VOL. 10, NO. 9, MAY 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

COMPARATIVE ANALYSIS OF VARIOUS WAVELETS FOR DENOISING COLOR IMAGES

Shalini Hema Rani S. and Godwin Premi M. S.
Department of ETCE, Sathyabama University, Chennai, India
E-Mail: Shalinihemarani@gmail.com

ABSTRACT

Wavelet transform has played an important role in Image processing task such as compression and restoration. Unlike most of existing denoising algorithms, using the curvelet makes it needless to hypothesize a statistical model for the noiseless image. This wavelet transform fails to represent the images, which has edges and treated them as smooth functions with discontinuity along curves. The curvelet transforms, where frame elements are indexed by scale, location and orientation parameters. This curvelet transform is designed to represent edges and other singularities along the curves which are more efficient than the traditional wavelet transform. Moreover, the curvelet transform and Gaussian filter are used for an effective image denoising system. This process will be based on the block-based noise estimation technique, in which an input image will be contaminated by the additive white Gaussian noise and filtering process to be performed by an adaptive Gaussian filter and curvelet transform. Coefficients of the Gaussian filter will be selected, as the functions of the standard deviation of the Gaussian noise will be estimated from an input noisy image. Denoising an image is carried out by processing an noisy image through Gaussian filter and using curvelet transform which gives better PSNR. The obtained PSNR values can be compared with that of many wavelet and curvelet in RGB regions. The renowned index Peak Signal to Noise Ratio (PSNR) and Root Mean Square Error (RMSE) demonstrate marked improvement of image denoising over other methods.

Keywords: wavelets, denoising, image, curvelets, FFT, filtering, ridgelets.

1. INTRODUCTION

Image distortion is often a common issue due to various types of noise. Gaussian noise, Salt and Pepper noise, Poisson noise, Speckle noise etc are fundamental noise types in case of images. The noise may originate from a noise source present in the vicinity of image capturing location or may be introduced due to imperfection/inaccuracy inherent in the image capturing devices like cameras. For example, lenses may be misaligned, focal length may be weak, scattering and other adverse conditions may also be present in the atmosphere, etc. This makes careful study of noise and noise approximation as an essential ingredient of image denoising. This leads to selection of proper noise model for image processing systems [1].

Noise gets introduced in images during image acquisition or transmission. This may be from Electronic or photometric sources. Blurring occurs due to imperfect image formation processes such as spreading of focal length, non-stationary camera placement etc., resulting in reduction of bandwidth in images [2].

To remove noise, traditionally, linear techniques were used. However, these techniques do not perform well in case of impulsive noise. They also obviously do not fit where nonlinear operation is required. Linear filters generally introduce noise during transmission process and image formation. Image signals deal with low and high frequency contents. Junctions, edges, corners and other fine details of an image are represented as high-frequency components. Hence these high frequency components are important for visual perception. However most of the linear filters have low pass filter characteristics and hence edges, lines and other fine details are lost due to filtering.

Several non-linear techniques have been proposed for image restoration both in spatial domain and multi scale (wavelet) domain. Multi scale approaches have proved to be superior [3-4].

Denoising an image is an important pre-processing technique for further processing such as edge detection or image segmentation.

The main aim of any denoising algorithm is to reduce noise levels by preserving the image features. At present, there are various image denoising algorithms available which are successful in denoising a noisy image, but inefficient in attaining better signal to noise ratio and/or retaining the image features.

2. RELATED WORK

Although the DWTs have established an impressive reputation as a tool for mathematical analysis and signal processing, which has the disadvantage of poor directionality. This undermined its usage in many applications. The complex wavelet transform have shown its improvement in directional selectivity. Though, the complex wavelet transform has not been widely used in the past, as it is difficult to design with perfect reconstruction properties and good filter characteristics. The dual-tree complex wavelet transform (DTCWT) proposed by Kingsbury [7], which added perfect reconstruction to the other attractive properties of complex wavelets. The 2-D complex wavelets are essentially constructed by using tensor-product one-dimensional (1-D) wavelets. The directional selectivity provided by complex wavelets in six directions is much better than that obtained by the classical DWTs in three directions. Though it has some limitations.

