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PRESSURE AND PRESSURE DERIVATIVE ANALYSIS FOR FRACTAL HOMOGENEOUS RESERVOIRS WITH POWER-LAW FLUIDS

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ABSTRACT

There are some approximations when we can say that a reservoir is homogeneous, even though this is not completely true, when the reservoir has some specific properties, from the well testing point of view, it can be considered as homogeneous. Other assumptions that are made when analyzing a reservoir is that the fluids inside the reservoir behave as Newtonian fluids, which we know is not truly an approximation since the reservoir is under conditions of temperature and pressure that do not allow the fluids to behave as a Newtonian fluid. Chang and Yorstos (1990) characterized a fracture network in a reservoir using fractal geometry which seems to reproduce better either fluid behavior inside the porous media or the network of natural fractures. The purpose of this paper is to apply the concept of fractal geometry to a homogeneous reservoir, and beyond that to take into account the fluid behavior on the pressure and pressure derivate log-log plot. The objective is to provide an analytical methodology for characterizing such systems. Since the reservoirs we are going to work with are homogenous, the supposition of a fractal reservoir is a good approximation, and so there were expressions developed for the calculation of the fractal parameters *Df* and θ, that were developed under the analysis of synthetic pressure transient tests. After the equations were proposed, they were tested with synthetic examples, and the results met the requirements satisfactorily.

Keywords: fractal dimension, conductivity index, flow behavior, homogenous, fractal reservoirs.

1. INTRODUCTION

Homogeneous reservoirs are the reservoirs that have been studied the longest since those are the ones that were found in the early years of the petroleum industry. Despite the fact that the conditions have changed, and we have more complex reservoir structures, there is a good approximation when its properties are constant with the distance, such as the permeability.

Different authors proposed models to explain the behavior of a reservoir, depending on its properties, for example, Odehand Yang (1979), and Ikokuand Ramey (1979a, 1979b) were the first ones who developed models for when there are non-Newtonian fluids inside infinite reservoirs; also, Chang and Yortsos (1990) characterized a fracture network in a reservoir using fractal geometry. They introduced the concepts of *Df* which represents the fractal dimension of a fractal network of fractures, and θ which characterizes the geometry and transport properties in the fracture network.

Throughout the previous studies of fractured reservoirs through fractal analysis, they have demonstrated a good approach and characterization, hence in the case where some properties are repeated in a non-trivial way, it can be said that the fractal representation is a good way to characterize a homogeneous reservoir, because properties are repeated at different scales, so they can be described as a power-law relation, which it is going to be shown. Most heavy oil follows the power-law behavior. Escobar (2012) presents several practical cases of non-Newtonian power-law behavior. For easier finding, a practical application when injecting foam is given by Escobar, Martinez and Montealegre (2010) for composite systems where the injected foam or EOR fluid in injected in an oil Newtonian bearing formation. If the injected fluid behaves as dilatant, the Martinez, Escobar and Cantillo (2011b) provide a practical interpretation methodology. If the power-law flows through a double-porosity medium, then Escobar, Zambrano and Giraldo introduced an interpretation methodology using unique features found on the pressure and pressure derivative plot. If the well is partially completed and spherical flow takes place, this special situation was handled from the interpretation point of view by Escobar, Martinez and Bonilla (2012).

This paper seeks to develop new equations based on key points that are observed in the pressure and pressure derivate dimensionless graphs, in the radial flow period for power law non-Newtonian fluids, in order to make new expressions that may facilitate the calculations for the parameters that were introduced by Chang and Yorstos (1990) to characterize thesekind of reservoirs, and at the same time see whether the flow behavior parameter affects these fractal parameters.

