



SINGLE SPEED GEAR BOX OPTIMIZATION USING GENETIC ALGORITHM

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ABSTRACT

Gears are used in almost all mechanical devices and they do several important jobs, but most important, they provide a gear reduction. This is vital to ensure that even though there is enough power there is also enough torque. Gear box has to produce maximum power with minimum weight. In many real-life problems, objectives under consideration conflict with each other, and optimizing a particular solution with respect to a single objective can result in unacceptable results with respect to the other objectives. Multi-objective formulations are realistic models for many complex engineering optimization problems. A reasonable solution to a multi-objective problem is to investigate a set of solutions, each of which satisfies the objectives at an acceptable level without being dominated by any other solution. In this paper, genetic algorithms developed specifically for a single speed gear box problem with multiple objectives.

Keywords: single speed gear box, design optimization, genetic algorithm, spur gear.

1. INTRODUCTION

Gears are machine elements used to transmit rotary motion between two shafts, normally with a constant ratio. Spur gears are the most common type of gear they have straight teeth and are mounted on parallel shafts. The main reason for the popularity of spur gears is their simplicity in design, easy manufacturer and maintenance. However due to their design spur gears create large stress on the gear teeth.

The development and use of optimization models is well established. However, the use of many models has been restricted in some fields of economic analysis where the problem is large in size and there are a large number of non-linear interactions. In most cases, the use of linear approximations or simplification of the model has been necessary in order to derive a solution.

Chaoli Sun *et al.* [1] have developed a modified particle swarm optimization with feasibility-based rules for mixed-variable optimization problems. Mehmet Bozca and Peter Fietkau [2] have proposed an empirical model based optimization of gearbox geometric design parameters to reduce rattle noise in an automotive transmission unit. Yong sheng Lian *et al.* [3] reviewed the recent progress in design optimization using evolutionary algorithms to solve real-world aerodynamic problems, such as design of turbo pump, compressor, and micro-air vehicles etc. Davoud Sedighzadeh and Ellips Masehian [4] have surveyed PSO methods and its application. Ming Chang [5] have developed a new proposal of using particle swarm optimization algorithms to solve multi-objective optimization problems was presented.

S Padmanabhan *et al.* [6] have evaluated worm and worm wheel gear pair with multi objectives such as maximizing power, efficiency and minimizing weight, center distance using Metaheuristic algorithms. Ruifeng Bo *et al.* [6] have analyzed, concept optimization problem based on an Ant Colony System (ACS) and by analyzing

the similarity between concept solving and Traveling Salesman Problem.

Zhou *et al.* [7] proposed an ant colony algorithm to solve the prematurity and unsteadiness problem in GA for job shop scheduling with the objective of minimization make span. S. M. Kannan *et al.* [8] have proposed a PSO for minimizing assembly variation in selective assembly problem. S Padmanabhan *et al.* [9] have evaluated bevel gear pair with multi objectives such as maximizing power, efficiency and minimizing weight, center distance using Metaheuristic algorithms. Majid Jaberipour *et al.* [10] have described about two new harmony search metaheuristic algorithms for engineering optimization problems with continuous design variables. Lin C *et al.* [11] proposed a new immune algorithm makes use of the PSO advantages to improve the mutation mechanism in the immune algorithm. Wang *et al.* [12] proposed an improved algorithm named bi-directional convergence ant colony optimization (ACO) algorithm based on the graphic definition of the job-shop problem. Davoud Sedighzadeh and Ellips Masehian [13] have surveyed PSO methods and its application. V. Savsani *et al.* [14] have evaluated minimum weight of a spur gear train using particle swarm optimization (PSO) and simulated annealing (SA).

In this paper, Single speed gear box optimization have concentrated on investigating the combined objective function, which minimizes the volume and centre distance and maximizes power and efficiency with the constrained bending and crushing stresses, displacement acting on the gear tooth with regard to space requirements.

2. GEAR BOX DESIGN

Gears have been around for hundreds of years and are as old as almost any machinery ever invented by mankind. Gears were first used in various construction jobs, water raising devices and for weapons like catapults. Nowadays gears are used on a daily basis and can be



found in most people's everyday life from clocks to cars rolling mills to marine engines. Gears are the most common means of transmitting power in mechanical engineering.

