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INTUITIONISTIC PENTAGONAL FUZZY NUMBER

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ABSTRACT

In this paper we define Intuitionistic Pentagonal Fuzzy Number and include basic arithmetic operations like addition, subtraction for Intuitionistic Pentagonal Fuzzy Number. We present examples for the above defined operations between Intuitionistic Pentagonal Fuzzy Numbers and also Score and Accuracy Function of an Intuitionistic Pentagonal Fuzzy Numbers. Finally we give examples for Intuitionistic Pentagonal Fuzzy Number.

Keywords: fuzzy numbers, triangular fuzzy number, pentagonal fuzzy number, intuitionistic pentagonal fuzzy number, score and accuracy function, fuzzy arithmetic operations.

1. INTRODUCTION

L. A. Zadeh introduced fuzzy set theory in 1965. Different types of fuzzy sets [10] are defined in order to clear the vagueness of the existing problems. A fuzzy number [11, 16], is a quantity whose values are imprecise, rather than exact as in the case with single-valued function. The concept of fuzzy numbers is the generalization of the concept of real numbers. D. Dubois and H. Prade had defined fuzzy number as a fuzzy subset of the real line. So far fuzzy numbers like triangular fuzzy numbers, trapezoidal fuzzy numbers [15], Pentagonal fuzzy numbers [14], Hexagonal, Octagonal, pyramid fuzzy numbers, Diamond fuzzy number [17] and Reverse order fuzzy numbers [8] have been introduced with its membership functions.

Intuitionistic fuzzy sets were first introduced by Atanassov [1, 2] as a generalization of fuzzy sets. Intuitionistic Fuzzy Sets evolved from non-satisfication of membership function in evaluation. He used two characteristic functions expressing the degree of membership and the degree of non membership of elements in a fuzzy set. Many studied Intuitionistic fuzzy sets [3, 4] followed by Atanassov and introduced Intuitionistic fuzzy number, triangular Intuitionistic fuzzy number and trapezoidal Intuitionistic fuzzy number. Also arithmetic operations [7,9] were defined for Intuitionistic Fuzzy Numbers. It has got many applications [5,6] in information science, decision making problems, medical diagnosis, and system failure and pattern recognition.

In this paper, we introduce Intuitionistic Pentagonal Fuzzy Number (IPFN) with its membership functions. Section one is the introduction and section two presents the basic definitions of fuzzy numbers, section three presents the definition of Intuitionistic fuzzy numbers and Intuitionistic Pentagonal Fuzzy Number. Section four presents, arithmetic operations on Intuitionistic Pentagonal Fuzzy Number and section five gives Score and Accuracy Function of an Intuitionistic Pentagonal Fuzzy Number.

2. BASIC DEFINITIONS

Definition 2.1:

A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0, 1]. A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x); x \in X\}$. Here $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_{\tilde{A}}(x)$ is called the membership value of $x \in X$ in the fuzzy set $\tilde{A}$. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2.2: (Fuzzy Number)

A Fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

(i) $\mu_{\tilde{A}}(x_o)$ is piecewise continuous

(ii) There exist at least one $x_o \in R$ with $\mu_{\tilde{A}}(x_o) = 1$.

(iii) $\tilde{A}$ must be normal and convex

Definition 2.3: (Triangular Fuzzy Number)

Triangular Fuzzy Number is defined as $\tilde{A}$ = {a,b,c}, where all a, b, c are real numbers and its membership function is given by,

$$\mu_{\tilde{A}_T}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & for\ a \le x \le b \\ \frac{(c-x)}{(c-b)} & for\ b \le x \le c \\ 0 & otherwise \end{cases}$$

Definition 2.4: (Trapezoidal Fuzzy Number)

A fuzzy set $\tilde{A}$ = (a, b, c, d) is said to trapezoidal fuzzy number if its membership function is given by where a ≤ b ≤ c ≤ d

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$$\mu_{\tilde{A}_{TR}}(x)=\begin{cases}0 & for\ x<a\\ \dfrac{(x-a)}{(b-a)} & for\ a\le x\le b\\ 1 & for\ b\le x\le c\\ \dfrac{(d-x)}{(d-c)} & for\ c\le x\le d\\ 0 & for\ x>d\end{cases}$$

