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INTUITIONISTIC DOUBLE LAYERED FUZZY GRAPH

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ABSTRACT

In fuzzy graph theory, double layered fuzzy graph and intuitionistic fuzzy graph have been defined already by different authors. In this paper Intuitionistic double layered fuzzy graph is defined with examples. Some of its theoretical concepts were studied using different concepts in IFG.

Keywords: double layered fuzzy graph, order of IFG, size of IFG, vertex degree of IFG.

1. INTRODUCTION

In 1965, Zadeh published his seminal paper on "Fuzzy Sets" which described fuzzy set theory and consequently fuzzy logic [12]. The purpose of Zadeh's paper was to develop a theory which could deal with ambiguity and imprecision of certain classes of sets in human thinking, particularly in the domains of pattern recognition, communication, information and abstraction.
Azriel Rosenfeld in 1975 introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles, connectedness etc. [5]. Zadeh in 1987 introduced the concept of fuzzy relations.
Mordeson in 1993 introduced the concepts of fuzzy line graphs and developed its basic properties. Sunitha and Vijayakumar discussed about the operations of union, join Cartesian product and composition on two fuzzy graphs [4]. The degree of a vertex in some fuzzy graphs was discussed by Nagoorgani and Radha [6]. Nagoorgani and Malarvizhi have defined different types of fuzzy graphs and discussed its relationships with isomerism in fuzzy graphs [3]. The degree of a vertex in some fuzzy graphs was introduced by A. Nagooor Gani and K. Radha [2]. The first definition of intuitionistic fuzzy relations and intuitionistic fuzzy graphs were introduced by Atanassov in 1986[16] and further studied in 2009[13]. The operations on IFG was introduces by R. Parvathi, M. G. Karunambigai and K. Atanassov [13]. Degree, Order and Size in IFG was introduced by A. NaggorGani and S. ShajithaBegum[14]. The double layered fuzzy graph was introduced by T. Pathinathan and J. Jesintharosline, they have examined some of the properties of DLFG [1]. The vertex degree of cartesian product of intuitionistic fuzzy graph is given by T. Pathinathan and J. Jesintharosline [14]. In this paper Intuitionistic double layered fuzzy graph is defined and illustrated with some examples. Some of its properties is also analysed using intuitionistic fuzzy graph concepts and operations. First we go through some of the basic definitions.

2. PRELIMINARIES

a) Fuzzy graph [4]

A fuzzy graph G is a pair of functions G: (σ,μ) where σ is a fuzzy subset of a non empty set S and μ is a symmetric fuzzy relation on σ . The underlying crisp graph of G: (σ,μ) is denoted by $G^*:(\sigma^*,\mu^*)$.

b) Intuitionistic fuzzy graph (IFG) [12]

An IFG is of the form G: (V, E) where

(i) V = {$v_1, v_2, v_3, \ldots, v_n$} such that $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and non - membership of the element $v_i \in V$ respectively, and

$0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$
for every $v_i \in V, (i = 1, 2, \ldots n)$ (1)

(ii) $E \subseteq V \times V$ where $\mu_2: E \to [0,1]$ and $\gamma_2: E \to [0,1]$ are such that

$$\mu_2(v_i, v_j) \le \mu_1(v_i) \wedge \mu_1(v_j) \quad (2)$$

$$\gamma_2(v_i, v_j) \le \gamma_1(v_i) \vee \gamma_1(v_j) \quad (3)$$

And $0 \le \mu_2(v_i v_j) + \gamma_2(v_i v_j) \le 1$ (4)

for every $(v_i, v_j) \in E, (i, j = 1, 2, \ldots, n)$.

Notations

The triplets $(v_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and non – membership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and non – membership of the edge relation $e_{ij} = (v_i, v_j)$ on V.

c) Vertex degree of IFG [13]

Let G = (V,E) be an IFG. Then the degree of a vertex v is defined by $d(v) = (d_\mu(v), d_\gamma(v))$ where $d_\mu(v) = \sum_{u \ne v} \mu_2(u, v)$ and $d_\gamma(v) = \sum_{u \ne v} \gamma_2(u, v)$.

