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A HOST-MORTAL COMMENSAL ECO-SYSTEM WITH HARVESTED HOST- PHASE PLANE ANALYSIS

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ABSTRACT

In this paper we discussed about the stability nature of the biological system containing two species commensal and the host where as the host species is being harvested at a constant rate. This model is developed by a couple of first order non-linear differential equations. The behavior of this biological system is observed based on the equilibrium points, threshold regions and threshold diagrams by taking existed values of the equilibrium points to the parameters in the dynamic model equations.

Keywords: commensalism interaction, mortal commensal, host species, threshold regions, threshold diagrams.

INTRODUCTION

Mathematical modeling of ecosystems was initiated by Lotka [10] and Volterra [20] and the general concepts of modeling have been presented in the treatises of Meyer [11], Cushing [5], Gause [7], Paul Colinvaux [12], Haberman [8], Pielou [13], Thompson [19], Freedman [6], Kapur [9] etc. Later Pattabhi Ramacharyulu, Acharyulu [1-4], Seshagiri Rao and kalyani [14-18] studied the behavior of different types of biological models like Ammensalism, Competition, Prey-Predator, Mutualisim, Commensalism interactions between two and three species with limited and unlimited recourses. The qualitative nature of the flourishing and mortal species with harvesting and replenishment to the species are also investigated.

The growth rate equations of this ecological commensalism interaction between two species where host is harvested at constant rate are couple of first order non linear ordinary differential equations. Further both species are considered in the natural limited resources. All the possible six equilibrium points for the model are identified and threshold results are stated followed by the identification of threshold regions via illustrations using known software. For the construction of the basic model equations we use the following notations

$N_1(t)$, $N_2(t)$: The populations of the commensal and host at time t.

d_1 : The mortal rate of the commensal species.

a_2 : The rate of natural growth of the host species.

$a_{ii}(i=1,2)$: The rate of decrease of the commensal and host due to the limitations of its natural resources.

a_{12}: The rate of increase of the commensal due to the support given by the host.

$e_1(=d_1/a_{11})$: The mortality coefficient of commensal species.

$c(=a_{12}/a_{11})$: The coefficient of the commensal.

$k_2(=a_2/a_{22})$: The carrying capacity of the host species.

$h_2(=a_{22}H_2)$: The coefficient of harvesting /migration of the host species.

H_2: The harvesting/migration of the host per unit time.

The defined above variables $N_1(t)$ and $N_2(t)$ as well as all the model parameters d_1, a_2, a_{11}, a_{12}, a_{22}, k_2, e_1, c, h_2, H_2 are assumed to be non-negative constants. Employing the terminology given above the equations for this model are given by the following system of non-linear coupled ordinary differential equations.

(i). Growth rate equation for the Mortal commensal species (S_1)

$$\frac{dN_1}{dt}=a_{11}N_1\left[-e_1-N_1+cN_2\right] \quad (1)$$

(ii). Growth rate equation for the Host species (S_2)

$$\frac{dN_2}{dt}=a_{22}\left[k_2N_2-N_2{}^2-H_2\right] \quad (2)$$

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The system under investigation has the following six equilibrium states given by $\frac{dN_1}{dt} = 0$; $\frac{dN_2}{dt} = 0$. These states are classified into two categories A and B.

(A) The States in which only the Host survives

(A.1) When $k_2^{\ 2} = 4H_2$ (A.1)

E_1: $\overline{N_1} = 0$, $\overline{N_2} = \frac{k_2}{2}$, This would exists only when $k_2^{\ 2} = 4H_2$

(A.2) When $k_2^{\ 2} > 4H_2$ (A.2)

E_2: $\overline{N_1} = 0$, $\overline{N_2} = \frac{k_2 + \sqrt{k_2^{\ 2} - 4H_2}}{2}$;

E_3: $\overline{N_1} = 0$, $\overline{N_2} = \frac{k_2 - \sqrt{k_2^{\ 2} - 4H_2}}{2}$

Both would exists when $k_2^{\ 2} > 4H_2$

<u>Note</u>: As H_2 increases and ultimately approaches to $\frac{k_2^{\ 2}}{4}$, the two equilibrium points E_2 and E_3 coincide with E_1.

