DESIGN AND ANALYSIS OF A NON LINEAR OPTIMIZATION INVENTORY MODEL FOR UNCERTAIN QUANTITY

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ABSTRACT

This paper considers a continuous review policy with controllable lead time and the partial backordering under the circumstance of uncertain quantity received. A Non Linear Programming Model is constructed with service level constraint. An optimal inventory control approach by the Lagrange multiplier method is applied. The lead time crashing cost is considered as an exponential function of lead time, while the order processing cost and lost sales rate are considered as logarithmic functions of capital investment. The objective of this study is to minimize the total relevant cost by simultaneously optimizing the order quantity, lost sales rate and order processing cost.

Keywords: inventory, non linear optimization,

1. INTRODUCTION

Inventory is a life blood of Supply Chain. Controlling inventory is a process and it is a method of total inventory management. An enormous research effort has been taken over this issue for the past few decades. There are two replenishment policies are often used in practice, those are continuous review and periodic review. Continuous review indicates that inventory status is continuously tracked and ordering can be done according to lot size when the level is reached entrusted inventory reorder point. While periodic review indicates that inventory status tracked at regular periodic intervals and reorder was made to raise the inventory level to the point at predetermined time. These inventory system policies are not comprehensive, but sufficiently provide solutions to problems concerning the safety of the inventory management system [1]. In business every operation can be reviewed on continuous basis for the purpose of monitoring and control. Being an important entity of business, inventory also requires continuous review for timely replenishment. In this context continuous review inventory system is an appropriate mathematical model to handle such problem [2]. Many companies have recognized the significance of response time as a competitive weapon and have used it as a means of differentiating themselves in the marketplace. In previous literature, lead time is viewed as a prescribed constant or a random variable. In many practical situations, lead time can be reduced at an added crashing cost; in other words, it is controllable. Recently, much attention has been paid to the lead time reduction .The service level constraint (SLC) is introduced into the inventory model to replace the shortage cost which implies that the stock-out level per cycle is bounded and the stock is available in a probabilistic or expected sense. Therefore, many authors [6], [7] replace the shortage cost by a condition on the service level.

A continuous review inventory system with controllable lead time and order processing cost, backorder price discount, uncertain received quantity with service level constraint is considered in this paper. It is also considered that the buyer offers backorder price discount to his customers with outstanding orders during the shortage period to secure customer orders. The expected annual total cost per unit time is minimized by simultaneously optimizing the order quantity, order processing cost, backorder price discount, and lead time with service level constraint using Lagrange Multiplier method.

2. LITERATURE REVIEW

Nita H. Shah and Hardik N. Soni [2] considers a continuous review inventory system for the inventory model involving fuzzy random demand, variable lead-time with backorders and lost sales. The triangular fuzzy number count upon lead-time to construct a lead-time demand. The expected shortages are calculated using credibility criterion. The authors provide a solution to find the optimal lead-time and the optimal order quantity along with the reorder point such that the total expected cost in the fuzzy sense has a minimum value.

Kuo-Chen HUNG [3], proposed an inventory model considering issues of crash cost and current value this article considers the time value of money of a continuous review inventory model with a mixture of backorders and lost sales, where lead time demand has a normal distribution. The author find the optimal order quantity at all lengths of lead time with components crashed at their minimum duration and a also formulated simple method to locate the optimal solution.

ArefGholami-Qadikolaei *et al.* [4] presents the mixed inventory backorder and lost sales involving four variables; order quantity, lead time, safety factor and backorder rate. The order lot have some defective items and the number of defective item is considered as a random variable with backorder rate is dependent on length of lead time through the amount of shortages .Optimal safety factor is obtained discretely with Negative exponential lead time crashing cost This study, first assumes the lead time demand follows a normal distribution and then relax the assumption about the form of the distribution function of the lead time demand and apply the minimax distribution free procedure to solve the problem.

M. Grida *et al.* [5] developed a constrained mathematical model, non-linear objective function and a number of bounding linear and nonlinear constraints, based on the results of the questionnaire, in order to minimize the total cost involving procurement, holding, ordering, and shortage costs while satisfying the given conditions. The authors determine the optimal orders quantity from the available suppliers and determine the suppliers' share factor, in order to minimize the total cost

Porteus [8] investigated the impact of capital investment in reducing ordering cost on the classical economic order quantity (EOQ) model for the first time. The author introduces three options for investing in quality improvements: (i) reducing the probability that the process moves out of control (ii) reducing setup costs and (iii) simultaneously using the two previous options. The optimal investment strategy is obtained using the specific form of the investment cost function.