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d) Order of IFG [13]

Let G = (V,E) be an IFG. Then the order of G is defined as $O(G) = (O\mu(G), O\gamma(G))$ where

$$O\mu(G) = \sum_{v \in V} \mu_1(v) \text{ and } O\gamma(G) = \sum_{v \in V} \gamma_1(v)$$

e) Size of IFG [13]

Let G = (V,E) be an IFG. Then the size of G is defined as $S(G) = (S\mu(G), S\gamma(G))$ where

$$S\mu(G) = \sum_{u \neq v} \mu_2(u,v) \text{ and } S\gamma(G) = \sum_{u \neq v} \gamma_2(u,v)$$

f) Double layered fuzzy graph [1]

Let $G:(\sigma,\mu)$ be a fuzzy graph with the underlying crisp graph $G^*:(\sigma^*,\mu^*)$. The pair $DL(G):(\sigma_{DL},\mu_{DL})$ is defined as follows. The node set of $DL(G)$ be $\sigma^* \cup \mu^*$. The fuzzy subset σ_{DL} is defined as $\sigma_{DL} = \begin{cases} \sigma(u) \text{ if } u \in \sigma^* \\ \mu(uv) \text{ if } uv \in \mu^* \end{cases}$

The fuzzy relation μ_{DL} on $\sigma^* \cup \mu^*$ is defined as

$$\mu_{DL} = \begin{cases} \mu(uv) \text{ if } u,v \in \sigma^* \\ \mu(e_i) \wedge \mu(e_j) \text{ if the edge } e_i \text{ and } e_j \textit{ have} \text{ a node in common between them} \\ \sigma(u_i) \wedge \mu(e_i) \text{ if } u_i \in \sigma^* \text{ and } e_i \in \mu^* \text{ and each } e_i \text{ is incident with single } u_i \\ \qquad \text{either clockwise or anticlockwise.} \\ 0 \text{ otherwise} \end{cases}$$

By definition $\mu_{DL}(u,v) \le \sigma_{DL}(u) \wedge \sigma_{DL}(v)$ for all u,v in $\sigma^* \cup \mu^*$. Here μ_{DL} is a fuzzy relation on the fuzzy subset σ_{DL}. Hence the pair $DL(G):(\sigma_{DL},\mu_{DL})$ is defined as double layered fuzzy graph (DLFG).

3. INTUITIONISTIC DOUBLE LAYERED FUZZY GRAPH (IDLFG)

Let $G:\langle (v_i,\mu_1,\gamma_1),(e_{ij},\mu_2,\gamma_2)\rangle$ be an intuitionistic fuzzy graph with the underlying crisp graph $G^*:(\sigma^*,\mu^*)$. The pair $IDL(G):\langle (v_i,\mu_{DL_1},\gamma_{DL_1}),(e_{ij},\mu_{DL_2},\gamma_{DL_2})\rangle$ is called the intuitionistic fuzzy graph and is defined as follows. The node set of $IDL(G)$ be $\langle \mu_{DL_1},\gamma_{DL_1}\rangle$. The fuzzy subset $\langle \mu_{DL_1}(u),\gamma_{DL_1}(u)\rangle$ is defined as

$$\langle \mu_{DL_1}(u),\gamma_{DL_1}(u)\rangle = \begin{cases} \langle \mu_1(u),\gamma_1(u)\rangle \text{ if } u \in \sigma^* \\ \langle \mu_2(uv),\gamma_2(uv)\rangle \text{ if } uv \in \mu^* \end{cases}$$

Where $0 \le \mu_{DL_1} + \gamma_{DL_1} \le 1$

The fuzzy relation $\langle \mu_{DL_2},\gamma_{DL_2}\rangle$ on $\sigma^* \cup \mu^*$ is defined as

$$\langle \mu_{DL_2}(uv),\gamma_{DL_2}(uv)\rangle = \begin{cases} \langle \mu_2(uv),\gamma_2(u,v)\rangle \text{ if } u,v \in \sigma^* \\ \langle \mu_2(e_i)\wedge\mu_2(e_j),\gamma_2(e_i)\vee\gamma_2(e_j)\rangle \text{ if the edge } e_i \text{ and } e_j \textit{ have} \text{ a node in common between them} \\ \langle \mu_1(u_i)\wedge\mu_2(e_i),\gamma_1(u_i)\vee\gamma_2(e_i)\rangle \text{ if } u_i \in \sigma^* \text{ and } e_i \in \mu^* \text{ and each } e_i \text{ is incident with single } u_i \\ \qquad \text{either clockwise or anticlockwise.} \\ 0 \text{ otherwise} \end{cases}$$

Here $0 \le \mu_2(uv) + \gamma_2(uv) \le 1$ for all u,v in $\sigma^* \cup \mu^*$.

Here $\langle \mu_{DL_2}(uv),\gamma_{DL_2}(uv)\rangle$ is a fuzzy relation on the fuzzy subset $\langle \mu_{DL_1}(u),\gamma_{DL_1}(u)\rangle$.

