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ANALYSIS OF POVERTY: USING FUZZY TRIANGULAR ANALYTICAL HIERARCHY PROCESS

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ABSTRACT

Grading the poor helps the government to establish a better policy to distribute resources more reasonably, and therefore provide a government aid to the deserving families more effectively. The traditional single-factor model (Income and consumption expenditure model) is not adequate, because poverty grade analysis involves various factors of different weights. Some factors cannot be analysed by classical algorithm namely income – expenditure and consumption model. In this paper we establish a multi-criteria decision model (MCDM). We use fuzzy triangular analytical hierarchy process (FTAHP) to analyse poverty. We determine the indexes of poverty grade according to maximum membership degree which is derived from the Fuzzy AHP- Fuzzy Triangular Numbers comparison criteria importance matrices. In this way we quantify the qualitative data.

Keywords: poverty, multi-criteria decision, fuzzy triangular analytical hierarchy process.

1. INTRODUCTION

Any tool to measure poverty has to consider the basic needs namely food, house and dress (Roti, Kapda and Makkan). By casual observations on one's living conditions we cannot easily decide on the level of financial status of a person. Majority of houses in a village have the same pattern, all wear almost the same style of dress and as the staple food is what is available during the season, there is not a big difference in food but there may be some difference in quantity and quality.

A fuzzy hierarchical analytic model and triangular fuzzy number are used to quantify the weights of the relative importance of the criteria. The relative weights are normalized and then fuzzy composite weights are calculated for the household performance in each criterion to identify the different categories. A case study from the rural villages in Nalanda district, Bihar, India is present to verify the methodology.

2. NEED FOR A FUZZY AHP APPROACH AND DEVELOPMENT

Fuzzy logic may be viewed as an attempt to communicate reason and make rational decisions in an environment of imprecision. Though the aim of the AHP is to capture the expert's knowledge, the conventional AHP still cannot reflect the human thinking style. Therefore, a fuzzy extension of AHP was developed to address and to solve imprecision inherent in the real world problem.

Fuzzy AHP method has been evolved from Multi-Criteria Decision Making process. Analytic Hierarchy Process (AHP) was introduced by Thoma L. Saaty in the year 1980. The major characteristic of the AHP method is the use of pair-wise comparisons, which are used to compare with respect to the various criteria, sub-criteria and alternatives to estimate criteria weights.

Van Laarhoven and Pedrycg introduced Fuzzy AHP in the year 1983. They proposed a method of fuzzy

judgement by comparison of the triangular fuzzy numbers. They also used fuzzy numbers with triangular membership function with simple operation laws and the logarithmic least squares method to obtain element sequencing. Later in the year 1985, J.J. Buckley extended Saaty's method to incorporate fuzzy comparison ratio by using fuzzy trapezoidal fuzzy numbers. In 1992, Da-Yong Chang introduced the extent analysis method on fuzzy AHP. In 1995 Again Chang proposed the principle for comparison between the elements of the fuzzy numbers. In 2002, Cebeci and Cengiz Kahraman compared some catering firms using four attributes and fuzzy AHP. Since then many scholars have engaged in the fuzzy extension of fuzzy AHP.

3. FUZZY APPROACH TO POVERTY ANALYSIS

a) Defining Poverty

A person who is poor implies poverty as lack of security, low wages, lack of employment opportunity, poor nutrition, poor access to safe drinking water, having too many children to feed, children being engaged in work to bring money to a family, poor educational opportunities, and misuse of resources etc. whereas, for a non-poor person poverty is a lack of income. There is a general consensus that poverty is multi-dimensional. This view is clearly expressed by the following definition given by the World Bank in the year 2002.

"Poverty is hunger. Poverty is lack of shelter. Poverty is being sick and not being able to see a doctor. Poverty is not being able to go to school and not knowing how to read. Poverty is not having a job, is fear for the future, living one day at time. Poverty is losing a child to illness brought by unclean water. Poverty is powerless, lack of representation and freedom."

It is in this context Mozaffar Qizilbash defines poverty as a vague concept [4]. Thus we propose to measure the degree of poverty incorporating © 2006-2015 Asian Research Publishing Network (ARPN). All rights reserved.