Ouyang *et al.* [9] discusses lead time and ordering cost reductions in continuous review inventory systems with partial backorders. The authors have simultaneously optimise the order quantity, ordering cost, reorder point and lead time, by assuming the lead time demand following the normal distribution, then they relax this assumption to consider the distribution free case where only the mean and variance of lead time demands are known.

Later, Chang *et al.* [10] presented lead time and ordering cost reduction problem in the single-vendor single-buyer integrated inventory model. The authors have formulated two models First model assumes that the ordering cost reduction has no relation to lead time crashing, second model assumes that the lead time and ordering cost reduction are interrelated. It has been shown that buyer lead time can be shortened at an extra crashing cost which depends on the lead time length to be reduced and the ordering lot size.

N. Kazemi *et al.* [11], discusses an inventory model with backorders in a fuzzy situation by employing two types of fuzzy numbers, trapezoidal and triangular. The optimal policy for the developed model is determined using the Kuhn–Tucker conditions after the defuzzification of the cost function with the graded mean integration (GMI) method by developing a full-fuzzy model with the input parameters and fuzzified decision variables.

M. Vijayashree *et al.* [12], discusses the integrated inventory model with controllable lead time with main focus on setup cost reduction for defective and non-defective items under investment for quality by assuming that the setup cost and process quality as logarithmic function. The mathematical model is derived to investigate the effects to the optimal decisions when investment strategies in setup cost reductions are adopted in order to minimize the total cost of both the vendor and purchaser.

3. ASSUMPTIONS AND NOTATIONS

a) Notations

D: Average demand per year
Q: Order quantity, a decision variable,
Y: Received quantity, a random variable
α: Bias factor, which is the expected amount received ÷ amount ordered, $0 < \alpha < 1$
A: Ordering cost per order, a decision variable
A_o: Original ordering cost (before any investment is made)
h: Inventory holding cost per unit per year
π: Fixed penalty cost per unit short
π_o: Marginal profit per unit
β : Fraction of the demand backordered during the stock out period, $0 \le \beta \le 1$, while the remaining fraction $(1-\beta) \equiv \breve{\beta}$ is lost sales, a decision variable
$\breve{\beta}$: Original fraction of the shortage that will be lost
θ : The interest rate per year
L: Length of lead time, a decision variable X
X: Demand during lead time, a random variable
$f(x)$: The probability distribution function (p.d.f) of X
$E(.)$: Expected value
x^+: Maximum value of x and 0, i.e., $x^+ = \max\{x, 0\}$
$E(X-R)^+$: Expected number of shortages per cycle = $\sigma\sqrt{L}\psi(k)$ where $\psi(k) = \varphi(k) - k[1-\Phi(k)]$, Where φ is the standard normal density function and Φ is the standard normal cumulative distribution function.

b) Assumptions

- Demand per unit , is assumed to be Normal with mean D
- The quantity received is uncertain and depends on the quantity ordered.
  The expected quantity received $E(Y/Q) = \begin{cases} \alpha Q, & \alpha < 1, \text{ when expected quantity < ordered quantity} \\ \alpha Q, & \alpha > 1, \text{ when expected quantity > ordered quantity} \end{cases}$
- Variance of the quantity received, $V(Y) = \sigma_o^2 + \sigma_1^2 Q^2$, $\sigma_o^2 >> \sigma_1^2$
  If $\sigma_1^2 = 0$, then the standard deviation of the quantity received is independent of quantity ordered. If $\sigma_o^2 = 0$, then the standard deviation of the quantity received is proportional to quantity ordered.

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- The lead time demand $X \sim N(DL, \sigma\sqrt{L})$, σ is the standard deviation of demand per unit, L is the Length of the lead time, D is the average demand per year.
- The reorder point r = expected demand during the lead time (DL)+ the safety stock(s_s) $= DL + k\sigma\sqrt{L}$, where k is the safety factor and satisfies P($X > r$) = q, q represents the allowable stock out probability.
- $\alpha Q > r$. Since total cost of the inventory system is independent of the reorder point r.
- The lead time Crashing Cost $R(L)$ is an exponential function of L is defined as

$$R(L) = \begin{cases} 0 & \text{if } L = L_o \\ e^{C/L} & \text{if } L_b \le L < L_o \end{cases}$$

C is a positive constant and L_o and L_b represent the existing and the shortest lead times, respectively.