Remark-3.1: Here the crisp graph G^* is a cycle and the above definition is applicable for n number of cycles.

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Example-3.1: Consider the intuitionistic fuzzy graph G, whose crisp graph G^* is a cycle with n = 3 vertices.

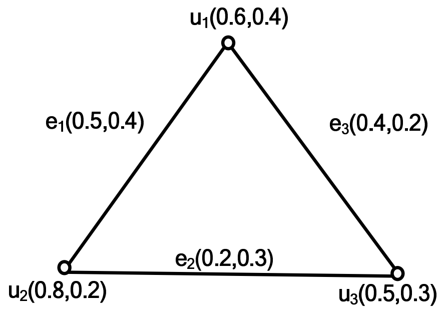

Figure-1. Intuitionistic fuzzy graph.

$$G:\left\langle (v_i,\mu_1,\gamma_1),(e_{ij},\mu_2,\gamma_2)\right\rangle$$

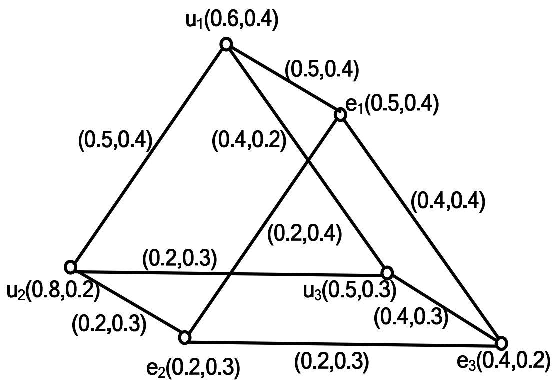

Figure-2. Intuitionistic double layered fuzzy graph.

$$IDL(G):\left\langle (v_i,\mu_{DL_1},\gamma_{DL_1}),(e_{ij},\mu_{DL_2},\gamma_{DL_2})\right\rangle$$

Consider the fuzzy graph with n = 4 vertices.

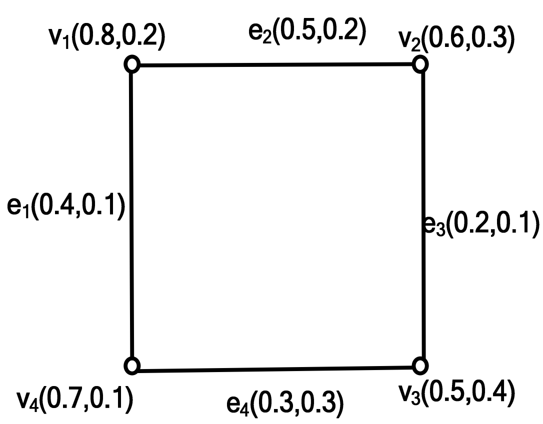

Figure-3 Intuitionistic fuzzy.

Graph $G:\left\langle (v_i,\mu_1,\gamma_1),(e_{ij},\mu_2,\gamma_2)\right\rangle$.

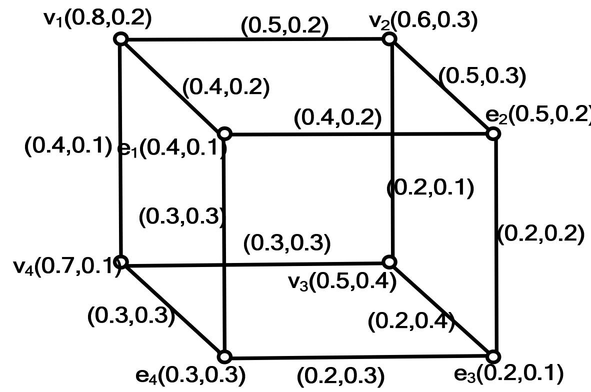

Figure-4. Intuitionistic double layered fuzzy.

graph $IDL(G):\left\langle (v_i,\mu_{DL_1},\gamma_{DL_1}),(e_{ij},\mu_{DL_2},\gamma_{DL_2})\right\rangle$.

Similarly we can get different intuitionistic double layered fuzzy graphs for a given intuitionistic fuzzy graph G, whose crisp graph is a cycle.

4. THEORETICAL CONCEPTS

Theorem-4.1: Order IDL(G) = Order(G) + Size(G), where G is an intuitionistic fuzzy graph.