2. MATHEMATICAL BACKGROUND

From the equations proposed by Beier (1994), where he defined some basic properties as permeability and porosity as a function of the radius; used a modified form of Darcy's law for power-law fluids in a homogeneous reservoir, and also taking into account the effective viscosity; a new partial differential equation was

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developed for transient flow of non-Newtonian fluids in a fractal reservoir:

$$\frac{\partial^2 P_D}{\partial {r_D}^2} + \left[df + \frac{1}{n} - \frac{d_f}{nd_s} - \frac{d_f}{d_s}\right]\frac{n}{r_D}\frac{\partial P_D}{\partial r_D} = \left(\frac{d_f}{d_s}\right)^2 r_D^{\left\{\left(\frac{d_f}{d_s}\right)(n+1)-d_f(n-1)-2\right\}} \frac{\partial P_D}{\partial t_D} \tag{1.a}$$

The objective was the observation of the different patterns and features found on the pressure and pressure derivative curves (see Figure-1) so expressions were obtained from an adequate treatment of such behaviors. The complete detail of the original work was presented by Salcedo (2015). The equation from which the model was proposed was presented by Chakrabarty, Farouq and Tortike (1993) for a finite wellbore radius:

$$\overline{P}_{D(I)} = \frac{K_V(\alpha r_D^{\beta})}{I^{3/2} K_{1-V(\alpha)}} \tag{1.b}$$

Where $\overline{P}_D$ represents the dimensionless pressure in the Laplace space, I is Laplace variable, and the γ, α, and β are defined by:

$$\gamma = \frac{(d_f / d_s)(n+1) - nd_f}{2} \tag{2}$$

$$\alpha = \frac{2\sqrt{I}(d_f / d_s)}{(d_f / d_s)(n+1) - d_f(n-1)} \tag{3}$$

$$\beta = \frac{(d_f / d_s)(n+1) - d_f(n-1)}{2} \tag{4}$$

$$v = 1 - \frac{d_s}{(n+1) - d_s(n-1)} \tag{5}$$

The graphs were made for variations in the fractal exponent, θ, and for variations in the fractal exponent, *Df*. There can be appreciate in Figure-1and Figure-2 the pressure behavior and the flow periods, however, for the model proposed the wellbore storage is neglected; there is only take into account the radial flow period; here the slope of the radial flow period is not ½, but the slope depends on the fractal parameters *Df* and θ.

2.1. Reservoirs with non Newtonian fluids

2.1.1 Fractal dimension, *Df*

Figures-2 and 3 display the effect of the fractal dimension on pressure and pressure derivative for a fractal homogeneous reservoir with constant values of θ=0.8, and *n*=0.9.

From Figure-3, it can be seen the fractal dimension, *Df,* modifies the slope values during radial flow regime, it can be seen clearly that the slope decreases as the fractal dimension increases; therefore, the maximum slope value is obtained when *Df*=1 and the lowest slope value is given when *Df*=2. It is also noted that this fractal parameter influences the starting point of the radial period.

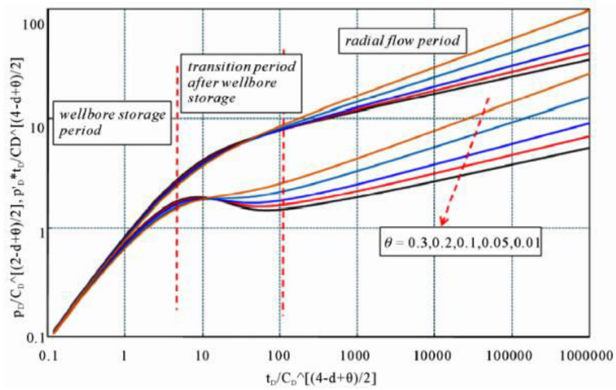

Figure-1. Pressure behavior of infinite reservoir with different fractal factor θ.

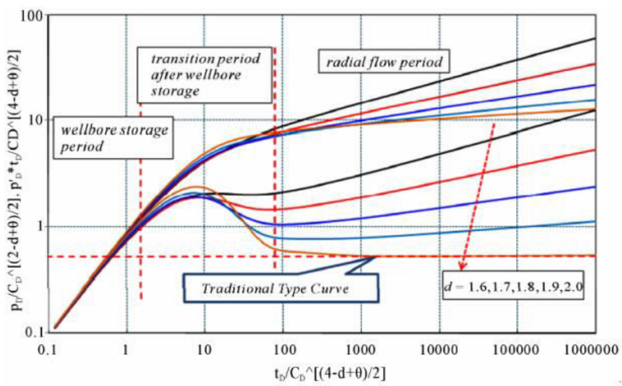

Figure-2. Pressure behavior of infinite reservoir with different fractal dimension *Df.*