- During the stock out period, a fraction β of the demand will be back ordered and the remaining fraction (1-β) = $\breve{\beta}$ will be lost.

4. MODEL FORMULATION

a) The objective function

According to Moon and Choi [14], If the buyers receive the same order quantity which he replenished with controllable lead time in a continuous review model, then Expected Annual Total Cost per cycle = Ordering Cost + Holding Cost + Stock Out Cost + Lead Time Crashing Cost

i.e. EAC

$$(Q,r,L) = A + h\left[\frac{Q}{2} + r - DL + (1-\beta)E(X-r)^+\right] + \bar{\pi}E(X-R)^+ + R(L) \quad (1)$$

Where $\bar{\pi} = \pi + (1-\beta)\pi_o$

Here we assume the model of S. Priyan [13], if the amount received is uncertain, then the quantity received Y, is a random variable with $E(Y/Q) = \alpha Q$. The total cost per cycle, from (1), with a variable lead time with Y units received is

$$\check{C}(Y,r,L) = A + h\frac{Y}{D}\left[\frac{Y}{2} + r - DL + (1-\beta)E(X-r)^+\right] + \bar{\pi}E(X-R)^+ + R(L) \quad (2)$$

$$= A + h\frac{Y}{D}\left[\frac{Y}{2} + r - DL + \breve{\beta}E(X-r)^+\right] + (\pi + \breve{\beta}\pi_o)E(X-R)^+ + R(L) \quad (3)$$

Expected Total Cost per cycle with variable lead time when the amount received is uncertain is

$$E\left(\check{C}(Y,r,L)\frac{\square}{Q}\right) = A + h\frac{\alpha Q}{2}\left[r - DL + \breve{\beta}E(X-r)^+\right] + \frac{h}{2D}\left[\sigma_o^2 + (\sigma_1^2 + \alpha^2)Q^2\right] + (\pi + \breve{\beta}\pi_o)E(X-R)^+ + R(L) \quad (4)$$

Where

$E(Y/Q) = \alpha Q$; $E(Y^2) = V(Y) + [E(Y)]^2 = \sigma_o^2 + (\sigma_1^2 + \alpha^2)Q^2$

We know that expected cycle time is $\frac{E(Y/Q)}{D} = \frac{\alpha Q}{D}$ (5)

Hence from (4) & (5) ,Expected Annual Total Cost with variable lead time when the uncertain amount received is,

$$\text{EAC}(Q,r,L) = \frac{AD}{\alpha Q} + h\left[r - DL + \breve{\beta}E(X-r)^+\right] + \frac{h}{2\alpha Q}\left[\sigma_o^2 + (\sigma_1^2 + \alpha^2)Q^2\right] + \frac{D}{\alpha Q}(\pi + \breve{\beta}\pi_o)E(X-r)^+ + \frac{D}{\alpha Q}R(L) \quad (6)$$

We assume that the Lead Time Demand X follows Normal Distribution with mean DL and standard deviation $\sigma\sqrt{L}$ ie., $X \sim N(DL, \sigma\sqrt{L})$, reorder point , $r = DL + k\sigma\sqrt{L}$, where k is the safety factor.

Expected Shortages at the end of the cycle time is given by

$$E(X-r)^+ = \int_r^{\infty}(x-r)f(x)\,dx = \sigma\sqrt{L}\psi(k), \text{ which occurs when } X > r \quad (7)$$

$$\text{EAC}(Q,r,L) = \frac{AD}{\alpha Q} + h\left[k\sigma\sqrt{L} + \breve{\beta}\sigma\sqrt{L}\psi(k)\right] + \frac{h}{2\alpha Q}\left[\sigma_o^2 + (\sigma_1^2 + \alpha^2)Q^2\right] + \frac{D}{\alpha Q}(\pi + \breve{\beta}\pi_o)\sigma\sqrt{L}\psi(k) + \frac{D}{\alpha Q}R(L) \quad (8)$$