Spur gears or straight-cut gears are the simplest type of gear. They consist of a cylinder or disk with the teeth projecting radially, and although they are not straight-sided in form, the edge of each tooth is straight and aligned parallel to the axis of rotation. These gears can be meshed together correctly only if they are fitted to parallel shafts.

a) Design of single speed gear box

This section describes about the design objectives, constraints, considered in this work. This work uses a combined objective function, which minimizes the volume and centre distance and maximizes power and efficiency.

Table-1. Specification of gear box.

Gear box	Single speed
Gear Ratio	3.5
Power	18 kW
Speed on Pinion	1200 rpm
Gear material	C45
Bending stress	140 N/mm ²
Crushing stress	500 N/mm ²
Young's modulus	2.15 N/mm ²
Density	7.85 x 10 ⁻⁶ kg/mm ³
Factor of safety	2

The objective functions considered in this work are given below and represents in equations (1), (2), (3) and (7).

- Maximization of power delivered by the gear box (f_1)
- Minimization of the over all weight – which is indirectly related to the volume of the gears (f_2)
- Maximization of the efficiency of the gear box (f_3)
- Minimization of the center distance between the input and output shafts (f_4).

Design constraints are to be considered for the design of above gear box is, crushing stress (8), bending stress (9), center distance (10), module (11) and number teeth (12) represented in eqns.

The objective functions are reduced in terms of design variables Power (P), module (m), number teeth on pinion (Z_1) and thickness of the gear (b). The following objective functions and design constraints are adopted from [15] and [16].

The complete problem is,

$$\text{Maximize } f_1 = P \quad (1)$$

$$\text{Minimize } f_2 = 8.172 \times 10^{-5} \times b \times (mZ_1)^2 \quad (2)$$

$$\text{Maximize } f_3 = 100 - P_L \quad (3)$$

$$P_L = \frac{50f}{\cos\Phi} \times \frac{(H_s^2 + H_t^2)}{(H_s + H_t)} \quad (4)$$

$$P_L = 4.256 \times \frac{(H_s^2 + H_t^2)}{(H_s + H_t)}$$

$$H_t = 1.28 \times \left\{ \left[\left(1 + \frac{2}{Z_1} \right)^2 - 0.883 \right]^{0.5} - 0.342 \right\} \quad (5)$$

$$H_s = 4.5 \times \left\{ \left[\left(1 + \frac{0.571}{Z_1} \right)^2 - 0.883 \right]^{0.5} - 0.342 \right\} \quad (6)$$

$$\text{Minimize } f_4 = 2.25 mZ_1 \quad (7)$$

Subject to,

$$mZ_1 \times (b)^{0.5} \times (P)^{-0.5} \geq 317.16 \quad (8)$$

$$(Z_1 + 8) \times m^2 \times b \times P^{-1} \geq 607.92 \quad (9)$$

$$m Z_1 \times P^{-1/3} \geq 53.648 \quad (10)$$

$$m^3 \times P^{-1} (Z_1 + 8) \geq 31.4523 \quad (11)$$

$$Z_i \in I, \text{ for } i = 14, 16, 18, 20, 22, 24, 26, 28 \quad (12)$$

and

Combined Objective Function =

$$\left[\left(\frac{\text{power}}{\max \text{ .power}} xNW_1 \right) + \left(\frac{\min \text{ .weight}}{\text{weight}} xNW_2 \right) + \left(\frac{\text{efficiency}}{\max \text{ .efficiency}} xNW_3 \right) + \left(\frac{\min \text{ .cent .dist}}{\text{cent .dist}} xNW_4 \right) \right] \quad (13)$$

Where NW_1, NW_2, NW_3 and $NW_4 = 0.25$.

3. GENETIC ALGORITHM

Genetic algorithms (GA) are an evolutionary optimisation approach which is an alternative to traditional optimization methods. GA is most appropriate for complex non-linear models where location of the global optimum is a difficult task. It may be possible to use GA techniques to consider problems which may not be modeled as accurately using other approaches. Therefore, GA appears to be a potentially useful approach.