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multidimensional aspects of deprivation into the definition.

b) Poverty Set: A matter of degree

Poverty Set can be defined as a matter of degree based on the fuzzy logic concept. In fuzzy logic a statement can be true to a certain degree. Therefore, the poor individual or a household are assigned a degree in relation to the membership functions. A poor person belonging to a given set in a varying degree is assigned with membership values 1 (the poorest person) and 0 (the non-poorest person). In mathematical terms it can be represented as follows: False: Truth value = 0, True: truth value =1, Uncertain: 0 < Truth value < 1.

c) Poor: A vague predicate

Poor is a vague predicate because, (i) It involves borderline cases (a person is not clearly poor and not clearly non-poor), (ii) It lacks sharp boundaries (along a hypothetical scale of well-being, an exact point at which a poor ceases to be poor does not really exist).

d) Review on fuzzy approach to analyse poverty

The studies on fuzzy poverty were made by Andréa Cerioli and Sergio Zani in 1990. Totally Fuzzy and Relative (TFR) approach was developed by Cheli and Lemmi and modified by Betti et al. (2005) in the form of an Integrated Fuzzy and Relative (IFR) approach to analyse the poverty and social exclusion. In the year 2002 Chiappero- Martinetti used the 1994 Italian household survey data to promote the methodology of the fuzzy set theory to measure well-being in the functionings and capabilities space. The implementation of this approach has been developed by a number of authors. Cheli and Betti (1999) and Betti et al. 2005 focusing more on the "time dimension", in particular utilising the tool of transition matrices. Afterwards, Betti and Verma (1999, 2002, and 2004) and verma and Betti (2002) refined the approach giving focus on capturing the multi-dimensional aspects, developing the concepts of "manifest" and "latent" deprivation to reflect the intersection and union of different dimensions.

4. FUZZY AHP – FUZZY TRIANGULAR APPROACH

In this study, FAHP is used to analyze the relative importance of each criterion and to evaluate each poor criterion in order to determine the positioning level of the socio-economic status of a person.

a) Fuzzy subset approach to Poverty analysis

Let us consider a set *E* of *n* individuals or households and let \underline{A} be a subset of *E* consisting of the poor, such that a fuzzy membership is given by $\mu_{\underline{A}}(x_i)$ where (i = 1, 2, 3, ..., n) denote for each individual or household in \underline{A} and $\mu : \underline{A} \rightarrow [0,1]$. Then the membership function for the poor is defined by

- 1) $\mu_{\underline{A}}(x_i) = 0$ if i^{th} individual is certainly not poor;
- 2) $\mu_{A}(x_{i}) = 1$ if i^{th} individual is poor;
- 3) $0 < \mu_A(x_i) < 1$ if i^{th} individual exhibits a

partial membership in the subset of A.

Fuzzy approach tries to answer: (i) How can we assign memberships to elements in a fuzzy set? (ii) How can the notion of fuzzy sets be applied to practical problems? The first question concerns the construction of a numerical scale for membership values in such a way that the scale satisfies some conditions imposed on rational measurement system. It is done through assigning membership function to the criteria and alternatives.

b) Fuzzy AHP-methodology

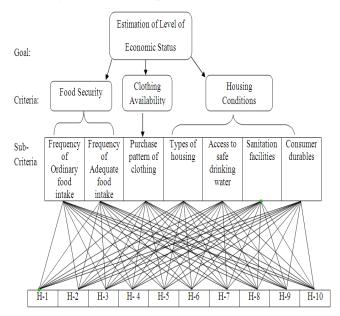


Figure-1. Alternatives :(Households) Hierarchy tree for ten households (H-1...H-10).

c) Computational procedures of fuzzy AHP

To assign the weights of criteria, sub-criteria and alternatives, we proceed as given below:

Step-1: Construction of the hierarchical structure with decision elements: criteria and sub-criteria. Each decision maker is asked to express relative importance of the decision elements in the same level with help of a reference scale values: 1-9 scale.