The relation between ordering cost, A, and the capital investment in ordering cost reduction, I_o, is given by

$$I_o(A) = \frac{1}{c_1}In\left(\frac{A_o}{A}\right) \text{for } 0 < A \le A_o \quad (9)$$

The relation between lost sales, $\breve{\beta}$, and capital investment in lost sales reduction, I_1, is given by

$$I_1(\breve{\beta}) = \frac{1}{c_2}In\left(\frac{\breve{\beta}_o}{\breve{\beta}}\right) \text{for } 0 < \breve{\beta} \le \breve{\beta}_o \quad (10)$$

Where c_1 and c_2 are the fraction of the reduction in A and $\breve{\beta}$ per dollar increase in investment, respectively. The total investment in ordering cost and lost sales rate reduction is

$$I(A,\breve{\beta}) = I_o(A) + I_1(\breve{\beta}) \quad (11)$$

The Objective is to minimize the sum of the investment in ordering cost, lost sales rate reduction and inventory relevant cost expressed in (8), (9), (10).

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$$EAC(Q,A,\breve{\beta},L)=\theta\left(I(A,\breve{\beta})\right)+EAC(Q,L)$$
$$=\theta\left((I_o(A)+I(\breve{\beta})\right)+\frac{AD}{\alpha Q}+h\left[k\sigma\sqrt{L}+\breve{\beta}\sigma\sqrt{L}\psi(k)\right]+\frac{h}{2\alpha Q}\left[\sigma_o^2+(\sigma_1^2+\alpha^2)Q^2\right]$$
$$+\frac{D}{\alpha Q}(\pi+\breve{\beta}\pi_o)\sigma\sqrt{L}\psi(k)+\frac{D}{\alpha Q}R(L) \tag{12}$$

for $0<A\leq A_o$, $0<\breve{\beta}\leq\breve{\beta}_o$.

b) The service level constraint

For a given safety factor which satisfies the probability that lead time demand at the buyer exceeds reorder point ie., $X>r$, the actual proportion of demands not met from stock should not exceed the desired value, m.

According to [15] the service level constraint can be given as

$$\frac{Expected demand shortages at the end of the cycle for a given safety factor}{Quantity available for satisfying the demand per cycle}\leq m$$

ie.,$\frac{\sigma\sqrt{L}\psi(k)}{Q}\leq m$, $L\in[L_b,L_0]$ (13)

c) Mathematical model

Thus the mathematical model of the problem is

$$Min\ EAC(Q,A,\breve{\beta},L)=\theta\left(\frac{1}{c_1}In\left(\frac{A_o}{A}\right)+\frac{1}{c_2}In\left(\frac{\breve{\beta}_o}{\breve{\beta}}\right)\right)+\frac{AD}{\alpha Q}+h\left[k\sigma\sqrt{L}+\breve{\beta}\sigma\sqrt{L}\psi(k)\right]+$$
$$\frac{h}{2\alpha Q}\left[\sigma_o^2+(\sigma_1^2+\alpha^2)Q^2\right]+\frac{D}{\alpha Q}(\pi+\breve{\beta}\pi_o)\sigma\sqrt{L}\psi(k)+\frac{D}{\alpha Q}R(L) \tag{14}$$

Subject to

$$\frac{\sigma\sqrt{L}\psi(k)}{Q}\leq m \quad \text{(SLC)}$$
$$0<A\leq A_o \text{ and } 0<\breve{\beta}\leq\breve{\beta}_o \tag{15}$$

5. SOLUTION PROCEDURE

a) The solution without SLC

By ignoring the constraints the following lemma can be proved [13]

Lemma 1. $EAC(Q,A,\breve{\beta},L)$ is a concave or convex function for fixed $Q,A,\breve{\beta}$ and $L\in[L_b,L_0]$.

