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NETWORK FLOW WITH FUZZY ARC LENGTHS USING HAAR RANKING

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ABSTRACT

Shortest path problem is a classical and the most widely studied phenomenon in combinatorial optimization. In a classical shortest path problem, the distance of the arcs between different nodes of a network are assumed to be certain. In some uncertain situations, the distance will be calculated as a fuzzy number depending on the number of parameters considered. This article proposes a new approach based on Haar ranking of fuzzy numbers to find the shortest path between nodes of a given network. The combination of Haar ranking and the well-known Dijkstra's algorithm for finding the shortest path have been used to identify the shortest path between given nodes of a network. The numerical examples ensure the feasibility and validity of the proposed method.

Keywords: Haar ranking, fuzzy numbers, ranking techniques, Dijkstra's algorithm, fuzzy shortest path problem.

1. INTRODUCTION

Solving mathematical models by using network fascinates many researchers and problems like assignment, transportation and shortest path based on networks have certainly garnered much attention. This article primarily focuses on the fuzzy based shortest path problem which is essential in many applications like communication, transportation, scheduling and routing. The objective of the shortest path problem is to find the path of minimum length between any pair of vertices. The arc length of the network may represent the real life quantities such as time, cost etc.. Algorithms for finding the shortest path in a network have been studied for a long time. The algorithms are still being improved [15 - 17]. Most of the time the parameters used in the algorithms are assumed to be deterministic. This may not be the case in the real time models. Fuzzy concepts [12] help us to overcome this difficulty and Dubois [1] was the first one to study the fuzzy shortest path problem. Klen [2] and Madhavi *et al.* [7] proposed a dynamic programming technique to solve the fuzzy shortest path problem. Chern [3] proposed an algorithm which concentrates on finding the important arc in the network. Li *et al.* [4] and Gen *et al.* [14] proposed algorithms involving neural networks and genetic algorithm respectively. Liu & Kao [5] investigated the network flows using fuzzy parameters. Hernandes *et al.* [6] used generic ranking to order the fuzzy numbers in the fuzzy shortest path problem. Ali Tajdin *et al.* [8] proposed an algorithm for the fuzzy shortest path problem in a mixed network.

The choice of ranking technique plays a vital role in decision making problems. The shortest path may vary depending on the ranking technique used. There is no unique ranking methodology of fuzzy numbers to decide on the shortest path of a given network. Furthermore, some of them give different ranking orders using different α-cuts. Yuan [9] formulated four criteria such as distinguish ability, rationality, fuzzy or linguistic presentation and robustness to evaluate the fuzzy ranking methods. Wang and Kerre [10] added three more properties for evaluating fuzzy ranking methods. Chien and *et al*. [11] extended the necessary characteristics of fuzzy ranking methods to eight. Decision makers must consider the various characteristics of ranking methods in determining whether the chosen fuzzy ranking method can support the features of the decision making problems. While studying fuzzy shortest path problem, the main difficulty is to find the suitable ordering of fuzzy numbers. We can avoid this by choosing Haar ranking [13] because it not only gives the Haar tuple to make use of the crisp value to decide the shortest path but also gives back the original fuzzy number using defuzzification to calculate the fuzzy distance of the shortest path. In this paper we utilized the Haar ranking which converts a given fuzzy number into a Haar tuple and applied the Dijkstra's algorithm in order to identify the shortest path of a given network. The fuzzzification from the defuzzied value is very easy using Haar ranking method and is unique. This paper is organised as follows: section 2 deals with the preliminaries that are required for formulating fuzzy shortest path problem, section 3 discusses the proposed algorithm and section 4 deals with numerical examples of the fuzzy shortest path problems. Finally, the conclusion part is given in section 5.

2. PREMILINARIES

Fuzzy set: The Fuzzy set can be mathematically constructed by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set [12].