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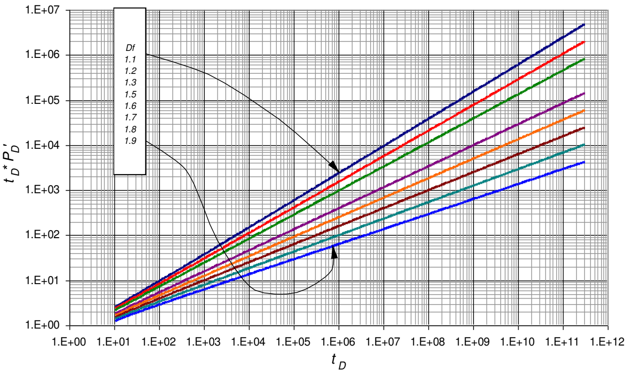

Figure-3. Influence of the fractal dimension on the derivative response for n=0.9 and θ=0.8.

An expression for the determination of *Df* was developed based upon observations of the slopes of the derivative response during radial flow regime on each curve. It can be seen from Figure-3 that as the value of*Df* increases, the slope of the radial flow curve decreases.

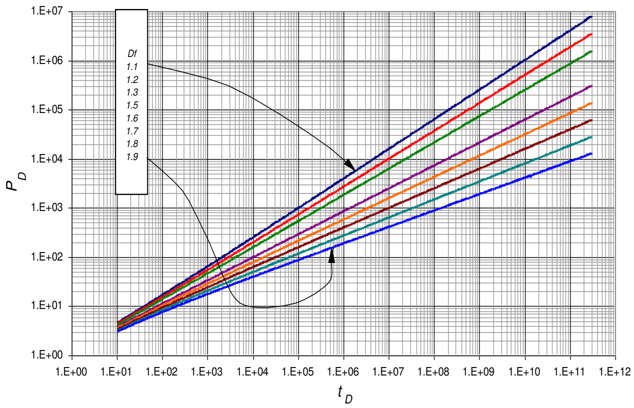

Figure-4. Influence of the fractal dimension on the pressure response for n=0.9 and θ=0.8.

$$D_f = A + (B*m) + (C*(m^2)) + D*(m^3) + E*(m^4) + F*(m^5) \quad (6)$$

Constants for Equation (6) areprovided in Table-1. Such constants depend on the other parameters of relevance as fluid index and fractal exponent.

Table-1. Coefficients for equation (6).

Constant	1	2	3	4	5
n	**0.7**	**0.6**	**0.7**	**0.8**	**0.9**
θ	**0.1**	**0.25**	**0.45**	**0.65**	**0.8**
A	2.55315687	3.00386681	2.98037107	2.98967231	2.94550367
B	-3.64422809	-5.00215566	-4.27467153	-3.78982827	-3.16581294
C	1.58318919	3.35049529	1.97197259	1.27441266	0.00890804
D	-0.77169156	-2.2615256	-1.23521372	-1.16673195	0.34370125
E	0.43989453	1.35537363	0.95786586	1.297862	0.03386865
F	-0.25202674	-0.55803035	-0.5279646	-0.75653443	-0.31678379

Since there are many constants (Table-1) which can be applied to Equation (6) and are dependent on the parameters*n* and θ, and we only are able to know*n*and the slope of the curves from the transient pressure test, there should be another selecting criterion to choose the group of constants that should be used for the best accuracy, depending on the known values. The slope of the radial flow was used to develop the new criterion, where some basic values were stablished and the slope found from the curve has to be fixed to the closest basic values shown in Table-2.

Table-2. Criteria for selecting the constants group for equation (6).

RANGE	LIMITS			SET NUMBER		
	LOW	MEDIUM	HIGH	Close to the low	Close to the Medium	Close to the high
1	0.24	0.355	0.47	1	2	3
2	0.26	0.385	0.51	2	2	3
3	0.28	0.41	0.54	2	3	4,3
4	0.31	0.445	0.58	2	2	5
5	0.33	0.465	0.6	2	2	5

Table-2 is used forchoosing the slope for the radial flow regime curve on a homogeneous fractal reservoir, and then subtracting the value with each limit on each range. The absolute smallest difference calculated is the limit and rangewe should choose, and then we have a look to the set number depending if it is close to the low, medium or high limits. If there are two group of constants suggested by Table-2, use the first number and in the case there is no information for that group of constants, then second group of constants suggested to be used. When the group of constants is chosen according to the described criterion, replace the values in Equation (6) to calculate the fractal dimension, *Df*.