(B) The co-existent states

(B.1) When $k_2^{\ 2} = 4H_2$ and $ck_2 > 2e_1$ (B.1)

E_4: $\overline{N_1} = \frac{ck_2 - 2e_1}{2}$, $\overline{N_2} = \frac{k_2}{2}$, This would exists when $k_2^{\ 2} = 4H_2$ and $ck_2 > 2e_1$

(B.2) When $k_2^{\ 2} > 4H_2$ (B.2)

E_5: $\overline{N_1} = \frac{c\left[k_2 + \sqrt{k_2^{\ 2} - 4H_2}\right]}{2} - e_1$,

$\overline{N_2} = \frac{k_2 + \sqrt{k_2^{\ 2} - 4H_2}}{2}$;

E_6: $\overline{N_1} = \frac{c\left[k_2 - \sqrt{k_2^{\ 2} - 4H_2}\right]}{2} - e_1$,

$\overline{N_2} = \frac{k_2 - \sqrt{k_2^{\ 2} - 4H_2}}{2}$

Both these exists only when $\frac{c\left[k_2 + \sqrt{k_2^{\ 2} - 4H_2}\right]}{2} > e_1$.

<u>Note</u> : As H_2 increases and ultimately approaches to $\frac{k_2^{\ 2}}{4}$, the two equilibrium points E_5 and E_6 coincide with E_4.

Based on the signs of derivatives of the basic model equations around the equilibrium points in the first quadrant, we have the following different regions shown in Table-1.

Table-1.

Signs of derivatives	**Threshold regions around the equilibrium points**			
	E_1	E_2 **and** E_3	E_4	E_5 **and** E_6
$\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} > 0$	-	I	I	I
$\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} < 0$	III(=II)	IV	IV	IV
$\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} < 0$	II(=III)	III	III	III
$\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} > 0$	I(IV)	II	II	II

where

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Region-I: In this region $\frac{dN_1}{dt} > 0$ an $\frac{dN_2}{dt} > 0$ $\Rightarrow \frac{dN_1}{dN_2} > 0$ then $N_1(t)$ is an increasing function of $N_2(t)$ and the trajectories move up and right.

Region-II: Here $\frac{dN_1}{dt} > 0$ and $\frac{dN_2}{dt} < 0 \Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$ and the trajectories move down and right.

Region-III: Here $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} < 0 \Rightarrow \frac{dN_1}{dN_2} > 0$ then $N_1(t)$ is an increasing function of $N_2(t)$ and the trajectories move down and left.

Region-IV: Here $\frac{dN_1}{dt} < 0$ and $\frac{dN_2}{dt} > 0 \Rightarrow \frac{dN_1}{dN_2} < 0$ then $N_1(t)$ is a decreasing function of $N_2(t)$ and the trajectories move up and left.

The following Table-2 shows the existing of the equilibrium points for the selected values of the model parameters a_{11}, e_1, c, a_{22}, k_2, H_2 in the basic equations.

Table-2.

Parameters	**Existing of the equilibrium points at the parameter values**			
	E_1	E_2 **and** E_3	E_4	E_5 **and** E_6
a_{11}	1.5	0.1	1.5	0.1
e_1	1.875	0.3	0.8	0.3
c	0.75	0.6	0.75	0.6
a_{22}	0.5	0.1	0.5	0.1
k_2	5	3	5	3
H_2	6.25	2	6.25	2

Category: A (When the harvesting rates are interdependent)

A.1 The threshold diagram for equilibrium point E_1

The threshold lines/ null clines given by $\frac{dN_1}{dt} = 0$, $\frac{dN_2}{dt} = 0$ divide the phase plane into three regions I , II and III in the first quadrant are shown in Figure-1 and the filed lines around E_1 in the threshold regions are shown in Figure-2.