Fuzzy number: The Fuzzy number is a fuzzy set whose membership function $\mu_{\tilde{A}}$ satisfies the following condition [12]

a) $\mu_{\tilde{A}}$ is a piecewise continuous function.
b) $\mu_{\tilde{A}}$ is a convex function.
c) $\mu_{\tilde{A}}$ is normal.($\mu_{\tilde{A}}(x_0)$ ■

Triangular fuzzy number: A Fuzzy number $\tilde{A}$ with membership function $\mu_{\tilde{A}}(x)$ of the form

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$\tilde{A} = \langle a\ b\ c \rangle$ is called Triangular Fuzzy number, where a, b and c are real numbers if $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, a \leq x < b \\ 1, x = b \\ \frac{c-x}{c-b}, b < x \leq c \end{cases}$$

Trapezoidal fuzzy number: A Fuzzy number with membership function $\mu_{\tilde{A}}(x)$ of the form $\tilde{A} = \langle a\ b\ c, d \rangle$ is called a Trapezoidal Fuzzy number, where a, b, c and d are real numbers if $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, a \leq x < b \\ 1, b \leq x \leq c \\ \frac{d-x}{d-c}, c < x \leq d \\ 0, otherwise \end{cases}$$

Arithmetic operations on triangular fuzzy number and Trapezoidal fuzzy number:

Addition:

$$\langle a_1,a_2,a_3\rangle + \langle b_1,b_2,b_3\rangle = \langle a_1+b_1,a_2+b_2,a_3+b_3\rangle, \langle a_1,a_2,a_3,a_4\rangle + \langle b_1,b_2,b_3,b_4\rangle = \langle a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4\rangle.$$

$$\text{Subtraction}\langle a_1,a_2,a_3\rangle - \langle b_1,b_2,b_3\rangle = \langle a_1-b_3,a_2-b_2,a_3-b_1\rangle, \langle a_1,a_2,a_3,a_4\rangle - \langle b_1,b_2,b_3,b_4\rangle = \langle a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1\rangle.$$

Haar Tuples: Consider a Trapezoidal fuzzy number $\tilde{A} = \langle a\ b\ c, d \rangle$. he Haar tuple of $\tilde{A}$ can be calculated using (1) by $H(\tilde{A}) = [\alpha, \beta, \gamma, \delta]$.

Where

$$\alpha = \frac{a+b+c+d}{4}, \beta = \frac{a+b-c-d}{4}, \gamma = \frac{a-b}{2}, \delta = \frac{c-d}{2} \quad (1)$$

Here is the average coefficient and the remaining coefficients are all called the detailed coefficients of the given trapezoidal fuzzy number. The same can be applied for triangular fuzzy numbers by extending the triangular fuzzy number into a trapezoidal fuzzy number using zero padding. The advantage of Haar ranking is that, the defuzzification can be done using the corresponding average and detailed coefficients at different levels which is not possible with any of the existing fuzzy ranking procedures.

Element wise operations of addition and subtraction of ordered tuples;

$$\text{Addition:}[\alpha_1,\beta_1,\gamma_1,\delta_1] + [\alpha_2,\beta_2,\gamma_2,\delta_2] = [\alpha_1+\alpha_2,\beta_1+\beta_2,\gamma_1+\gamma_2,\delta_1+\delta_2].$$
$$\text{Subtraction:}[\alpha_1,\beta_1,\gamma_1,\delta_1] - [\alpha_2,\beta_2,\gamma_2,\delta_2] = [\alpha_1-\alpha_2,\beta_1-\beta_2,\gamma_1-\gamma_2,\delta_1-\delta_2].$$

Ranking function: Let and are two fuzzy numbers. If the first element of Haar tuple of is less than the first element of Haar tuple of then $\tilde{A} \preccurlyeq$. That is if $H(\tilde{A}) = [\alpha_1, \beta_1, \gamma_1,$ and $H(\tilde{B}) = [\alpha_2, \beta_2, \gamma_2,$ then

i) $\tilde{A} \preccurlyeq \tilde{B}$ whenever $\alpha_1 \leq \alpha_2$.

ii) $\tilde{A} \succcurlyeq \tilde{B}$ whenever $\alpha_1 \geq \alpha_2$.

iii) $\tilde{A} \approx \tilde{B}$ whenever $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2, \delta_1 = \delta_2$.