Thus for fixed $[L_b,L_0]$ the first order partial derivatives of $EAC(Q,A,\breve{\beta},L)$ with respect to $Q,A,\breve{\beta}$, we obtain

$$\frac{\partial EAC(Q,A,\breve{\beta},L)}{\partial Q}=\frac{-AD}{\alpha Q^2}-\frac{h\sigma_o^2}{2\alpha Q^2}+\frac{h}{2\alpha}\left[\sigma_1^2+\alpha^2\right]-\frac{D}{\alpha Q^2}(\pi+\breve{\beta}\pi_o)\sigma\sqrt{L}\psi(k)-\frac{D}{\alpha Q^2}R(L) \tag{16}$$

$$\frac{\partial EAC(Q,A,\breve{\beta},L)}{\partial A}=\frac{-\theta}{c_1 A}+\frac{D}{\alpha Q} \tag{17}$$

And

$$\frac{\partial EAC(Q,A,\breve{\beta},L)}{\partial\breve{\beta}}=\frac{-\theta}{c_2\breve{\beta}}+h\sigma\sqrt{L}\psi(k)+\frac{D}{\alpha Q}\pi_o\sigma\sqrt{L}\psi(k) \tag{18}$$

and the second order partial derivatives

$$\frac{\partial^2 EAC(Q,A,\breve{\beta},L)}{\partial Q^2}>0,\ \frac{\partial^2 EAC(Q,A,\breve{\beta},L)}{\partial A^2}>0\ \frac{\partial^2 EAC(Q,A,\breve{\beta},L)}{\partial\breve{\beta}^2}>0.$$

By setting (16), (17) and (18) equal to zero, we get

$$Q=\left[\frac{2D[A+\frac{h\sigma_o^2}{2D}+(\pi+\breve{\beta}\pi_o)\sigma\sqrt{L}\psi(k)+R(L)}{h[\sigma_1^2+\alpha^2]}\right]^{\frac{1}{2}}. \tag{19}$$

$$A=\frac{\theta\alpha Q}{c_1 D} \tag{20}$$

$$\breve{\beta}=\frac{\theta\alpha Q}{c_2[h\alpha Q+\pi_o D]\,\sigma\sqrt{L}\psi(k)}$$

And (21)

Lemma 2. For fixed $\in[L_b,L_0]$, the Hessian matrix of EAC$(Q,A,\breve{\beta},L)$ is positive definite at point

$(Q^*,A^*,\breve{\beta}^*)$ when $\frac{\pi+\breve{\beta}^*\pi_o}{Q^{*2}c_2\breve{\beta}^{*2}}>\frac{1}{c_1^2A^*}$ and $\frac{h\sigma_0^2}{Q^{*2}}>\frac{\theta}{c_1\alpha}$.

By [14] we get the above result by ignoring the constraints. Hence $Q^*,A^*,\breve{\beta}^*$ is the optimal solution such that EAC$(Q,A,\breve{\beta},L)$has minimum cost.

Now we consider the constraints $0<A\leq A_0$ and $0<\breve{\beta}\leq\breve{\beta}_0$. From equations (20) and (21), if$c_1,c_2,\ h,\ \alpha,\ \theta,\ D,\ L,\ \sigma,\ \pi_o$and $\psi(k)$are positive then A^* , $\breve{\beta}^*$ are also positive. Also, if $A^*<A$ and $\breve{\beta}^*<\breve{\beta}_0$, then $(Q^*,A^*,\breve{\beta}^*)$ is an interior optimal solution for a given $L\in[L_b,L_0]$. If $A^*<A_0$ and$\breve{\beta}^*\geq\breve{\beta}_0$, then the decision maker may decide against offering the price discount. Conversely, if $A^*\geq A_0$ and$\breve{\beta}^*<\breve{\beta}_0$., in this situation, the optimal ordering cost is the original ordering cost, i.e., $A^*=A_0$ Finally, if $A^*\geq A_0$ and $\breve{\beta}^*\geq\breve{\beta}_0$ then we should not make any investment .Then the optimat ordering cost is the original cost ie., $A^*=A_0$ and $\breve{\beta}^*=\breve{\beta}_0$

b) The solution with SLC

Consider the Lagrange Multiplier then the new model is

$$Min\, EAC(Q,A,\breve{\beta},L) = \theta(c_1 In\left(\frac{A_o}{A}\right)) + c_2 In(\frac{\breve{\beta}_o}{\breve{\beta}})) + \frac{AD}{\alpha Q} + h[k\sigma\sqrt{L} + \breve{\beta}\sigma\sqrt{L}\psi(k)] +$$

$$\frac{h}{2\alpha Q}[\sigma_o{}^2 + (\sigma_1{}^2 + \alpha^2)Q^2] + \frac{D}{\alpha Q}(\pi + \breve{\beta}\pi_o)\,\sigma\sqrt{L}\psi(k) + \frac{D}{\alpha Q}R(L) + \lambda[mQ - \sigma\sqrt{L}\psi(k)] \quad (22)$$

$$\text{Such that } 0 < A \le A_o \text{ and } 0 < \breve{\beta} \le \breve{\beta}_o \quad (23)$$