Proof: As the node set of IDL(G) is $\sigma^* \cup \mu^*$ and the fuzzy subset $\left\langle \mu_{DL_1}(u),\gamma_{DL_1}(u)\right\rangle$ on

$\sigma^* \cup \mu^*$ is defined as

$$\left\langle \mu_{DL_1}(u),\gamma_{DL_1}(u)\right\rangle = \begin{cases} \left\langle \mu_1(u),\gamma_1(\mathrm{u})\right\rangle & \text{if } u \in \sigma^* \\ \left\langle \mu_2(uv),\gamma_2(uv)\right\rangle & \text{if } uv \in \mu^* \end{cases}$$

$$\text{Order IDL(G)} = \sum_{u \in VUE} \left\langle \mu_{DL_1}(u),\gamma_{DL_1}(u)\right\rangle$$

(by definition 2.4)

$$= \sum_{u\in V} \sigma_{DL}(u) + \sum_{u \in E} \sigma_{DL}(u)$$

$$= \sum_{u\in V} \left\langle \mu_1(u),\gamma_1(\mathrm{u})\right\rangle + \sum_{u\in E}\left\langle \mu_2(uv),\gamma_2(uv)\right\rangle$$ by

definition of $\left\langle \mu_{DL_1},\gamma_{DL_1}\right\rangle$

= Order (G) + Size (G). by defintitions 2.4 and 2.5

Theorem-4.2:

$$Size\ IDL(G) = 2\text{size (G)} + \sum_{e_i,e_j\in E} \left\langle \mu_2(e_i)\wedge\mu_2(e_j),\gamma_2(e_i)\vee\gamma_2(e_j)\right\rangle$$

where G is a intuitionistic fuzzy graph and i, j $\in$ N.

Proof: $Size\ IDL(G) = \sum_{u,v \in VUE} \langle \mu_{DL_2}(uv), \gamma_{DL_2}(uv) \rangle$

(by defenition 2.5)

$$= \sum_{u,v \in V} \langle \mu_2(uv), \gamma_2(u,v) \rangle + \sum_{e_i,e_j \in E} \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \vee \gamma_2(e_j) \rangle + \sum_{u_i \in V, e_i \in E} \langle \mu_1(u_i) \wedge \mu_2(e_i), \gamma_1(u_i) \vee \gamma_2(e_i) \rangle$$

(u_i is in one of the end node of e_i in the third summation)

$$= \text{size (G)} + \sum_{e_i,e_j \in E} \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \vee \gamma_2(e_j) \rangle + \sum_{u_i \in V, e_i \in E} \langle \mu_2(e_i), \gamma_2(e_i) \rangle$$

Since in the third summation, we are considering only one vertex in each edge either clockwise or anticlockwise direction, its membership value is less than the membership value of the vertices and non membership value is more than the non membership value of the vertices.

$Size\ DL(G)$

$$= \text{size (G)} + \sum_{e_i,e_j \in E} \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \vee \gamma_2(e_j) \rangle + size \text{ (G)}$$

$$= 2\text{size (G)} + \sum_{e_i,e_j \in E} \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \vee \gamma_2(e_j) \rangle$$

Theorem-4.3: Let G be an intuitionistic fuzzy graph then.

$$d_{IDL(G)}(u) = \begin{cases} d_G(u) + \langle \mu_1(u_i) \wedge \mu_2(e_i), \gamma_1(u_i) \vee \gamma_2(e_i) \rangle \text{ if } u \in \sigma^* \\ \sum_{e_i,e_j \in \mu^*} \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \vee \gamma_2(e_j) \rangle \\ \quad + \langle \mu_1(u_i) \wedge \mu_2(e_i), \gamma_1(u_i) \vee \gamma_2(e_i) \rangle \text{ if } u \in \mu^* \end{cases}$$

Proof: By definition 2.3, we have

$$d_G(u) = (\sum_{u \neq v} \mu_2(u,v), \sum_{u \neq v} \gamma_2(u,v))$$

Case-i: Let $u \in \sigma^*$, then

$$d_{IDL(G)}(u) = \sum_{u \neq v} \langle \mu_{DL_2}(uv), \gamma_{DL_2}(uv) \rangle$$

$$= \sum_{u \neq v} \langle \mu_2(uv), \gamma_2(uv) \rangle + \langle \mu_1(u_i) \wedge \mu_2(e_i), \gamma_1(u_i) \vee \gamma_2(e_i) \rangle$$

($\because$ in the first summation the vertices which are adjacent in G is also adjacent in IDLFG)

$$= d_G(u) + \langle \mu_1(u_i) \wedge \mu_2(e_i), \gamma_1(u_i) \vee \gamma_2(e_i) \rangle$$