Step-2: Collect the score of pair wise comparison and form pair wise comparison matrices for each of the n decision makers. It is done at each level using the scale response on the questionnaire. How important is one element when it is compared with the other element?

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Step-3: Construction of a fuzzy judgement Matrix which are represented by the positive triangular numbers.

Step-4: Fuzzification is done by normalizing the triangular weights.

Step-5: Calculation of the fuzzy Centre Membership values.

Step-6: Computation of the composite weight and finally obtaining the ranking of the households into poverty category.

d) Construction of triangular numbers

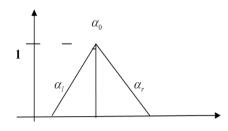


Figure-2. Triangular fuzzy numbers.

The triangular number is represented by the three parameters such as α_l , α_0 and α_r where α_l denotes the smallest possible value, α_0 the most promising value and α_r the largest possible value respectively.

Since each number in the pair wise comparison represents the subjective judgement opinion of the decision maker is a vague judgement. Therefore, the fuzzy numbers work the best to consolidate the fragmented judgement of the expert opinions. The fuzzy triangular number is determined by the following formula as defined:

$$\alpha = (\alpha - 1, \alpha, \alpha + 1), \forall \alpha = 2, ..., 8 and 1 = (1, 1, 1) and 9 = (9, 9, 9)$$

where \sim tilde symbol represents the fuzziness involved in the judgement system.

e) Fuzzy numbers

A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0, 1]. A fuzzy set \underline{A} in a universe of discourse X is defined as the following set of pairs: $\underline{A} = \{ (x, \mu_A(x); x \in X) \}$, $\mu_A: X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set \underline{A} and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set \underline{A} . These membership grades are often represented by real numbers ranging from [0, 1].

Definition of fuzzy number

A Fuzzy number A is a fuzzy set on the real line R, must satisfy the following conditions.

(i) $\mu_{\underline{A}}(x_o)$ is piecewise continuous (ii) There exist at least

one
$$x_o \in R$$
 with $\mu_{\underline{A}}(x_o)^{=1}$

(iii) A must be normal and convex

f) Definition triangular fuzzy number

Triangular Fuzzy Number is defined as $\underline{A} = \{\alpha_l, \alpha_0, \alpha_r\}$ where α_l, α_0 and α_r are real numbers and its membership function is defined by

$$\mu_{A}(x) = \begin{cases} \frac{(x - \alpha_{1})}{(\alpha_{0} - \alpha_{1})} & \text{if } x \in [\alpha_{1}, \alpha_{0}) \\ 1 & \text{if } x = \alpha_{0} \\ \frac{(x - \alpha_{r})}{(\alpha_{0} - \alpha_{r})} & \text{if } x \in (\alpha_{0}, \alpha_{r}] \\ 0 & \text{otherwise} \end{cases}$$

g) Definition of fuzzy centre value

Let $\underline{\mathcal{C}}$ be a fuzzy number and $\mu_{\underline{c}}$ be its membership function the for a given fuzzy number $\underline{\mathcal{C}}$, let α_0 be a core element of $\underline{\mathcal{C}}$ such that

$$F_{c} = \alpha_{0} - \frac{1}{2} \int_{\alpha_{1}}^{\alpha_{0}} \mu_{c}(x) dx + \frac{1}{2} \int_{\alpha_{0}}^{\alpha_{r}} \mu_{c}(x) dx$$

Where, $x = \frac{x - \alpha_{1}}{\alpha_{0} - \alpha_{1}}$ and $x = \frac{x - \alpha_{r}}{\alpha_{0} - \alpha_{r}}$, then F_{c} is

called a fuzzy centre value of \mathcal{L} .