Let us find the first and second order partial derivative of (22) with respect to $Q, A, \breve{\beta}, L$ and equate it to zero, we get

$$Q = \left[\frac{D[A + \frac{h\sigma_0{}^2}{2D} + (\pi + \breve{\beta}\pi_0)\sigma\sqrt{L}\psi(k) + R(L)}{\frac{h}{2}[\sigma_1{}^2 + \alpha^2] + m\alpha[\frac{hk}{\psi(k)} + h\breve{\beta} + \frac{D}{\alpha Q}(\pi + \breve{\beta}\pi_0)]}\right]^{\frac{1}{2}}, \quad (24)$$

$$A = \frac{\theta\alpha Q}{c_1 D}, \quad (25)$$

$$\breve{\beta} = \frac{\theta\alpha Q}{c_2[h\alpha Q + \pi_0 D]\,\sigma\sqrt{L}\psi(k)}, \quad (26)$$

And

$$\lambda = \frac{hk}{\psi(k)} + h\breve{\beta} + \frac{D}{\alpha Q}(\pi + \breve{\beta}\pi_o) \quad (27)$$

we find that

$\frac{\partial^2 EAC(Q,A,\breve{\beta},L)}{\partial Q^2} > 0$, $\frac{\partial^2 EAC(Q,A,\breve{\beta},L)}{\partial A^2} > 0$ $\frac{\partial^2 EAC(Q,A,\breve{\beta},L)}{\partial \breve{\beta}^2} > 0$. Now

$$\frac{\partial^2 EAC(Q,A,\breve{\beta},L)}{\partial L^2} = -h\left[\frac{\sigma k}{3L^{\frac{3}{2}}} + \frac{\sigma\breve{\beta}\psi(k)}{3L^{\frac{3}{2}}}\right] - \frac{D}{3L^{\frac{3}{2}}\alpha Q}(\pi + \breve{\beta}\pi_o)\,\sigma\psi(k) + \frac{\lambda\sigma\psi(k)}{3L^{\frac{3}{2}}} \quad (28)$$

$$\frac{\partial^2 EAC(Q,A,\breve{\beta},L)}{\partial L^2} \ge 0, \text{ if } \lambda > \frac{hk}{\psi(k)} + h\breve{\beta} + \frac{D}{\alpha Q}(\pi + \breve{\beta}\pi_o) \quad (29)$$

Thus $EAC(Q,A,\breve{\beta},L)$ is convex in L for a given $Q, A, \breve{\beta}$ if $\lambda > \frac{hk}{\psi(k)} + h\breve{\beta} + \frac{D}{\alpha Q}(\pi + \breve{\beta}\pi_o)$. Now

$$\frac{\partial^2 EAC(Q,A,\breve{\beta},L)}{\partial L^2} \le 0, \text{ if} \le \frac{hk}{\psi(k)} + h\breve{\beta} + \frac{D}{\alpha Q}(\pi + \breve{\beta}\pi_o).$$

Therefore, $EAC(Q,A,\breve{\beta},L)$ is concave in L for a given $Q, A, \breve{\beta}$, the minimum total cost occurs at the end points of the interval $[L_b, L_0]$ if $\lambda \le \frac{hk}{\psi(k)} + h\breve{\beta} + \frac{D}{\alpha Q}(\pi + \breve{\beta}\pi_o)$ Hence the minimum total cost occurs at the end points of the interval $[L_b, L_0]$.

If we consider the constraint equation (23) then we have the optimal ordering cost and optimal lost sales as discussed in solution without SLC.

6. CONCLUSIONS

In this paper, with the consideration of partial backorder under the situation of uncertain quantity received and the controllable lead time, a continuous review policy is set and a Mathematical Model of a Non Linear Programming with service level constraint is derived. A Lagrangian approach is applied to find the optimal solution. Further study is to extend to address the whole supply chain, developing the inventory control approaches for the multi-echelon decision makers with controllable lead time and service level constraint. Also to consider the problem of multi items and permissible delay in payments in this model.

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