VOL. 10, NO. 9, MAY 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

In 1999, an anisotropic geometric wavelet transforms, named ridgelet transform, which was proposed by Candès and Donoho[10]. The ridgelet transform is optimal at representing straight-line singularities. Though, global straight-line singularities are rarely observed in real applications. To analyse these local line or curve singularities, an idea is to consider a partition of the image, and then apply the ridgelet transform to the obtained sub images. These block ridgelet based transform, which is named as curvelet transform, was proposed by Candès and Donoho in 2000.

In 2007, an SURE-LET based wavelet transform was proposed by Ajay Boyat and Brijendra Kumar Joshi [10]. Here, in this paper a new approach to image denoising that is especially useful when redundant or non-orthonormal transforms are involved. The main drawback of this paper is PSNR (Peak Signal to Noise Ratio) obtained will be less.

In 2013, a Haar based wavelet transform was proposed by Ajay Boyat and Brijendra Kumar Joshi [11]. Here, in this paper an image denoising algorithm based on combined effect of Haar wavelet transform and median filtering. The algorithm removes most of the noise from the image and maintains the quality. The level of wavelet decomposition was restricted to three. The main drawback of this paper is when the noise level is higher, then the PSNR value will be less (i.e.)original image cannot be obtained.

Hanlei Dong [12] in this paper, bi-orthogonal wavelet transform combined with median filtering, an image-denoising method was presented. The result shows that noise of the image is removed effectively. This method has better denoising effect than single wavelet thresholding method or median filtering method. The main drawback of this paper is PSNR (Peak Signal to Noise Ratio) obtained will be less.

Manoj Gabhel, Aashish Hiradhar [15] ,in this paper different approaches of wavelet based image denoising methods was proposed. Magnetic resonance (MR) images are routinely used in medical diagnosis. Denoising of these images to enhance their quality & Clinical parameter is an active area of research. This paper presents the wavelet-based image denoising and noise suppression in MRI images. The performance of denoising is evaluated in terms of PSNR, MSE and MAE.

3. PROPOSED WORK

In this paper, initial efforts at image denoising based on a Bior , Haar, Surelet and Curvelet transform. Here, the results were compared with various wavelet transform to see which transform gives better result for image denoising. The objective is to decrease a mean square error (MSE) and to increase a peak signal to noise ratio (PSNR) in db, by adding a white Gaussian noise. Initially a noisy image is obtained by degrading it by adding additive Gaussian noise (most common type of noise). Then the algorithm is implemented, which firstly passes through a wavelet transforms and the output of which is then applied to gaussian or median filter. The resultant image is denoised and it retains the important image information. Gaussian noise is a type of a statistical noise in which the amplitude of the noise follows that of a Gaussian (normal) distribution. In Gaussian noise, the power spectral density is normally distributed. The probability density function of a Gaussian random variable Z [1] is given by;

$$P(z) = \frac{1}{2\pi\sigma} e^{\frac{-(Z-\mu)^2}{2\sigma^2}} \quad (1)$$

Where Z represents gray level, μ is the average value of Z, σ is its standard deviation, and σ_2 is variance. Image Reduction is one of the important image processing tasks. The important analysis of image is to reduce the noise effectively from the image. The denoising algorithm depends on the type of noise corrupting the image. A number of non linear filters have been developed to remove Gaussian noise.

Gaussian filters has the properties of having no overshoot to a step function input which minimizes the rise and fall time. This behavior of Gaussian filter has minimum group delay. It is considered as the ideal time domain filter, just as the ideal frequency domain filter The one-dimensional Gaussian filter has an impulse response [1] given by;

$$g(x) = \sqrt{\frac{a}{\pi}} e^{-ax^2} \quad (2)$$

Curvelets are based on multiscale ridgelet combined with a spatial band pass filtering operation to isolate different scale. Like ridgelets, curvelets occur at all locations, scales and orientations. However, while ridgelet have global length and variable widths, curvelets in addition to variable width have a variable length and hence have variable anisotropy. The width and length at fine scales are related by a scaling law width length and so the anisotropy increases with decreasing scale like a power law. Recent work shows thresholding of discrete curvelet coefficients provide near optimal N–term representations of otherwise smooth objects with discontinuities along C^2 curves. Thus, for understanding curvelet, knowledge about ridgelet transform is required.