Definition 2.5: (Pentagonal Fuzzy Number)

A Pentagonal Fuzzy Number (PFN) of a fuzzy set $\tilde{A}$ is defined as $\tilde{A}_P$ = {a, b, c, d, e}, and its membership function is given by,

$$\mu_{\tilde{A}_P}(x)=\begin{cases}0 & for\ x<a,\\ \dfrac{(x-a)}{(b-a)} & for\ a\le x\le b\\ \dfrac{(x-b)}{(c-b)} & for\ b\le x\le c\\ 1 & x=c\\ \dfrac{(d-x)}{(d-c)} & for\ c\le x\le d\\ \dfrac{(e-x)}{(e-d)} & for\ d\le x\le e\\ 0 & for\ x>e\end{cases}$$

3. INTUITIONISTIC FUZZY NUMBER

Definition 3.1: (Intuitionistic Fuzzy Set)

Let X denote a universe of discourse, then the intuitionistic fuzzy set A in X is given by , $A_I=\{0\le\mu_A(x)+\gamma_A(x)\le 1, x\in X\}$ where $\mu_{AI}(x),\ \gamma_{AI}(x): X\to[0,1]$ are functions such that it represents the degree of membership and degree of non-membership functions of the element $x\in X$ respectively.

Definition 3.2: (Intuitionistic Fuzzy Number)

An Intuitionistic Fuzzy Set A_I is called an Intuitionistic Fuzzy Number if it satisfies the following conditions,

1. A_I is normal, i.e. there exists at least two points $x_0, x_1\in R$ such that $\mu_A(x_0)=1$ and $\gamma_A(x_1)=1$.
2. A_I is convex, i.e. its membership function is fuzzy convex and its non membership function is concave.
3. Its membership function is upper semicontinuous and its non-membership function is lower semicontinuous and the set A_I is bounded.

Definition 3.3: (Intuitionistic Triangular Fuzzy Number)

An Intuitionistic triangular Fuzzy Number of a Intuitionistic Fuzzy set is $\tilde{A}$ is defined as $\tilde{A}_{IT}=\{(a_1,b_1,c_1)(a_2,b_2,c_2)\}$, where all $(a_1,b_1,c_1)(a_2,b_2,c_2)$ are real numbers and its membership function $\mu_{\tilde{A}_{IT}}(x)$,non-membership function $\gamma_{\tilde{A}_{IT}}(x)$ is given by,

$$\mu_{\tilde{A}_{IT}}(x)=\begin{cases}\dfrac{(x-a_1)}{(b_1-a_1)} & for\ a_1\le x\le b_1\\ \dfrac{(c_1-x)}{(c_1-b_1)} & for\ b_1\le x\le c_1\\ 1 & x=b_1\\ 0 & otherwise.\end{cases}$$

$$\gamma_{\tilde{A}_{IT}}(x)=\begin{cases}\dfrac{(b_1-x)}{(b_1-a_2)} & for\ a_2\le x\le b_1\\ \dfrac{(x-b_1)}{(c_2-b_1)} & for\ b_1\le x\le c_2\\ 0 & x=b_1\\ 1 & otherwise.\end{cases}$$

Definition 3.4: (Intuitionistic Trapezoidal Fuzzy Number)

An Intuitionistic trapezoidal Fuzzy Number of a Intuitionistic Fuzzy set is $\tilde{A}$ is defined as $\tilde{A}_{ITR}=\{(a_1,b_1,c_1,d_1)(a_2,b_2,c_2,d_2)\}$, where all $(a_1,b_1,c_1)(a_2,b_2,c_2)$ are real numbers and its membership function $\mu_{\tilde{A}_{ITR}}(x)$,non-membership function $\gamma_{\tilde{A}_{ITR}}(x)$ is given by,