Defuzzification: Given a Haar tuple $H(\tilde{A}) = [\alpha, \beta, \gamma, \delta]$, the corresponding fuzzy number can be calculated using (1) and is unique.

3. PROPOSED ALGORITHM

Step-1: Convert the given fuzzy numbers into Haar tuples by calculating its average and detailed coefficients.

Step-2: Use the element wise addition and subtraction operation and apply the Dijkstra's algorithm to the network model to identify the fuzzy shortest path.

Step-3: Convert the obtained shortest path into fuzzy number by using defuzzification in order to get the fuzzy distance of the shortest path.

4. NUMERICAL EXAMPLES

As an example to illustrate the algorithm, we consider a small network with six nodes and eight directed arcs used in Liu and Kao [15] given in Figure-1. The problem is to find the shortest path from starting node 1 to the ending node 6 where the arc lengths are having the trapezoidal fuzzy number as its distance.

As per step-1 of our algorithm we calculate the Haar tuples of the fuzzy distance for the arcs. For example, if, we consider the fuzzy distance $\langle 10\ 13\ 17$ between the nodes 2 and 3, the Haar tuple can be calculated using (1) as [15,-3.5,-1.5,-1.5]. The other fuzzy numbers can be converted into Haar tuples by applying the same procedure repeatedly.

Using Dijkstra's algorithm, we consider the starting value of node 1 as [0,0,0,0] and all other nodes as having the distance as . Let us also fix the source set S = {node 1}. From node 1, we can either reach node 2 or node 3 for which the path 1-2 has

$$\{[0,0,0,0] + [20,-5,-5,-5] = [20,-5,-5,-5]\}$$

as its length and the path 1-3 has

$$\{[0,0,0,0] + [62.5,-5.25,-5,-2.5] = [62.5,-5.25,-5,-2.5]\}$$

as its length. The average coefficients of both the Haar tuples suggest that the path 1-2 is the shortest path. From Table-1 we can see that the second row and the second column are now fixed before we proceed further. We also update the source set S = {node 1, node 2}. From the source set S, again by evaluating Haar tuple for all available paths, we can identify the next node to be included in the shortest path. There are now two paths to consider namely 1-2-3 of length [59.5,-8,-6.5,-7.5] and 1-2-5 of length [78,-9.5,-6.5,-7.5]. The third and fifth columns of Table-1 are updated with the corresponding values and we fix the third row and the third column entry of the table with the least average coefficient as 59.5. Therefore, the updated source set is S = {node 1, node 2, node 3}. From S, there are two paths 1-2-3-5 and 1-2-4 of lengths [68.5,-8.5,-7,-8] and [74.5,-11.5,-8]. Applying the procedure repeatedly, we can identify that the shortest path as **1-2-3-5-6** from Table-1.

The average coefficients of both the Haar tuples suggest that the path 1-2 is the shortest path using Dijkstar's algorithm. From Table-1 we can see that the second row and the second column is fixed to proceed further. We also update the source set S = {node 1, node 2}.

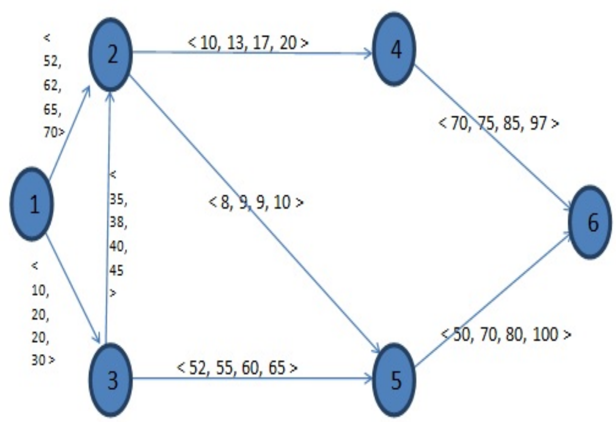

Figure-1. Network diagram: Example-1.

Table-1. Dijkstra's algorithm for Example-1.

Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Selected node	Path
[0 0 0 0]	-	-	-	-	-	1	1
	[20,-5,-5,-5]	[62.25,-5.25,-5,-2.5]	-	-	-	2	1-2
		[59.5,-8,-6.5,-7.5]	-	[78,-9.5,-6.5,-7.5]	-	3	1-2-3
			[74.5,-11.5,-8,-9]	**[68.5,-8.5,-7,-8]**	-	5	1-2-3-5
			[74.5,-11.5,-8,-9]		[143.5,-23.5,-17,-18]	4	1-2-3-4,1-2-3-5
					[143.5,-23.5,-17,-18]	6	**1-2-3-5-6,**1-2-3-4.

Once we identify the shortest path, the corresponding fuzzy distance can be calculated by applying defuzzification using Haar as {103, 137, 149, 18 shown in Table-2. We arrive at this value by considering the distance of the shortest path, that is, **(143.5,-23.5,-17,-18**) and using the definition of Haar tuples

$$\alpha = \frac{a+b+c+d}{4} = 143.5, \beta = \frac{a+b-c-d}{4} = -23.5, \gamma = \frac{a-b}{2} = -17, \delta = \frac{c-d}{2} = -18$$

By solving the above equations we can get {103, 137, 149, 18 as the corresponding fuzzy distance of the shortest path**.** The result matches with the shortest path obtained by Liu *et al*.[5] and Amit *et al.*[18].

Table-2. Defuzzification and the fuzzy distance of the shortest path of Example-1.

Node	Haar tuple	Fuzzy distance	Fuzzy shortest path from node 1
1	[0,0,0,0]	⟨0,0,0,0⟩	1
2	[20,-5,-5,-5]	⟨10,20,20,30⟩	1-2
3	[59.5,-8,-6.5,-7.5]	⟨45,58,60,75⟩	1-2-3
4	[74.5, -11.5,-8,-9]	⟨55,71,77,95⟩	1-2-3-4
5	[68.5,-8.5,-7,-8]	⟨53,67,69,85⟩	1-2-3-5
6	[143.5,-23.5,-17,-18]	⟨103,137,149,185⟩	1-2-3-5-6

The following example considered in Figure-2. illustrates a network with seven nodes and ten directed arcs given in Amit *et al.* [18]. The objective is to find the shortest path between the nodes 1 and 7. Consider the fuzzy distance between the nodes 2 and 4 given by ⟨2 11 . The Haar tuple obtained using the unnormalized Haar wavelet of the fuzzy number is given by [8.25,-1.75,-4.5,10]. The other fuzzy numbers can be converted into Haar tuples by applying the same procedure repeatedly. Using Dijkstra's algorithm, we consider the starting value of node 1 as [0,0,0,0] and all other nodes are having the distance as. Let us also fix the source set S = {node 1}. From node 1, we can either reach node 2 or node 4 for which the path 1-2 has $\{[0,0,0,0] + [9,0,-3,9] = [9,0,-3,9]\}$ as its length and the path 1-4 $\{[0,0,0,0] + [17.75,1.25,-6,16.5] = [17.75,1.25,-6,16.5]\}$ has as its length .The average coefficients of both the Haar tuples suggest that the path 1-2 is the shortest path using Dijkstar's algorithm. From Table-3 we can see that the second row and the second column is fixed to proceed further. We also update the source set S = {node 1, node 2}.

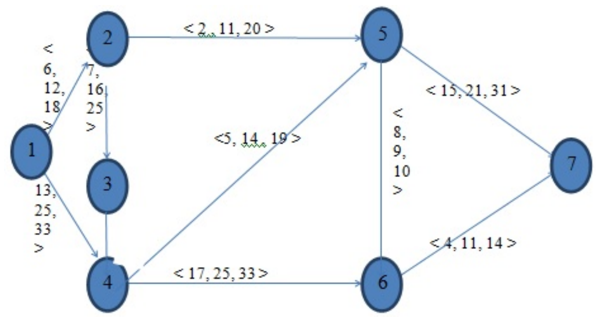

Figure-2. Network diagram: example 2.