Therefore, for fuzzy triangular Numbers [α_l , α_0 , α_r] and its fuzzy Centre value is derive by

$$F_{c} = \alpha_{0} - \frac{1}{4}(\alpha_{0} - \alpha_{1}) + \frac{1}{4}(\alpha_{r} - \alpha_{0})$$
$$= \frac{\alpha_{0}}{2} + \frac{1}{4}(\alpha_{1} + \alpha_{r})$$

h) Construction of fuzzy pair-wise comparison matrix (Fuzzification)

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$$\tilde{M} = [\tilde{a}_{ij}]_{m \times n} = \begin{cases} \tilde{a}_{ij} = \frac{E_i}{E_j} & \text{How importance more (less) is } E_i \text{ w.r.t } E_j \\ \tilde{a}_{ij} = 1 & \text{Every element has the same importance} \\ \tilde{a}_{ji} = \frac{1}{\tilde{a}_{ij}} & \text{if } E_i \text{ is } \tilde{a}_{ij} \text{ times more (less) importance than } E_j, \text{ otherwise vice versa} \end{cases}$$

where, E_i and E_j are the criteria compared one over the other and a_{11} are the values assigned to the criteria.

i) Establishment of scale

- 1. If a criterion on the Left is more important than the one matching on the Right, assign actual judgments value to the Left criterion.
- 2. If a criterion on the Left is less important than the one matching on the Right, assign the reciprocal value to the right criterion.
- 3. While comparing one household with the other, we relate one activity over another by favouring the highest possible affirmation.
- j) Comparison Judgement matrix is defined as follows:

Consider a triangular fuzzy comparison matrix expressed by

$$\mathbf{M} = [\alpha_{ij}]_{m \times n} = \begin{bmatrix} 1 & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & 1 & \cdots & \alpha_{2n} \\ \vdots & \vdots & 1 & \cdots \\ \vdots & \vdots & \ddots & 1 \\ \alpha_{m1} & \vdots & \ddots & 1 \end{bmatrix}$$
(1)

k) Normalization of the fuzzy comparison judgments to obtain fuzzy the weights

$$\mathbf{M}_{z} = \left[\alpha_{z,y}\right]_{z \times z} = \begin{bmatrix} \frac{w_{1}}{w_{1} + w_{1}} & \frac{w_{1}}{w_{1} + w_{2}} & \cdots & \frac{w_{1}}{w_{1} + w_{z}} \\ \frac{w_{2}}{w_{2} + w_{1}} & \frac{w_{2}}{w_{2} + w_{2}} & \cdots & \frac{w_{2}}{w_{2} + w_{z}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{w_{z}}{w_{z} + w_{1}} & \vdots & \cdots & \frac{w_{z}}{w_{z} + w_{z}} \end{bmatrix}$$

$$(2)$$

where

 $w_{i} = \sum_{j=1}^{n} \alpha_{ij} = \left(\sum_{j=1}^{n} \alpha_{l_{ij}}, \sum_{j=1}^{n} \alpha_{0_{ij}}, \sum_{j=1}^{n} \alpha_{r_{ij}}\right), i = 1, 2, 3 \dots n$ and α_{ij} is the fuzzy triangular numbers. This can also be expressed as $w_{1} = w_{1} \otimes \left[w_{1} \oplus w_{2}\right]^{-1}$ Next step we sum up each row of the above normalized matrix of M by interval fuzzy arithmetic operations then row sums divided to n.

5. CASE STUDY

We selected a random sample of 10 households from Shahpur Village, Nalanda District, Bihar, India from the available data by field work done by us. They are represented by household- 1, household-2 ... household -10 of are five members respectively.

Table-1. The fuzzy comparison judgments with regard to
the overall goal.

Criteria	Scale Reference Values	Fuzzy Triangular Number	Relative Importance (Linguistic Variable)		
Income	9	(9,9,9)	Extremely Important		
Food	9	(9,9,9)	Extremely Important		
Clothing	8	(7,8,9)	Very very Important		
Housing	9	(9,9,9)	Extremely Important		
Access to Health	8	(7,8,9)	Very very Important		
Education	9	(9,9,9)	Extremely Important		
Social Status	7	(6,7,8)	Very Important		
Right to Information	8	(7,8,9)	Very very Important		

Table-2	Comparison	Judgement matri	x basic need.

CRITERIA	Food	Clothing	Housing
Food	(1,1,1)	(9,9,9)	(9,9,9)
Clothing	(1/9,1/9,1/9)	(1,1,1)	(7,8,9)
Housing	(1/9,1/9,1/9)	(1/9,1/8,1/7)	(1,1,1)

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Table-3. Main certria weights.