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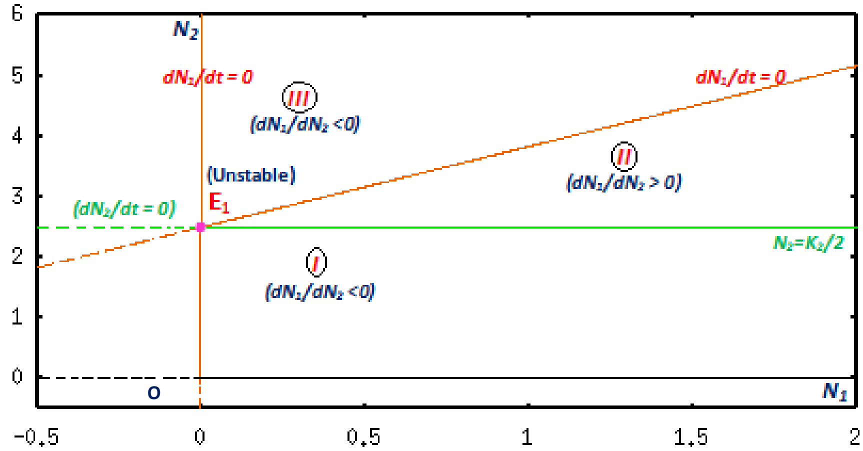

Figure-1. Threshold regions.

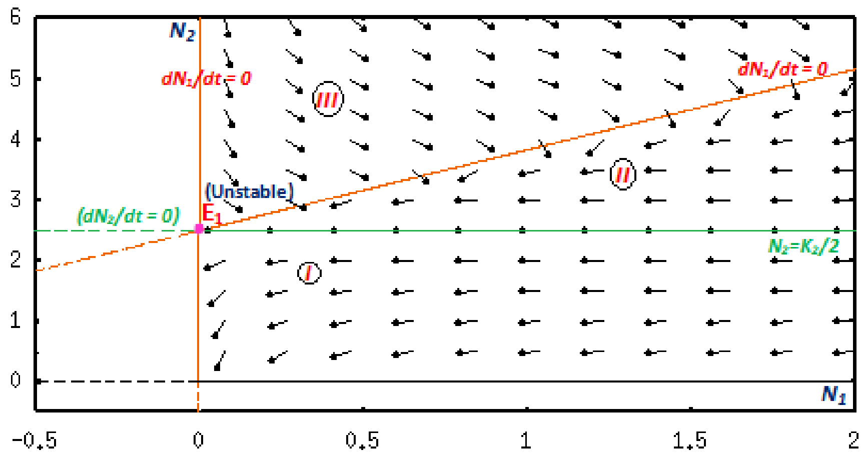

Figure-2. Threshold diagram for E_1.

Note: The defined regions above III (=II), II (=III) and I (=IV) are same for the equilibrium point E_1.

A.2 The threshold diagram for equilibrium points E_2 and E_3

In this case the null clines divide the phase plane into four regions I, II, III and IV in the first quadrant are shown in Figure-3.

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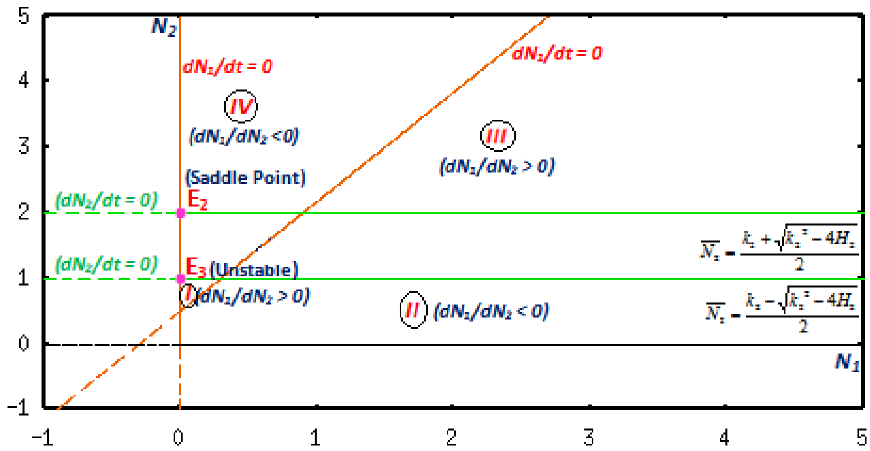

Figure-3. Threshold regions.

Figure-4 shows the direction of the field lines and the trajectories in the threshold regions for $a_{11} = 0.1$, $e_1 = 0.3$, $c = 0.6$, $a_{22} = 0.1$, $k_2 = 3$, $H_2 = 2$.