2.1.2 Fractal exponent, θ

Figures-5 and 6 shows the effect of conductivity index on the transient pressure and pressure derivative behavior, respectively, in a homogeneous reservoir when *Df*=1.7, *n*=0.8.

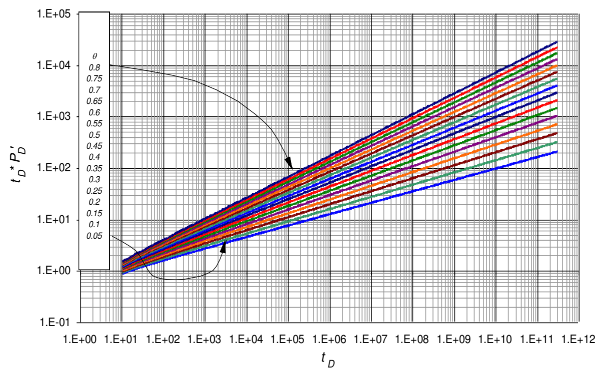

Figure-5. Influence of the fractal exponent on the derivative response for n=0.8 and Df=1.7.

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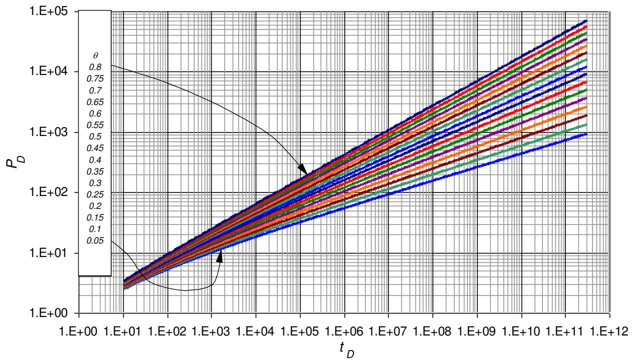

Figure-6. Influence of the fractal exponent on the pressure response for *n*=0.8 and *Df*=1.7.

It can be observed in Figure-4 and Figure-5 that as θ increases, the slope of the radial flow curve also increases. This means that the slope is going to have the highest value when θ=0.8, and the smallest value when θ=0.05.

From the behavior of the slopes for the radial flow on each curve for the derivate response, an equation for calculating the parameter θ was developed, where the constants involved in the equation are shown in Table-3, the equation is given by:

$$\theta = \frac{A+(B\times D_f)+(C\times m)+(D\times m^2)}{1+(E\times D_f)+(F\times m)} \quad (7)$$

The constants for the equation presented above depend on the parameteralready calculated, *Df*.

Table-3. Criteria for selecting the constants group for Equation (7).

	Aplication Range				
Constants	$1.1 \le Df < 1.3$	$1.3 \le Df < 1.5$	$1.5 \le Df < 1.7$	$1.7 \le Df < 1.9$	$1.9 \le Df < 2$
A	-2.505909	-2.735492	-3.139084	-3.924515	-3.465497
B	1.0793887	1.2324648	1.4723092	1.8962281	1.659823
C	2.7891117	2.8195132	2.9457691	3.2843087	2.9891235
D	0.117351	0.2714158	0.4089926	0.5385585	0.2719455
E	0.0640428	0.1091703	0.1710793	0.2767539	0.1844073
F	-1.087506	-1.136276	-1.223146	-1.396272	-1.300146

2.2. Reservoirs with Newtonian fluids

2.2.1 Fractal dimension, *Df*

Figures-2 and 3 display the effect of the fractal dimension on pressure and pressure derivative for a fractal homogeneous reservoir with constant values of θ=0.8, and *n*=1.

From the Figure-3, it can be seen that the fractal dimension, *Df,* modifies the slope values during radial flow regime, it can be seen clearly that the slope decreases as the fractal dimension increases; therefore, the maximum slope value is obtained when *Df*=1 and the lowest slope value is given when *Df*=2. It is also noted that this fractal parameter influences the starting point of the radial period.