Continuous ridgelet transform

The ridgelet transform of two-dimensional function F(x, y) allows the sparse representation of both smooth function and straight edges by superposition of ridgelet function. For every $\in L^2$ (R^2)[13], ridgelet coefficients R_f (a,b,θ) are obtained by an inner product with the frame like function $\Psi_{a,b,\theta}$ (x), which is wavelet in transverse orientation constant along the line x1cosθ + x2sinθ = constant. For every a>0, each b ϵ R and θ ϵ [0,2π), the bivariate ridgelet function, $\Psi_{a,b,\theta}$ [13],[14] is given by

$$\frac{1}{\sqrt{a}} \Psi\left(\frac{x\cos\theta + y\sin\theta - b}{a}\right) \quad (3)$$

This function is constant on the lines $x_1\cos\theta + x_2 \sin\theta$ =constant. Transverse of these ridgelet is a wavelet.

VOL. 10, NO. 9, MAY 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

Given a bivariate function f(x), the ridgelet coefficients is given by [5]

$$R_f(a,b,\theta) = \int \Psi_{a,b,\theta(x)}\, f(x)\,dx \tag{4}$$

The reconstruction formulae is given by [5]

$$f(x) = \int_0^{2\pi}\int_{-\infty}^{\infty}\int_0^{\infty} R_f(a,b,\theta)\,\Psi_{a,b,\theta(x)}\,\frac{da}{a^3}\,db\,\frac{d\theta}{4\pi} \tag{5}$$

The above equation is valid for both integral and square integral. Furthermore, this formula is stable as one has a Parseval relation given by [5]

$$\int |f(x)|^2 dx = \int_{-\infty}^{\infty}\int_0^{\infty} \left|R_f(a,b,\theta)\right|^2 \frac{da}{a^3}\,db\,\frac{d\theta}{4\pi} \tag{6}$$

Hence like the wavelet or Fourier transforms the identity can expresses the fact that one can represent any arbitrary function as a continuous superposition of ridgelets.

Radon transform

A basic tool for calculating ridgelet coefficients is to view ridgelet analysis as a form of wavelet analysis in Radon domain. The Radon transform of an object f is the collection of line integrals indexed by (θ,t) ϵ [0,2π] €×R [6].

$$R_f(\ \ ,t) = \int f(x1,x2)\,\delta(x_1\cos\theta + x_2\sin\theta - t)\,dx_1 dx_2 \tag{7}$$

where δ is the Dirac distribution. The ridgelet coefficient $R_f(a,b,\theta)$ [6] of an object f is given by analysis of Radon transform via

$$R_f(a,b,\theta) = \int Rf(\theta,t)\,a^{-1/2}\,\psi\left(\frac{(t-b)}{a}\right)dt \tag{8}$$

Hence ridgelet transform is precisely the application of a one-dimensional (1-D) wavelet transform to the slices of the Radon transform where the angular variable θ is constant and t is varying.

Discrete Curvelet Transform of continuum function

The discrete curvelet transform of a continuum function $f(x_1,x_2)$ makes use of dyadic sequence of scales, and a bank of filters ($P0f,\Delta1f,\Delta2f$,..) with the property that the pass band filter is concentrated near the frequencies [2^2s,2^(2s+1)] e.g. [14].

$$\Delta_s = \psi_{2s} * f, \qquad \psi_{2s}(\xi) = \psi(2^{-2s}\xi) \tag{9}$$

In wavelet theory, one uses a decomposition into dyadic sub- bands [2^s,2^(s+1)]. In contrast, the sub-bands used in the discrete curvelet transform of continuum functions have the nonstandard form [2^2s,2^(2s+1)]. This is nonstandard feature of the discrete curvelet transform well worth remembering.

The curvelet decomposition follows the sequence of below steps.