Table-6. Sub criteri	weights:	housing.
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WI	W2	W3	RELATIVE INTENSITY IMPORTANT WEIGHTS
19	8.111	1.222	
19	9.111	1.236	
19	10.111	1.253	
W1+W2	W1+W3	AVERAGE	CENTRE WEIGHTS- FUZZY MEMBERSHIP VALUES
0.652674	0.938133	0.795403	
0.675892	0.938921	0.807406	0.807603 (FOOD SECURITY)
0.700823	0.939571	0.820197	
W2+W1	W2+W3	AVERAGE	
0.299177	0.86619	0.582684	
0.324108	0.880545	0.602327	0.601772 (CLOTHING)
0.347326	0.892173	0.61975	
W3+W1	W3+W2	AVERAGE	
0.060429	0.107827	0.084128	
0.061079	0.119455	0.090267	0.090625 (HOUSING)
0.061867	0.13381	0.097839	

Table-4. Sub criteria weights: food.

W1	W2	RELATIVE INTENSITY IMPORTANT WEIGHTS
1.111	10	
1.111	10	
1.111	10	
W1+W2	AVERAGE	CENTRE WEIGHTS- FUZZY MEMBERSHIP VALUES
0.099991	0.099991	
0.099991	0.099991	0.099991
0.099991	0.099991	
W2+W1	AVERAGE	
0.900009	0.900009	
0.900009	0.900009	0.900009
0.900009	0.900009	

W2+W1	W2+W3	AVERAGE	
0.299177	0.86619	0.582684	
0.324108	0.880545	0.602327	0.601772 (CLOTHING)
0.347326	0.892173	0.61975	

WI	W2	W3	W4	W5	RELATIVE INTENSITY IMPORTANT WEIGHTS
37	28.111	13.222	8.472	1.458	
37	28.111	15.222	9.365	1.49	
37	28.111	17.222	10.388	1.531	
W1+W2	W1+W3	W1+W4	W1+W5	AVERAGE	CENTRE WEIGHTS- FUZZY MEMBERSHIP VALUE
0.56826	0.68238	0.780788	0.960266	0.747924	
0.56826	0.708514	0.798016	0.961289	0.75902	0.75903853 (TYPES OF HOUSE)
0.56826	0.736729	0.813688	0.962089	0.770191	
W2+W1	W2+W3	W2+W4	W2+W5	AVERAGE	
0.43174	0.6201	0.730175	0.94835	0.682591	
0.43174	0.64872	0.750107	0.949664	0.695058	0.695111547 (SAFE DRINKING WATER)
0.43174	0.68011	0.768417	0.950692	0.70774	
W3+W1	W3+W2	W3+W4	W3+W5	AVERAGE	
0.263271	0.31989	0.560017	0.896224	0.509851	
0.291486	0.35128	0.619108	0.910843	0.543179	0.542161019 (LIGHT SOURCE)
0.31762	0.3799	0.670273	0.921949	0.572435	
W4+W1	W4+W2	W4+W3	W4+W5	AVERAGE	
0.186312	0.231583	0.329727	0.846946	0.398642	
0.201984	0.249893	0.380892	0.862736	0.423876	0.424470021 (SANITATION)
0.219212	0.269825	0.439983	0.87692	0.451485	
W5+W1	W5+W2	W5+W3	W5+W4	AVERAGE	
0.037911	0.049308	0.078051	0.12308	0.072088	
0.038711	0.050336	0.089157	0.137264	0.078867	0.079218884 (CONSUMERABLE DURABLES
0 030734	0.05165	0 103776	0.153054	0.087053	

Composite weights of the main criteria and sub-criteria

Table-7. Composite criteria relative weights.

ORDINARY FOOD INTAKER	0.080753032
ADEQQUATE FOOD INTAKE	0.726849968
CLOTHING	0.36212954
TYPES OF HOUSE	0.068787867
ACCESS TO SAFE DRINKING WATER	0.062994484
SANITATION	0.038467596
CONSUMERABLE DURABLES	0.007179211

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Table-8. Survey data normalized weights (All the centre values are only considered).