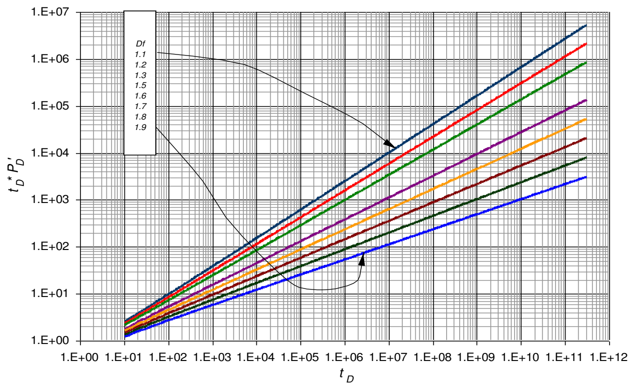

Figure-7. Influence of the fractal dimension on the derivative response for *n*=1 and θ=0.8.

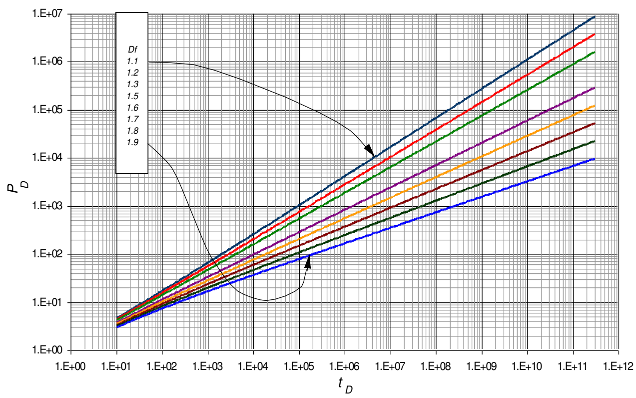

Figure-8. Influence of the fractal dimension on the pressure response for *n*=1and θ=0.8.

An equation for the determination of parameter *Df* was found from observing theradial flow regime slope of each curve for the derivate response. It can be seen from the Figure-5 that as the value of *Df* increases, the slope of the radial flow curve decreases. The obtained equation is given by:

$$D_f = A+B\times m+C\times m^2+D\times m^3+E\times m^4+F\times m^5 \quad (8)$$

It can be seen from the graph that as the value of *Df* increases, the slope of the radial flow curve decreases.

The constants for the equation presented in Table-4 depend on the other parameters we are interested in; those are the fluid index, and the fractal exponent.

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Table-4. Coefficients for equation (8).

Contant	1	2	3	4	5
n	**1**	**1**	**1**	**1**	**1**
θ	**0.1**	**0.25**	**0.45**	**0.65**	**0.8**
A	2.1023483	2.25269008	2.45384029	2.65707953	2.81245258
B	-2.09948218	-2.25267439	-2.46651168	-2.70517205	-2.91574824
C	0.00883457	0.03242381	0.11465493	0.31160477	0.5918897
D	-0.05940761	-0.14207932	-0.38111403	-0.8771319	-1.5207173
E	0.11636895	0.25386346	0.5926728	1.20927262	1.94228908
F	-0.14923089	-0.24224806	-0.43698308	-0.74679768	-1.08351081

As for the case of Equation (6), there are many constants in Table-3 that can be used in Equation (7), depending on the parameters n and θ, and the onlyknown variable is *n*and the slope of the pressure derivative curves obtained for the transient pressure test, there is a need of another selecting criterion to choose the group of constants that should be used for the best accuracy, depending on the known values. The slope of the radial flow was used to develop the new criterion, where some basic values were stablished and the slope obtained from the curve has to be fixed to the closest basic values shown in Table-5.

Table-5. Criteria for selecting the constants group.

RANGE	LIMITS			SET NUMBER		
	LOW	MEDIUM	HIGH	Close to the low	Close to the Medium	Close to the high
1	0.1	0.29	0.48	1	3	3
2	0.16	0.335	0.51	2	3	3
3	0.23	0.39	0.55	2	2	4
4	0.28	0.43	0.58	2	2	5
5	0.32	0.46	0.6	3	2	5

Table-5 is used for calculating the slope of the derivative curve during radial flow regime of a homogeneous fractal reservoir, and then subtracting the value with each limit on each range. The absolute smallest difference calculated is the limit and range to be chosen, and then going to the set number depending if it is close to the low, medium or high limits. If there are two groups of constants suggested by the table. Use the first number and in the case of unknown the information for that group of constants, then use the second group of constants.