Case-ii: Let $u \in \mu^*$, then

$$d_{IDL(G)}(u) = \sum_{u \neq v} \langle \mu_{DL_2}(uv), \gamma_{DL_2}(uv) \rangle$$

$$= \sum_{e_i,e_j \in \mu^*} \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \vee \gamma_2(e_j) \rangle + \mu_{DL}(u_i, e_i)$$

$$= \sum_{e_i,e_j \in \mu^*} \langle \mu_2(e_i) \wedge \mu_2(e_j), \gamma_2(e_i) \vee \gamma_2(e_j) \rangle + \langle \mu_1(u_i) \wedge \mu_2(e_i), \gamma_1(u_i) \vee \gamma_2(e_i) \rangle$$

Remark-4.1:

If G is a strong IFG then IDL(G) is also a strong intuitionistic fuzzy graph.

6. CONCULSION AND FUTUREWORK

In this paper, we have defined a new intuitionistic fuzzy graph namely intuitionistic double layered fuzzy graph and illustrated with some examples. Further work will lead to the application in IDLFG.

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REFERENCES

[1] T. Pathinathan and J. Jesintha Rosline. 2014. “Double layered fuzzy graph”, Annals of Pure and Applied Mathematics, Vol. 8, No. 1, pp. 135-143.

[2] A. Nagoorgani and J. Malarvizhi. 2009. “Properties of μ - complement of a fuzzy graph”, Inter. Journal of Algorithms, Computing and Mathematics, Vol. 2, No. 3, pp. 73 - 83.

[3] M. S. Sunitha and A. Vijayakumar. 2002. “Complement of a fuzzy graph”, Indian Journal of Pure and Applied mathematics, Vol. 33, No. 9, pp. 1451-1464.

[4] A. Rosenfeld, Fuzzy graphs, in: L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura,(editors), Fuzzy sets and its application to cognitive and decision process, Academic press, New York (1975) pp. 77 – 95.

[5] A. Nagoorgani and K. Radha. 2009. “The degree of a vertex in some fuzzy graphs”, Inter. Journal of Algorithms, Computing and Mathematics, Vol. 2, No. 3, pp. 107 - 116.

[6] R. T. Yeh and S. Y. Bang. 1975. Fuzzy relations, fuzzy graphs and their applications to clustering analysis, in: L.A. Zadeh, K.S. Fu, K. Tanaka and M.

Shimura,(editors), Fuzzy sets and its application to cognitive and decision process, Academic press, New York, pp. 125 – 149.

[7] A. Nagoorgani and M. Basheed Ahamed. 2003. “Order and size in fuzzy graphs”, Bulletin of Pure and Applied Sciences, 22E No. 1, pp. 145–148.

[8] J. N. Mordeson. 1993. Fuzzy line graphs, Pattern Recognition Letter, Vol. 14, pp. 381–384.

[9] J. N. Mordeson and P. S. Nair. 2011. “Fuzzy graphs and Fuzzy hypergraphs”, Physica Verlag Publication, Heidelbserg, Second edition.

[10]K. R. Bhutani. 1989. “On Automorphisms of Fuzzy Graphs”, Pattern Recognition Letter, 9 pp. 159–162.

[11] L. A. Zadeh. 1965. “Fuzzy sets”, Information control 8, pp. 338–353.

[12] R. Parvathi, M. G. Karunambigai and K. Atanassov. 2009. “Operations on Intuitionistic Fuzzy Graphs”, Proceedings of IEEE International Conference on Fuzzy Systems (FUZZ – IEEE), pp. 1396 – 1401.

[13] A. Naggor Gani and S. Shajitha Begum. 2010. “Degree, Order and Size in Intuitionistic fuzzy graphs”, International Journal of Algorithms, Computing and Mathematics, Vol. 3, No. 3, pp. 11-16.

[14]T. Pathinathan and J. Jesintha Rosline. 2014. “Vertex degree of Cartesian product of intuitionistic fuzzy graph”, International Journal of Scientific and Engineering Research, Vol. 5, No. 9, pp. 224 – 227.

[15]T. Pathinathan and J. Jesintha Rosline. 2014. “Characterization of fuzzy graphs into different categories using arcs in fuzzy graph”, Journal of Fuzzy set valued analysis pp. 1-6.

[16] K. T. Atanassov. 1986. “Intuitionistic fuzzy sets”, fuzzy sets and systems Vol. 20, pp. 87-96.

[17]T. Pathinathan and J. Jesintha Rosline. 2014. “Matrix Representation of Double layered fuzzy graph and its properties”, Annals of Pure and Applied Mathematics, Vol. 8, No. 2, pp. 51 – 58.