In a genetic algorithm, a population of strings (called chromosomes or the genotype of the genome),



which encode candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem, evolves toward better solutions. Traditionally, solutions are represented in binary as strings of 0s and 1s, but other encodings are also possible. The evolution usually starts from a population of randomly generated individuals and happens in generations. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), and modified (recombined and possibly randomly mutated) to form a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. If the algorithm has terminated due to a maximum number of generations, a satisfactory solution may or may not have been reached.

The GA consists of four main stages: evaluation, selection, crossover and mutation. The *evaluation* procedure measures the fitness of each individual solution in the population and assigns it a relative value based on the defining optimization criteria. Typically in a non-linear programming scenario, this measure will reflect the objective value of the given model. The *selection* procedure randomly selects individuals of the current population for development of the next generation. Various alternative methods have been proposed but all follow the idea that the fittest have a greater chance of survival. The *crossover* procedure takes two selected individuals and combines them about a crossover point thereby creating two new individuals. Simple reproduction can also occur which replicates a single individual into the new population. The *mutation* procedure randomly modifies the genes of an individual subject to a small mutation factor, introducing further randomness into the population. The algorithm as follows,

1. Generate random population of n chromosomes (Generation).
2. Evaluate the fitness $f(x)$ of each chromosome x in the population (Evaluation).
3. Create a new population by repeating following steps until the new population is complete (New population)
 - i. Select two parent chromosomes from a population according to their fitness (Selection)
 - ii. With a crossover probability cross over the parents to form a new offspring (children). (Crossover)
 - iii. With a mutation probability mutate new offspring at each locus (position in chromosome). (Mutation)
Place new offspring in a new population.
4. Use new generated population for a further run of algorithm.
5. If the end condition is satisfied, stop, and return the best solution in current population.
Or Go to Evaluation step.

This iterative process continues until one of the possible termination criteria is met: if a known optimal or acceptable solution level is attained; or if a maximum number of generations have been performed. Generally, the last of these criteria applies as convergence slows to the optimal solution.

b) GA for gear box design optimization

The size of population adopted in this work is 20. For an illustration of the implementation of the population size of '5 strings' is taken and it is given below. The range of module is taken as 4 to 8 mm.

i. Generation

Mixed integer generation is used for the control variables. The module, thickness, number of teeth in pinion, the power, maximum power, minimum weight, maximum efficiency, minimum cone distance and COF are represented in a control string. Module, thickness of gear pair & power are represented as continuous variables within limits. The number of teeth is represented as discrete variable. The control string will be as per the eqn (14).

$$X = [P, b, Z_1, m, f_1, f_2, f_3, f_4, COF] \quad (14)$$

The strings are having the variables, which satisfy all the constraints the corresponding objective functions and COF.

ii. Tournament selection

The tournament selection provides a selective pressure by holding a tournament competition among individuals. The best individual (the winner) from this group is selected as parent. That is, any two strings are randomly selected from this population and the COF value is compared. The string having the lowest COF will be stored in the new mating pool.

For example if the string 1 and 3 are selected randomly, string 3 is stored in the new mating pool. If any string having the lowest COF is selected more than one time, it will be stored that much time in the pool. This process is repeated until the mating pool for generating new offspring is filled. Tournament selection is used as selection mechanism in order to avoid premature convergence.

iii. Crossover

The crossover performs the crossover variable-wise using crossover operator. It creates children solutions in proportion to the difference in parent solutions. The two properties which give its search power are,

- The extend of children solution is in proportion to the parent solutions.
- Near parent solutions are more likely to be chosen as children solutions than solutions distant from parents.

The new child strings obtained after the crossover. The new values of the child strings have to be validated along with the existing variables for constraint



satisfaction and COF has to be updated. If there is any constraint violation, the process has to be repeated. The parents and their variables are randomly selected and the

process is repeated until 80% of the pool got crossover to generate new offspring

Table-2. Numerical generation of the control strings for GA.