- **sub-band decomposition:** The objects is decomposed into sub-bands[9],[14]

$$f \mapsto (P_0 f, \Delta_1 f, \Delta_2 f, \ldots) \tag{10}$$

- **Smooth partitioning:** Each sub-band is windowed into "squares" of an appropriate scale[9],[14] (of side length $\sim 2^{-s}$)

$$\Delta_s f \mapsto (wQ\Delta_s f)\,Q \epsilon Q_s \tag{11}$$

- **Renormalization:** *R*esulting square is renormalized to unit scale[9], [14].

$$gQ = 2^{-s}\left(T_Q\right)^{-1}(wQ\Delta_s f), \qquad Q \epsilon Q_s \tag{12}$$

- **Ridgelet analysis:** Each square is then analyzed in the ortho-ridgelet system[9],[14].

$$\alpha_\mu = |< gQ, \rho\lambda >, \qquad \mu = (Q,\lambda)| \tag{13}$$

In this definition, the two dyadic sub bands $[2^{2s}, 2^{2s+1}]$ and $[2^{2s+1}, 2^{2s+2}]$ are merged before applying the ridgelet and radon transform.

Process flow

Denoising procedure followed here is performed by taking wavelet/ curvelet transform of the noisy image and then applying Filters(Median/ Gaussian filter) to eliminate noisy coefficients. The proposed algorithm is as follows:

Step-1: Apply noise to the image.
Step-2: Apply wavelet/curvelet transform to noisy image
Step-3: Apply Median/ Gaussian filter on noisy image
Step-4: Apply inverse transform to the filtered image to transform image from transform domain to spatial domain.

The algorithm flow of proposed method is shown in the below Figure-1.

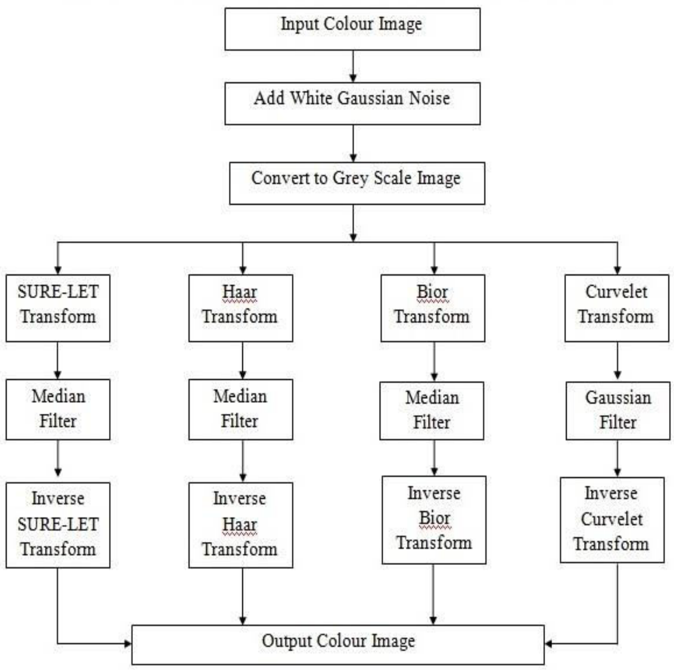


Figure-1. Process flow.

VOL. 10, NO. 9, MAY 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

4. PERFORMANCE CALCULATION

In this paper, a comparative study of wavelets was presented for denoising a noisy image by various filters and the wavelet transforms. The noise level in an image can be estimated by calculating the PSNR value. Therefore, performance can be measured by comparing the PSNR values of various wavelets.

MSE [1] is defined as:

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j) - K(i,j)]^2 \quad (14)$$

The PSNR [2] is defined as:

$$PSNR = 10.\log_{10}\left(\frac{MAX}{MSE}\right)$$

$$PSNR = 20.\log_{10}\left(\frac{MAX}{\sqrt{MSE}}\right)$$

$$PSNR = 20.\log_{10}(MAX_1) - 10\log_{10}(MSE) \quad (15)$$

PSNR value should be high and MSE value should be less for better denoising effect.