$$\mu_{\tilde{A}_{ITR}}(x)=\begin{cases}\dfrac{(x-a_1)}{(b_1-a_1)} & for\ a_1\le x\le b_1\\ 1 & for\ b_1\le x\le c_1\\ \dfrac{(d_1-x)}{(d_1-c_1)} & for\ c_1\le x\le d_1\\ 0 & otherwise.\end{cases}$$

$$\gamma_{\tilde{A}_{ITR}}(x)=\begin{cases}\dfrac{(b_1-x)}{(b_1-a_2)} & for\ a_2\le x\le b_1\\ 0 & for\ b_1\le x\le c_1\\ \dfrac{(x-c_1)}{(d_2-c_1)} & for\ c_1\le x\le d_2\\ 1 & otherwise.\end{cases}$$

Definition 3.5: (Intuitionistic Pentagonal Fuzzy Number)

An Intuitionistic Pentagonal Fuzzy Number of a Intuitionistic Fuzzy set is $\tilde{A}$ is defined as $\tilde{A}_{IP}=\{(a_1,b_1,c_1,d_1,e_1)(a_2,b_2,c_2,d_2,e_2)\}$, where all

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$(a_1, b_{1,} c_1, d_{1,} e_1)(a_2, b_{2,} c_2, d_{2,} e_2)$ are real numbers and its membership function $\mu_{\tilde{A}_{IP}}(x)$, non-membership function $\gamma_{\tilde{A}_{IP}}(x)$ are given by,

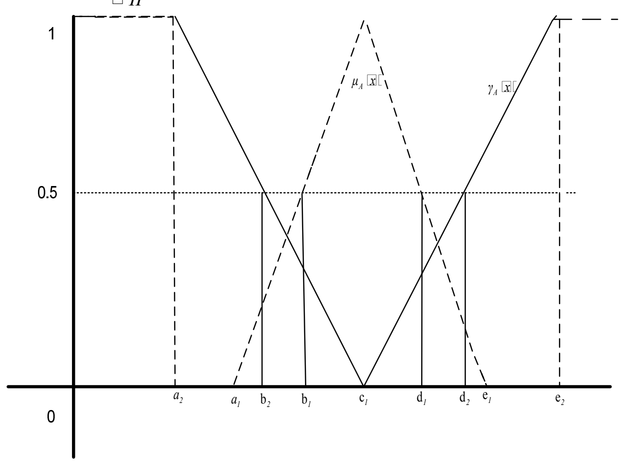

Figure-1. Graphical representation of Intuitionistic Pentagonal Fuzzy Number (IPFN).

$$\mu_{\tilde{A}_{IP}}(x) = \begin{cases} 0 & for\ x < a_1, \\ \dfrac{(x - a_1)}{(b_1 - a_1)} & for\ a_1 \le x \le b_1 \\ \dfrac{(x - b_1)}{(c_1 - b_1)} & for\ b_1 \le x \le c_1 \\ 1 & x = c_1 \\ \dfrac{(d_1 - x)}{(d_1 - c_1)} & for\ c_1 \le x \le d_1 \\ \dfrac{(e_1 - x)}{(e_1 - d_1)} & for\ d_1 \le x \le e_1 \\ 0 & for\ x > e_1 \end{cases}$$

$$\gamma_{\tilde{A}_{IP}}(x) = \begin{cases} 1 & for\ x < a_2, \\ \dfrac{(b_2 - x)}{(b_2 - a_2)} & for\ a_2 \le x \le b_2 \\ \dfrac{(c_1 - x)}{(c_1 - b_2)} & for\ b_2 \le x \le c_2 \\ 0 & x = c_1 \\ \dfrac{(x - c_1)}{(d_2 - c_1)} & for\ c_2 \le x \le d_2 \\ \dfrac{(x - d_2)}{(e_2 - d_2)} & for\ d_2 \le x \le e_2 \\ 1 & for\ x > e_2. \end{cases}$$

