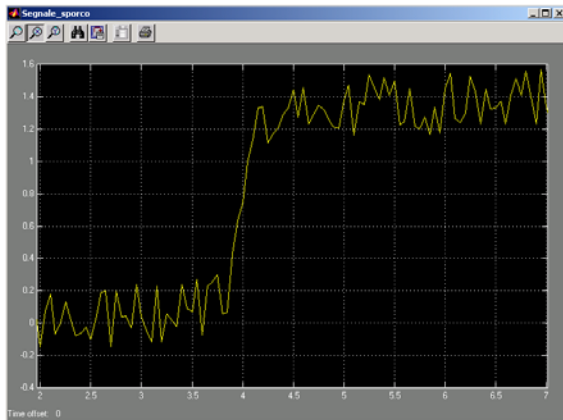


Figure-7. Stick input signal in presence of strong signal noise during a flare maneuver.

Figure-8 shows the response of the control (purple line) superimposed the sampled input signal (yellow line). It can be noted that in this case, not only the signal noise has been attenuated, but also that the filter is able to maintain the original stick input curve.

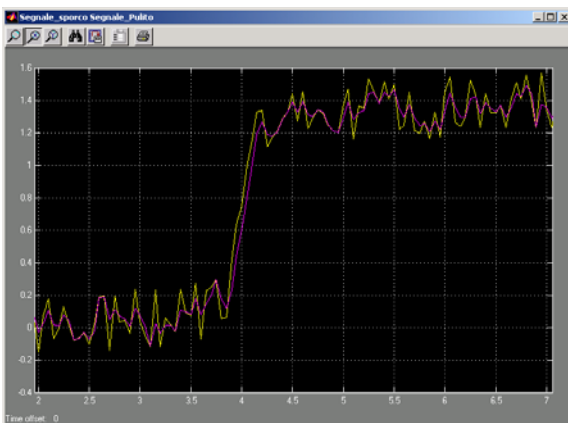


Figure-8. Input signal (yellow) and filtered output (purple) in the presence of a strong noise during a flare maneuver.

CONCLUSIONS

The introduction of fuzzy logic in a The DFCS FBW gives large benefits in input noise control, in noise bandwidth reduction and in the identification of channels and sensors faults and improving the In this way, the advantages of fuzzy logic with are combined with the high reliable current control technology. Furthermore, since the fuzzy logic filters are purely software, it is possible to introduce them as simple software upgrades in the DCFS computer.

The reduction of the signal noise by oversampling is not convenient. Far better results can be undoubtedly obtained by increasing the FBW DCFS operating frequency. Finally, when possible, it is convenient to have more sensors in serial arrangement

connected to a single DA converter in order to reduce the noise by hardware.

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