5. RESULTS

This paper presents color image denoising and its analysis using various wavelets like Bior wavelet, Haar wavelet , Surelet and Curvelet transform with Median and Gaussian filter. In this image an additive noises like Gaussian noise with different noise levels σ = 40,60, 80, 100, 120,140 etc is added.

Table-1 reports the PSNR results obtained by various denoising methods, the best results being shown in boldface.

Table-2 shows the comparison of various denoising methods and its MSE values with different noise levels. Here, the MSE value of curvelet is less when compared to other denoising methods.

6. CONCLUSIONS

Image denoising, using wavelet techniques are effective because of its ability to capture the energy of signal in a few high transform values, when natural image is corrupted by Gaussian noise. This paper presents a comparative analysis of various image denoising techniques (BIOR, SURELET, HAAR, CURVELET) using wavelet transforms. A lot of combinations have been applied in order to find the best method that can be followed for denoising intensity images. From the PSNR and MSE values as shown in tables, it is clear that Curvelet transform giving better results under different noise variance conditions for color images. The Comparative graph for PSNR is given below.

Table-1. Comparison of PSNR values of various wavelets.

Sigma	**SURELET transform & Median filter** (PSNR)	**Biorwavelet & Median filter** (PSNR)	**Haar transform & Median filter** (PSNR)	**Curvelet transform & Gaussian filter** (PSNR)
40	13.4334	26.4218	27.5334	27.9669
60	10.0067	23.4377	25.0856	25.8418
80	7.5322	21.2132	23.0683	24.0486
100	5.6530	19.3777	21.3401	22.4497
120	4.0309	17.7868	19.9028	21.1319
140	2.7418	16.5362	18.7430	19.9968

ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

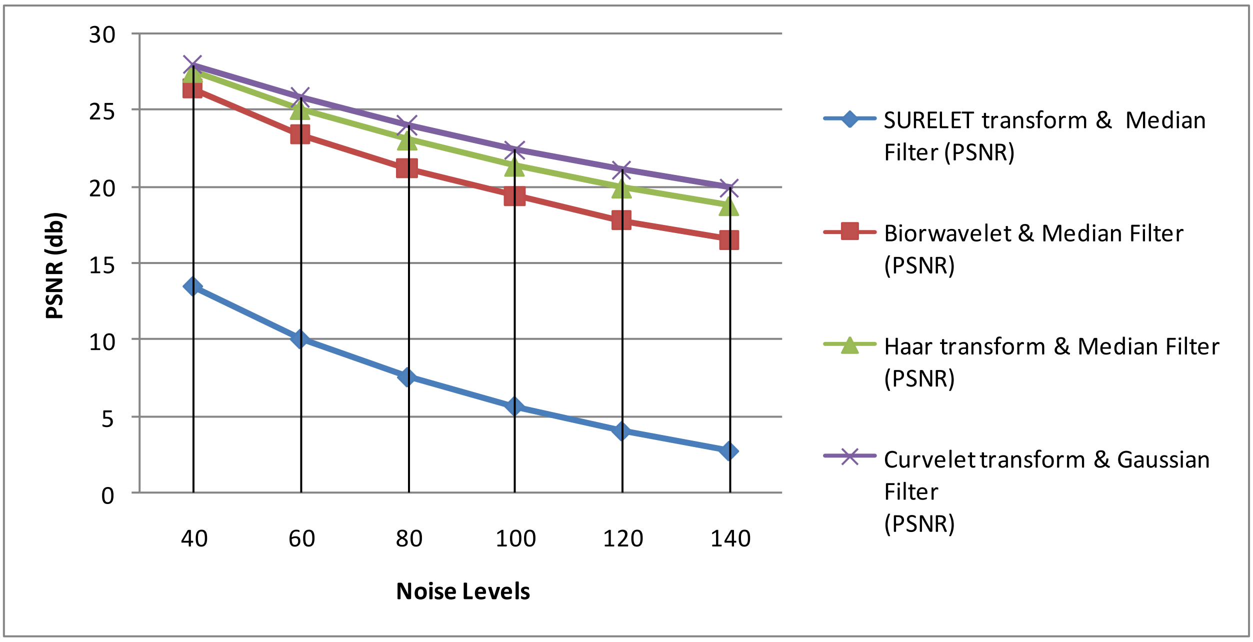
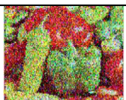
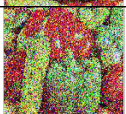
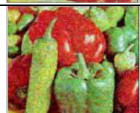
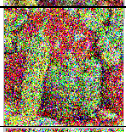
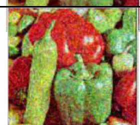
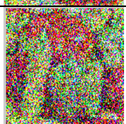
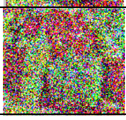
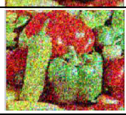