Once the group of constants is chosen according to the described criterion, replace the values on Equation (8) to calculate the fractal dimension, *Df*.

2.2.2 Fractal exponent, *θ*

Figures-9and 10 shows the conductivity index effect on the transient pressure behavior in a homogeneous reservoir when *Df*=1.7 and *n*=1.

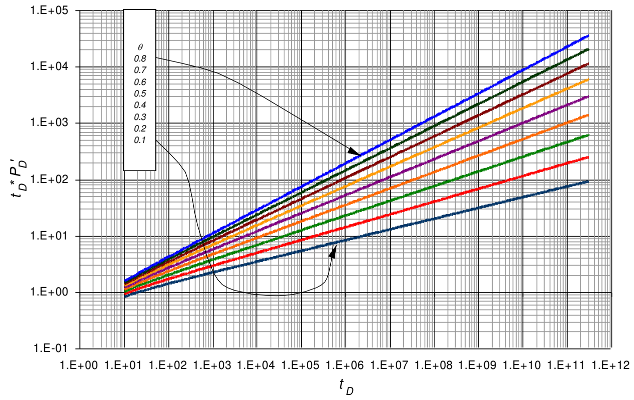

Figure-9. Influence of the fractal exponent on the derivative response for *n*=1 and *Df*=1.7.

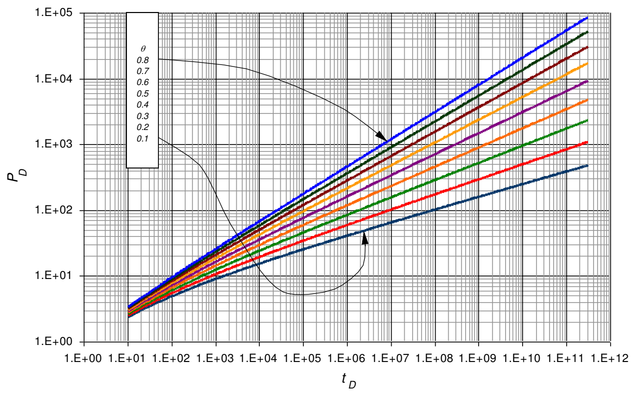

Figure-10. Influence of the fractal exponent on the pressure response for *n*=1and *Df*=1.7.

According to the Figure-9 and Figure-10, it can be seen that the slope of the radial flow curve increases as θ increases, it means that the slope is going to have the biggest value when θ=0.8, and is going to have the smallest value when θ=0.05.

Relating the slopes for the radial flow on each curve for the derivate response, it was developed an equation for calculating the parameter θ, where the constants involved in the equation are shown in Table-4, the equation is given by:

$$\theta = \frac{A + (B \times D_f) + (C \times m) + (D \times m^2)}{1 + (E \times D_f) + (F \times m)} \quad (9)$$

The constants for the equation presented above depend on the parameter already calculated, Df.

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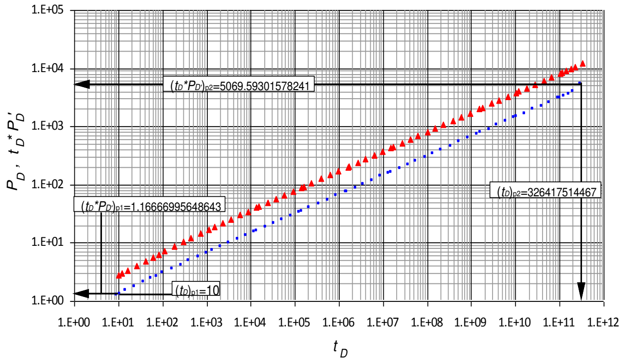

Figure-11. Pressure and pressure derivative vs. time log-log plot for example 1.

3. EXAMPLES

In the original work, Salcedo (2015) tested the developed equations with 92 synthetic examples, 60 for reservoirs containing non Newtonian fluids, and 32 for reservoirs containing Newtonian fluids. Only two of them are presented here for practical purposes.

3.1. Synthetic example 1

A pressure test was run in a homogeneous fractal reservoir, used as input data of the information provided in the second column of Table-7. Pressure and pressure derivative versus time data are provided in Figure-6. It is required to characterize this test.