4. ARITHMETIC OPERATIONS ON INTUITIONISTIC PENTAGONAL FUZZY NUMBER

4.1 Addition of two intuitionistic pentagonal fuzzy numbers

If $\tilde{A}_{IP} = \{(a_1, b_{1,} c_1, d_{1,} e_1)(a_2, b_{2,} c_2, d_{2,} e_2)\}$ and $\tilde{B}_{IP} = \{(a_3, b_{3,} c_3, d_{3,} e_3)(a_4, b_{4,} c_4, d_{4,} e_4)\}$ are two Intuitionistic Pentagonal Fuzzy Numbers then,

$\tilde{A}_{IP} + \tilde{B}_{IP} = \{(a_1 + a_3, b_1 + b_3, c_1 + c_3, d_1 + d_3, e_1 + e_3)(a_2 + a_4, b_2 + b_4, c_2 + c_4, d_2 + d_4 e_2 + e_4)\}$. Example: If $\tilde{A}_{IP} = \{(1,3,4,6,7)(1,3,5,7,8)\}$ and $\tilde{B}_{IP} = \{(-1,2,3,4,7)(2,4,5,7,8)\}$ then $\tilde{A}_{IP} + \tilde{B}_{IP} = \{(0,5,7,10,14)(3,7,10,14,18)\}$.

4.2 Subtraction of two intuitionistic pentagonal fuzzy numbers

If $\tilde{A}_{IP} = \{(a_1, b_{1,} c_1, d_{1,} e_1)(a_2, b_{2,} c_2, d_{2,} e_2)\}$ and $\tilde{B}_{IP} = \{(a_3, b_{3,} c_3, d_{3,} e_3)(a_4, b_{4,} c_4, d_{4,} e_4)\}$ are two Intuitionistic Pentagonal Fuzzy Numbers then,

$$\tilde{A}_{IP} - \tilde{B}_{IP} = \{(a_1 - e_3, b_1 - d_3, c_1 - c_3, d_1 - b_3, e_1 - a_3)(a_2 - e_4, b_2$$

Example: If $\tilde{A}_{IP} = \{(0,1,5,7,8)(2,3,5,7,8)\}$ and $\tilde{B}_{IP} = \{(0,1,2,6,7)(1,3,4,5,8)\}$ then $\tilde{A}_{IP} - \tilde{B}_{IP} = \{(-7,-5,-3,5,8)(-6,-2,1,4,7)\}$.

5. SCORE FUNCTION AND ACCURACY FUNCTION

Definition 5.1: (Score Function)

Score function of an Intuitionistic Fuzzy Number is defined as $S(A_I) = \mu_{AI}(x) - \gamma_{AI}(x)$.

Definition 5.2: (Accuracy Function)

Accuracy function of an Intitiuonistic Fuzzy Number is defined as $H(A_I) = \mu_{AI}(x) + \gamma_{AI}(x)$.

Definition 5.3: (Score Function of an Intuitionistic Pentagonal fuzzy number)

Score function of an Intuitionistic Pentagonal Fuzzy Number $\tilde{A}_{IP} = \{(a_1, b_{1,} c_1, d_{1,} e_1)(a_2, b_{2,} c_2, d_{2,} e_2)\}$ is defined as

$$S(A_{IP}) = (\mathrm{a}_1 - a_2 + \mathrm{b}_1 - b_2 + \mathrm{c}_1 - c_2 + d_1 - d_2 + e_1 - e_2)/5$$

. Where $S(A_{IP}) \in [-1,1]$.

Example: If

$\tilde{A}_{IP} = \{(0.2, 0.3, 0.4, 0.6, 0.7)(0.2, 0.3, 0.4, 0.6, 0.7)\}$

$\tilde{B}_{IP} = \{(0.2, 0.3, 0.5, 0.6, 0.8)(0.2, 0.3, 0.5, 0.6, 0.8)\}$

then $S(\tilde{A}_{IP}) = S(B_{IP}) = 0$ and we cannot compare $\tilde{A}_{IP}$ and B_{IP}. In order to solve this we move to accuracy function.

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Definition 5.4: (Accuracy Function of an Intuitionistic Pentagonal fuzzy number)

Accuracy function of an Intuitionistic Pentagonal Fuzzy Number $\tilde{A}_{IP} = \{(a_1, b_1, c_1, d_1, e_1)(a_2, b_2, c_2, d_2, e_2)\}$ is defined as

$H(A_{IP}) = (a_1 + a_2 + b_1 + b_2 + c_1 + c_2 + d_1 + d_2 + e_1 + e_2)/5$.