Generation	P	b	Z ₁	m	F1	F2	F3	F4	COF
1	19.583	26.58	16	4.244	19.583	10.00693	97.88525	152.784	1
2	19.583	26.58	16	4.244	19.583	10.00693	97.88525	152.784	0.966676
3	18.599	20	20	4.194	18.599	11.48952	98.30621	188.73	0.987705
4	18.599	20	20	4.194	18.599	11.48952	98.30621	188.73	0.958951
5	18.599	20	20	4.194	18.599	11.48952	98.30621	188.73	0.958951

iv. Mutation

Newly generated offspring undergo mutation operation. Like in the crossover operator, the probability distribution can also be a polynomial function, instead of a normal distribution. The mutation probability is taken as 0.2. That is 20 % of the offspring, which is having higher value of COF will undergo this operation, to improve their fitness. In this operator the shape of the probability distribution is directly controlled by the external parameter η_m and distribution is not dynamically changed with generations. Newly generated individuals replace their parents and forms the parents for the next generation.

Stopping criteria: This COF followed in this work, is to stop the computation after reaching the required number of iterations. The maximum number of iterations adopted here is 10. For each iteration, the population is generated continuously by 100 times.

4. RESULTS AND DISCUSSION

Initially, the input values are generated randomly with their variable limits. If the generated values satisfy the design constraints, then the values of objective functions f1, f2, f3 and f4 are computed along with COF. The optimum values of objective function and design variables corresponding to the minimum COF value obtained by the GA.

Table 3. Results of gearbox using GA.

Parameters	4 to 5 mm	5 to 6 mm	6 to 7 mm	7 to 8 mm
Module (mm)	4.065	5.354	6.162	7.055
Thickness(mm)	26.87	20.99	16.01	14.35
No. of teeth in pinion	18	16	16	16
Power (kW)	19.351	19.756	19.966	19.649
Weight (kg)	13.22	14.16	12.51	14.35
Efficiency (%)	97.86	97.88	97.88	97.88
Center distance (mm)	164.63	192.74	194.10	253.98

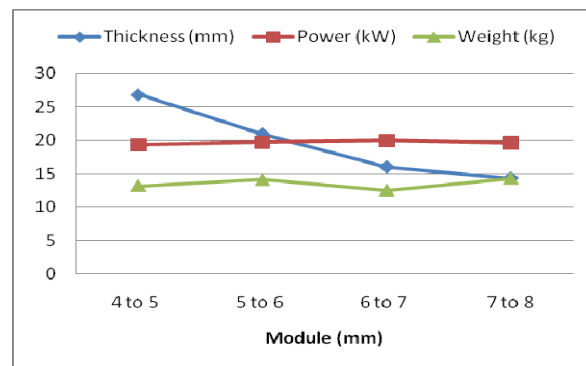


Figure-1. Comparison of GA results.

5. CONCLUSIONS

The above results indicate that the GA is highly capable of minimizing the weight of gear box without deviating all the framed constraints. Weight reduction reduces the amount of material consumed while manufacturing the gears. The material consumption is directly proportional to the cost. If the designer of automobile and machine tool gearboxes takes the benefit of optimization of GA in to account while designing, the manufacturers can confidently face the aggressive environment of producing the gearboxes at reasonable cost. As a future work, the GA can be evaluated by optimizing various gear design like helical, worm and bevel, epicyclic gear train and gear box etc.

Notations

- GA = Genetic Algorithm
- Hs = Specific sliding velocity at start of approach action
- Ht = Specific sliding velocity at end of recess action.
- P = Power transmitted in kW
- i = Gear (or) transmission ratio
- z₁, z₂ = Number of teeth in pinion, gear mm
- ρ = Density of the material in kg/mm³
- E = Young's modulus in N/mm²
- σ_c = Induced crushing stress in N/mm²
- [σ_c] = Allowable crushing stress in N/mm²



σ_b = Induced bending stress in N/mm²
 $[\sigma_b]$ = Allowable bending stress in N/mm²
 a = Centre distance between shafts in mm
 b = Thickness of gear and pinion in mm
 PL = Percentage of power loss
 $[M_t]$ = Design twisting moment in N-mm
 η = Percentage efficiency
 y = Form factor
 f = Average coefficient of friction
 m = Normal Module in mm
 Φ = Pressure angle in

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