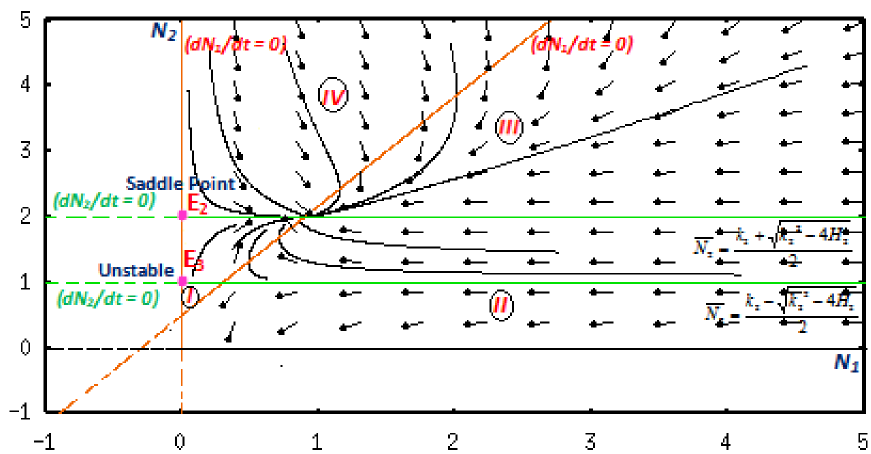

Figure-4. Threshold diagram for E_2 and E_3.

Category B (The co-existent States)

(B.1) The threshold diagram for equilibrium point E_4

In this case the threshold lines divide the phase plane into four regions I, II, III and IV in the first quadrant (i.e., $N_1 \geq 0$, $N_2 \geq 0$) are shown in Figure-5.

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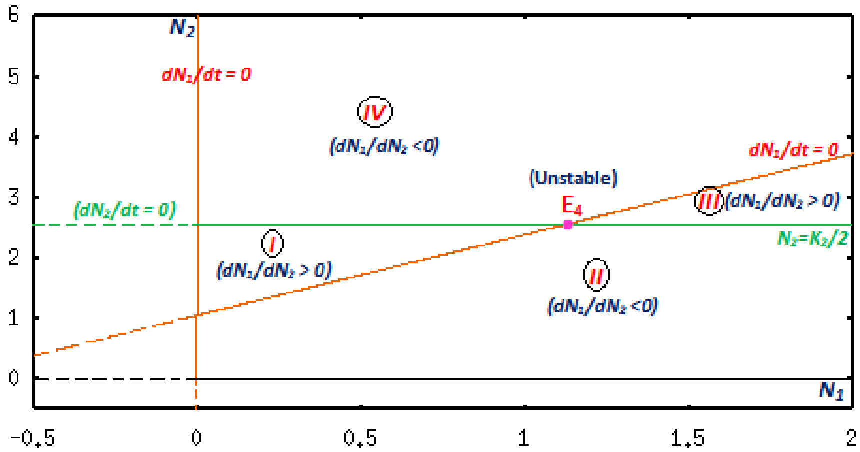

Figure-5. Threshold regions.

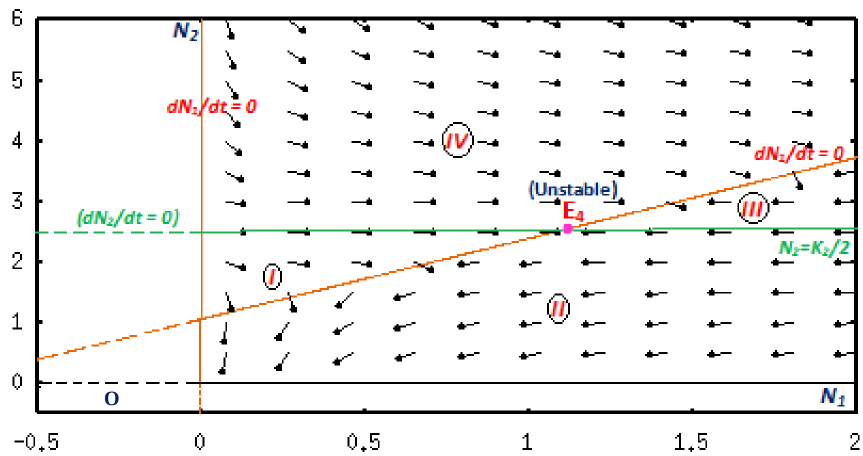

Figure-6. Threshold diagram for E_4.