Table-3. Dijkstra's algorithm for Example-2.

Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Selected node	Path
[0,0,0,0]	-	-	-	-	-	-	1	1
	[9,0,-3,9]	-	[17.75,1.25,-6,16.5]	-	-		2	1-2
		[21,-.5,-7.5,21.5]	[17.75,1.25,-6,16.5]	**[17.25,-1.75,-7.5,19]**	-		5	1-2-5
		[21,-.5,-7.5,21.5]	**[17.75,1.25,-6,16.5]**		[24,0,-8,24]	[34,-.5,-10.5,34.5]	4	1-2-5,1-4
		[21,-.5,-7.5,21.5]			[24,0,-8,24]	[34,-.5,-10.5,34.5]	3	1-2-5,1-4,1-2-3
					[24,0,-8,24]	[34,-.5,-10.5,34.5]	6	1-2-5-6,1-4,1-2-3
						[31.25,.25,-11.5,31]	7	1-2-5-6-7,1-4,1-2-3.

From the source set S, again by evaluating Haar tuple for all available paths, we can identify the next node to fix. There are three such paths to consider namely 1-2-3 of length [21,-.5,-7.5, 21.5] , 1-2-5 of length [17.25,-1.75,-7.5,19] and 1-4 of length [17.75,1.25,-6,16.5]. The third and fifth columns of the Table-1 are updated with the corresponding values and we fix the third row and the third column entry of the table with the least average coefficient as 17.25. Therefore, the updated source set is S = {node 1, node 2, node 5}. From S, there are four paths 1-2-5-6,1-2-5-7,1-2-3 and 1-4 of lengths [24,0,-8,24],[34,-.5,-10.5,34.5],[21,-.5,-7.5,21.5] and [17.75,1.25,-6,16.5] respectively. Applying the procedure repeatedly, we can identify that the shortest path as **1-2-5-6-7** from Table-4**.**

Table-4. Defuzzification and the fuzzy distance of the shortest path of Example-2.

Node	Haar tuples	Fuzzy distance	Fuzzy shortest path from node 1
1	[0,0,0,0]	⟨0,0,0,0⟩	1
2	[9,0,-3,9]	⟨6,12,18⟩	1-2
3	[21,-.5,-7.5,21.5]	⟨13,28,43⟩	1-2-3
4	[17.75,1.25,-6,16.5]	⟨13 25 33⟩	1-4
5	[17.25,-1.75,-7.5,19]	⟨8 23 38⟩	1-2-5
6	[24,0,-8,24]	⟨16 32 48⟩	1-2-5-6
7	[31.25,.25,-11.5,31]	⟨20 43 62⟩	1-2-5-6-7

Once we identify the shortest path, the corresponding fuzzy distance can be calculated as ⟨20 43 62) using Table-4. In order to get the fuzzy distance **⟨20 43 62)** we apply the concept of defuzzification using Haar. The distance of the shortest path is **[31.25,.25,-11.5,31]**. Let the fuzzy number corresponding to the distance of the shortest path be (a b c), by definition of Haar tuples

$$\alpha = \frac{a+b+c+d}{4} = 31.25, \beta = \frac{a+b-c-d}{4} = .25, \gamma = \frac{a-b}{2} = -11.5, \delta = \frac{c-d}{2} = 31$$

By solving the above equations we get **(20 43 62).** The result matches with Liu *et al*. [5] and Amit *et al*. [18] shortest path.

5. CONCLUSIONS

In this work, we have given a methodology to find the fuzzy shortest path between nodes of a network based on the Haar ranking and Dijkstra's algorithm. The proposed method is easy to implement because of the nature of the Haar ranking and the natural flow based on Dijkstar's algorithm makes the implemented technique an efficient one. The validation of the proposed algorithm is done by verifying the result obtained with the result got using existing approaches and the numerical results assert that the proposed approach can be implemented for any fuzzy shortest path problem. The idea can be extended to extract shortest path in a network with intutionistic fuzzy arc lengths.

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