Example: If

$\tilde{A}_{IP} = \{(0.2, 0.3, 0.4, 0.6, 0.7)(0.2, 0.3, 0.4, 0.6, 0.7)\}$

$\tilde{B}_{IP} = \{(0.2, 0.3, 0.5, 0.6, 0.8)(0.2, 0.3, 0.5, 0.6, 0.8)\}$

then $H(\tilde{A}_{IP}) = 0.88$ $H(B_{IP}) = 0.96$. now we can compare $\tilde{A}_{IP}$ and B_{IP}.

Definition 5.5: (Comparison of score and accuracy function)

1. If $S(A_{IP}) < S(B_{IP})$ then $A_{IP} \prec B_{IP}$.

Example:

If

$\tilde{A}_{IP} = \{(0.23, 0.28, 0.31, 0.39, 0.4)(0.32, 0.33, 0.42, 0.49, 0.6)\}$

$B_{IP} = \{(0.32, 0.38, 0.41, 0.45, 0.51)(0.34, 0.4, 0.45, 0.53, 0.56)\}$

$S(A_{IP}) = -0.11$, $S(B_{IP}) = -0.0084$.

$S(A_{IP}) < S(B_{IP})$ therfore $A_{IP} \prec B_{IP}$.

2. If $S(A_{IP}) > S(B_{IP})$ then $A_{IP} \succ B_{IP}$.

Example:

If

$\tilde{A}_{IP} = \{(0.48, 0.51, 0.54, 0.6, 0.65)(0.32, 0.48, 0.5, 0.55, 0.6)\}$

$B_{IP} = \{(0.38, 0.41, 0.53, 0.55, 0.6)(0.4, 0.42, 0.5, 0.51, 0.55)\}$

3. If

$S(A_{IP}) = S(B_{IP})$ and $H(\tilde{A}_{IP}) < H(B_{IP})$ *then*

$A_{IP} \prec B_{IP}$.

Example: $\tilde{A}_{IP} = \{(0.2, 0.3, 0.35, 0.4, 0.45)(0.2, 0.3, 0.35, 0.4, 0.45)\}$

$B_{IP} = \{(0.25, 0.35, 0.45, 0.48, 0.5)(0.25, 0.35, 0.45, 0.48, 0.5)\}$

4. If

$S(A_{IP}) = S(B_{IP})$ and $H(\tilde{A}_{IP}) > H(B_{IP})$ *then*

$A_{IP} \succ B_{IP}$.

Example: $\tilde{A}_{IP} = \{(0.25, 0.35, 0.45, 0.48, 0.5)(0.25, 0.35, 0.45, 0.48, 0.5)\}$

$\tilde{B}_{IP} = \{(0.2, 0.25, 0.3, 0.32, 0.35)(0.2, 0.25, 0.3, 0.32, 0.35)\}$

5. If

$S(A_{IP}) = S(B_{IP})$ and $H(\tilde{A}_{IP}) = H(B_{IP})$ *then*

$A_{IP} \approx B_{IP}$.

Example: $\tilde{A}_{IP} = \{(0.25, 0.35, 0.45, 0.48, 0.5)(0.25, 0.35, 0.45, 0.48, 0.5)\}$

$\tilde{B}_{IP} = \{(0.25, 0.35, 0.45, 0.48, 0.5)(0.25, 0.35, 0.45, 0.48, 0.5)\}$

6. CONCLUSIONS

The concept of fuzzy numbers and their utility aspects have been studied by researchers in recent times. Categorization of fuzzy numbers and their related properties have initiated new notions and approaches. In this paper, the Intuitionistic Pentagonal Fuzzy Number have been introduced with arithmetic operations. Using a few examples, we have explained the relevant arithmetic operations, Score and Accuracy of an Intuitionistic Pentagonal Fuzzy Number. We observe that fuzzy number concepts could be applied to many real life problems.

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