(B.2) The threshold diagram for equilibrium points E_5 and E_6

The threshold lines in this case divide the phase plane into four regions I , II , III and IV in the first quadrant are shown in Figure-7.

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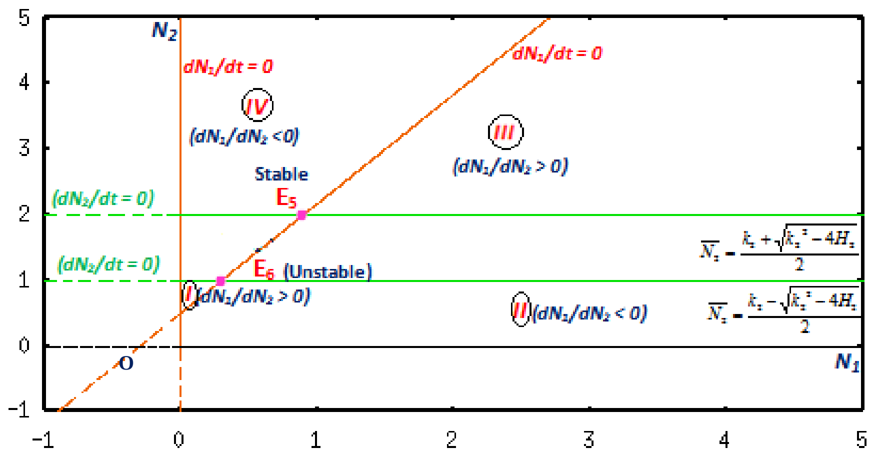

Figure-7. Threshold regions.

The direction of the field lines in the threshold regions and trajectories around the equilibrium points E_5 and E_6 for $a_{11} = 0.1$, $e_1 = 0.3$, $c = 0.6$, $a_{22} = 0.1$, $k_2 = 3$, $H_2 = 2$ are shown in Figure-8.

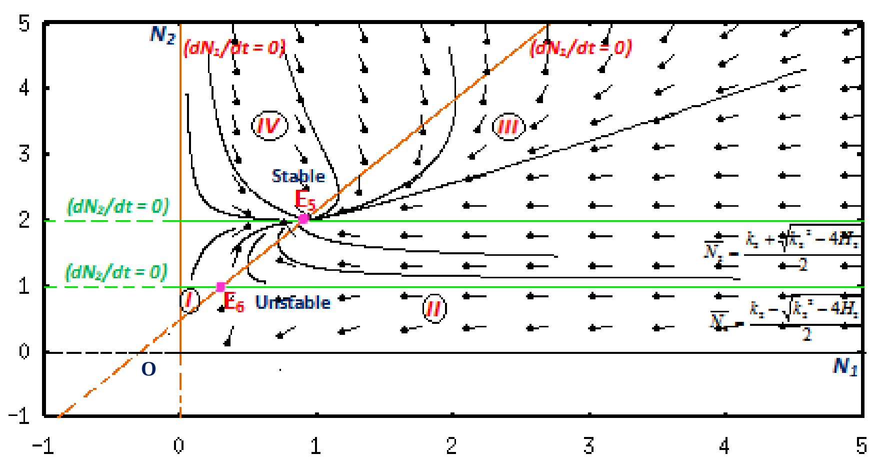

Figure-8. Threshold diagram for E_5 and E_6 .

CONCLUSIONS

i) The situation for the higher harvesting rates H_2 of the host species there is no interaction between the species because the host species extinct first and then automatically the dependent commensal species also extinct.

ii) The case when the higher mortal rates for the commensal species the biological system will not exists longer because the commensal is going to extinct first.

Open problems

The following problems are of interest to investigate in these basic model equations

a) Situations involving flourishing and delayed commensalism as they occur are of interest in nature at times.

b) Constant migration/ Immigration of the commensal and immigration/ migration of the host species in the model equations.

c) Immigration of the both commensal and the host at (a). Constant (b). Variable rates.

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