Solution: The below information was read from Figure-11;

$((t_D*P_D')_r)_{P1}$= 1.16667 $(t_{Dr})_{P1}$=10
$(t_D*P_D')_{r2})_{P2}$=5069.59302 $(t_{Dr})_{P2}$=326417514467
n=0.7

First, we need to compute the value of the slope, m, of the radial flow regime on the pressure derivative curve which resulted to be 0.546736824. With this value, we go to Table-2, and subtract the value of the slope from the limits low, medium and high, and we look for the smaller absolute difference. We got that it is the 5th range close to the low limit, and with this range and limit, the proposed equation constants are number 2. With the group of constants proposed, we replace the values on equation 6, and we obtain Df = 1.597128734.

With this value of Df, refer to Table-3, for the value of constants for Equation (7). It can be seen that the value of Dffits on the 3th application range, so those values are replaced into Equation (7) to obtain θ=0.33019.

Synthetic example 2

A pressure test was run in a homogeneous fractal reservoir using as input data the information provided in the second column of Table-7. Pressure and pressure derivative versus time data are provided in Figure-11. It is required to characterize this test.

Solution: The below information was read from Figure-7;

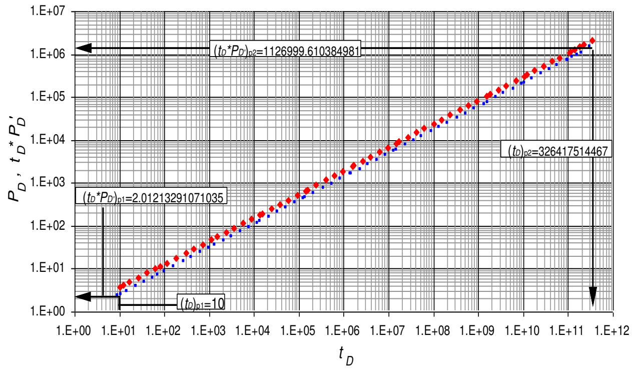

Figure-12. Pressure and pressure derivative vs. time log-log plot for example 2.

$((t_D*P_D')_r)_{P1}$= 2.012133 $(t_{Dr})_{P1}$=10
$(t_D*P_D')_{r2})_{P2}$=1126999.6103 $(t_{Dr})_{P2}$=326417514467
n=1

4. COMMENTS ON THE RESULTS

The results obtained with the proposed model were close to the results that are obtained with a simulator. However it is important that in order to get the values closer with the real value, the procedure proposed has to be followed as it was shown in the methodology.

It is important to have at least 4 decimals so the calculations are more accurate, and especially in the case when calculating the conductivity index, which is a extremely sensible parameter.

5. CONCLUSIONS

a) New correlations for the determination of the fractal parameters for a reservoir with a power-law fluid (heavy oil) which can be approximated to a homogeneous system were introduced in this work and satisfactorily tested with synthetic examples.

b) It was observed that the dimensionless pressure response followed the power-law behavior; hence, the assumption made in the paper by Chakrabarty et al. (1993), that the reservoir was fractal, was tested.

c) Due to the fractal dimension is an indicator that shows how disorder the reservoir is, when making the synthetic examples, it was observed that the proposed model approaches in a more significantly to the actual data when the medium is ordered, it means when Df is small.

d) The fact that the values of the fractal dimension for the proposed study varied between 1 and 2 indicates that the pore distribution within the reservoir was considered for variations in areal spaces.

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e) The fractal dimension, *Df*, istheparameter that affects in a greater way the calculations which is clearly observed in the graphs since the slope of the plot changes more with thevariation of *Df*, thanwiththe variation of θ.

Nomenclature

B	Volumetric factor, rb/STB
Df	Fractal dimension
k	Permeability, md
h	Formation thickness, ft
$\bar{P}$	Average reservoir pressure, psia
P	Pressure, psi
q	Flow rate, STB/D
t	Time, hr
r	Radius, ft
s	Skin factor

Greeks

Δ	Change, drop
ϕ	Porosity, fraction
θ	Conductivity index
μ	Viscosity, cp

Suffices

f	External
s	Gas
D	Initial
$P1$	Initial point
$P2$	Final point
$r2$	Late radial

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