ACTUAL DEMONSTRATION RELATIVE WEIGHTS OF THE HOUSES (FROM SURVEY)											
H-1	H-2	H-3	H-4	H-5	H-6	H-7	H-8	H-9	H-10		
	Food Security with its sub -Criteria										
0.3483744	0.3483744	0.3483744	0.7045759	0.7045759	0.7045759	0.3627697	0.366885	0.7414519	0.3700423		
0.2851043	0.4845689	0.317919	0.4658982	0.665308	0.665308	0.665308	0.665308	0.665308	0.1199697		
				Clot	hing						
0.1840371	0.1818504	0.1825217	0.3581408	0.4069233	0.1870748	0.1854089	0.3527095	0.449946	0.2613875		
			Housing	Conditions	with its sub	-criteria					
0.1626252	0.5170087	0.1753593	0.3970072	0.7446258	0.6245483	0.7412294	0.6245483	0.6375143	0.3755336		
0.2792117	0.1437421	0.455605	0.6202113	0.6441112	0.6364555	0.6364555	0.5853793	0.5853793	0.2565491		
0.219179	0.2191789	0.6104939	0.6104939	0.6147712	0.6284954	0.6284954	0.6284954	0.6284954	0.2119018		
0.2178782	0.4617576	0.6540837	0.3260666	0.7518402	0.6389416	0.586295	0.6222532	0.6280103	0.1128736		

Composite weight of the households

Table-9. Results from fuzzy AHP triangular numbers.

	H-1	H-2	H-3	H- 4	H-5	H-6	H-7	H-8	H-9	H-10
Ordinary Food Intake	0.028	0.028	0.028	0.056	0.056	0.056	0.029	0.029	0.059	0.029
Adequate Food Intake	0.207	0.352	0.231	0.338	0.483	0.483	0.483	0.483	0.483	0.087
Clothing	0.066	0.065	0.066	0.129	0.147	0.067	0.067	0.127	0.162	0.094
Type of Housing	0.011	0.035	0.012	0.027	0.051	0.042	0.050	0.042	0.043	0.025
Access to Safe drinking water	0.017	0.009	0.028	0.039	0.040	0.040	0.040	0.036	0.036	0.016
Sanitation	0.008	0.008	0.023	0.023	0.023	0.024	0.024	0.024	0.024	0.008
Consumer Durables	0.001	0.003	0.004	0.002	0.005	0.004	0.004	0.004	0.004	0.000
Aggregated Fuzzy Weights	0.340	0.502	0.394	0.617	0.808	0.720	0.699	0.749	0.815	0.262
Ranking based on FAHP	9	7	8	6	2	4	5	3	1	10

Table-10. Result and interpretation: Poverty categories.

Very Poor	Almost Very Poor	Poor	Rather Poor	Almost Rather Poor	Non-Poor
H-9, H-5	H-8, H-6	H-7, H-4	H-2	H-3, H-1	H-10

a) Result and interpretation: Poverty categories

From the fuzzy AHP and Triangular fuzzy number analysis of poverty, it is clear that the problem of identifying the poor takes a combination of many process factors. Household-9 with weight (0.815) and household-5 with weight (0.808) are very poor, household- 8 with weight (0.749) and house-6 with weight (0.720) are almost very poor, household-7 with weight (0.699) and household-4 with weight (0.647) are poor, household-2 with weight (0.502) is rather poor, household-3 with weight (0.394), household-1 with weight (0.340) are

almost rather poor and household-10 with weight (0.262) is non poor.

6. CONCLUSIONS AND FUTURE WORK

We have used inherent fuzziness and captured the level of poverty of the ten households. Our result shows that impreciseness is accounted as measureable factor using fuzzy AHP and Triangular Numbers approach. With help of this method we can easily position one's level of poverty. With this method we can overcome the dichotomy existing in the traditional method of analysing ©2006-2015 Asian Research Publishing Network (ARPN). All rights reserved.



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poverty. Fuzzy set theory can be propagated as further scope to address the real world problem.

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