Figure-2. Comparative graph for PSNR having different variances.

Table-2. Comparison of MSE values and test results of various wavelets.

Sigma	**SURELET transform & Median filter** (MSE)		**Haar transform & Median filter** (MSE)		**Bior wavelet & Median filter** (MSE)		**Curvelet &Gaussian filter** (MSE)	
40		2.9508e+03		114.9556		148.2		104.1
60		6.4927e+03		201.7896		294.6		169.5
80		1.1480e+04		320.9926		491.8		256.1
100		1.7697e+04		477.6540		750.4		370.1
120		2.5705e+04		665.0216		1.0e+3		501.2
140		3.4586e+04		868.5311		1.4e+3		650.8

REFERENCES

[1] R. C. Gonzalez and R. E. Woods. 2002. "Digital Image Processing," second ed., Prentice Hall, Englewood, Englewood, Cliffs, NJ.

[2] J. Patil and S. Jadhav. 2013. "A Comparative Study of Image Denoising Techniques," International Journal of Innovative Research in Science, Engineering and Technology, Vol. 2, No. 3, March.

[3] S. Mallet. 1998. "A Wavelet Tour of Signal Processing," Academic Press, New York, USA.

[4] V. Zlokolica, A. Pizurica and W. Philips. "Wavelet-Domain Video Denoising Based on Reliability

ARPN Journal of Engineering and Applied Sciences

Measures" Circuits and Systems for Video Technology.

[5] Jean-Luc Starck, Emmanuel J. Candès and David L. Donoho. 2002. The Curvelet Transform for Image Denoising. IEEE Transactions on image processing, Vol. 11, June 2002.

[6] A. Averbuch, R. R. Coifman, D. L. Donoho, M. Israeli, and J. Waldn. 2000. "Polar FFT, rectopolar FFT, and applications, Stanford Univ., Stanford, CA, Tech. Rep.

[7] Miller M. and Kingsbury. 2008. Nick Image Denoising Using Derotated. Complex Wavelet Coefficients." IEEETrans. Image Processing Vol. 17, No.9, pp. 1500-1511.

[8] Candes E. J. 1999.On the representation of mutilated Sobolev functions, Dept. Statist., Stanford Univ., Stanford, CA, Tech. Rep.

[9] E. J. Candès and D. L. Donoho. 1999. Curvelets—A surprisingly effective nonadaptive representation for objects with edges, in Curve and Surface Fitting. Nashville, TN: Vanderbuilt Univ. Press.

[10] Thierry Blu and Florian Luisier. 2007. - The SURE-LET Approach to Image Denoising, IEEE.

[11] Ajay Boyat and Brijendra Kumar Joshi. 2013. - Image Denoising using Wavelet Transform and Median Filtering, IEEE.

[12] Hanlei Dong and Fan Wang Image-denoising based on Bior Wavelet Transform and Median Filter, IEEE.

[13] E. J. Candès. 1999. "Harmonic analysis of neural netwoks," Appl. Computer. Harmon. Anal., Vol. 6, pp. 197–218.

[14] A. Cohen, C. Rabut and L. L. Schumaker. 1999. Eds., Nashville, ―Ridgelets: The key to higher-dimensional intermittency? Phil. Trans. R. Soc. Lond. A., Vol. 357, pp. 2495–2509.

[15] Manoj Gabhel and Aashish Hiradhar. 2014. Comparative Analysis of Various Denoising Techniques for MRI Image Using Wavelet, International Journal of Science and Research (IJSR), 2014.