



## SWARM INTELLIGENCE OPTIMIZATION OF WORM AND WORM WHEEL DESIGN

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### ABSTRACT

The problem of optimization of gears is difficult to solve. Indeed, the objectives are many and they require a multicriteria optimization. The compromise between the various objectives is not easy to find because the optimization criteria are often contradictory. In addition, there are a large number of design variables, and the majority of these variables takes discrete values. Worm and worm wheel drives are normally used for non-parallel, non-intersecting, right angle gear drive system where high reduction ratios are involved, though they are also employed with low to medium speed reducers in many applications. In this paper an attempt has been made to optimize worm and worm wheel design using conventional tool LINGO and heuristics as Particle Swarm Optimization (PSO) algorithms. A combined objective function which maximizes the Power, Efficiency and minimizes the overall Weight, Centre distance has been considered in this model.

**Keywords:** worm and worm wheel, gear pair design, LINGO, particle swarm optimization.

### 1. INTRODUCTION

Most real world optimization problems involve the optimization task of more than single objective function, which, in turn, requires a significant computational effort. In this context, the use of evolutionary algorithms for multi-objective optimization has significantly grown for a decade, giving rise to a wide variety of meta-heuristic algorithms. The main research direction is to improve both the computational efficiency of the algorithm and distribution of the obtained non-dominated optimal solutions.

Carroll and Johnson [1] have presented an optimal design technique to obtain compact and standard spur gear meshes with an objective of minimizing centre distance. Savage *et al* [2] have described the optimal design of an enclosed parallel shaft spur gear reduction. The object of the design was to determine a small, light weight transmission with a long service life. Deb [3] has proposed a Non-Sorted Genetic Algorithm II for optimizing multi speed gear box which consider multi objectives such as maximizing the power and minimizing the total volume of the gear. Tsay and Tseng [4] have applied a multiple optimization method to reduce the level of kinematical errors of helical gear train and investigated an optimal gear tooth modifications. Prayoonrat *et al* [5] have presented an algorithm to design and optimize multi spindle gear trains for the non speed-change type in which the designer may choose minimum overall centre distance, minimum overall size, minimum gear volume, or other desirable criteria, such as maximum gear contact or overlap ratios to optimize gear trains. Yoon and Rao [6] have presented a novel method to minimize the static transmission error using cubic splines for gear tooth profiles. Innocenti [7] has proposed a new approach to the efficiency evaluation of single or multidegree freedom of gear trains. Rosic [8] has

proposed an analytical and computer aided procedure for the multi criteria design of gear train transmission system.

The problem of worm and worm wheel design optimization is difficult to solve because it involves multiple objectives and large number of variables. Therefore a reliable and robust optimization technique will be helpful in obtaining optimal solution for the problems. This paper has made an attempt to use the potential of LINGO and PSO to solve the worm and worm wheel design problem.

### 2. PROBLEM DESCRIPTION

This section describes about the design objectives, constraints, considered in this work. A test problem has been considered in this work. This problem deals with the optimal design of worm and worm wheel. This work uses a combined objective function, which minimizes the volume and centre distance and maximizes power and efficiency.

#### a) Design of worm and worm wheel

A worm and worm wheel design has to be optimized considering the following data,

Power	=	22.5kW
Speed of the pinion	=	1440 rpm
Speed ratio (or) Gear ratio	=	24

#### i) Objective functions for worm and worm wheel

The following Objective functions have been considered in this model. The equation 1, 2, 3 and 4 represent the maximization of Power, minimization of Weight, maximization of Efficiency and minimization of Centre distance. The efficiency equation (3) has been adopted from Dudley.

$$\text{Maximize } f_i = P \text{ where } , P^{(L)} \leq P \leq P^{(U)} \quad (1)$$



Minimize,  $f_2 = \text{Weight} =$

$$\frac{\pi (qm_x)^2}{4} \times (12.5 + 0.09 Z_2) \times m_x \times \rho_1 + \frac{\pi (Z_2 m_x)^2}{4} \times b \times \rho_2 \quad (2)$$

$$\text{Maximize, } f_3 = 100 \times \frac{\cos \phi_n - f \tan \lambda}{\cos \phi_n + f \cot \lambda} \quad (3)$$

$f =$  coefficient of friction = 0.08

$$\text{Minimize, } f_4 = \frac{d_1 + d_2}{2} = \frac{qm_x + Z_2 m_x}{2} \quad (4)$$

## ii) Design constraints for worm and worm wheel

**Bending stress:** The tooth breakage is caused by fatigue due to repeated bending stresses. To safeguard the tooth against the breakage, the gear should have adequate bending strength. i.e., the induced bending stress when transmitting a torque should be lesser than the allowable bending stress.

**Crushing stress:** To avoid the surface failures of the tooth profile like Pitting, Surface Abrasion, Seizure etc., and to have the satisfactory life, gear should have the wear resistance. i.e., the induced crushing stress should be lesser than the allowable crushing stress.

**Gear ratio:** The gear ratio should be a constant and it should be equal to the ratio between the number of teeth in gear and the number of teeth in pinion. The eqn. (5) represents this constraint [10]:

$$i = 24 = Z_2/Z_1 \text{ (or) } d_2/d_1 \quad (5)$$

**Centre distance between the shafts:** Centre distance between the shafts 'a' should be greater than the minimum centre distance 'a<sub>min</sub>' to assure the required clearance between the tip of the pinion tooth and the root of the gear tooth and vice versa. This constraint is represented by the eqn. (6)

$$a \geq a_{\min} \quad (6)$$

**Module:** The module 'm' obtained through the optimization process should be greater than the minimum module to assure the proper transmission of rotational motion. This constraint is represented by equation (7)

$$m_x \geq m_x \quad (7)$$

### Variable bounds:

- Axial module is varied from 4 to 5mm, 5 to 6mm, 6 to 7mm and 7 to 8mm in the range of 0.001mm.
- Number of teeth in worm wheel- 24, 42, 72, 96.
- Power is varied from 22.5 to 24.5 kW in the range of 0.001mm.

## b) Proposed design objective function

The gear pair design problem has four different parameters in the objectives considered in this work. i.e.,

power, weight of material, efficiency and center distance. Since all these parameters are on different scales, these factors are to be normalized to the same scale. For maximizing criterion value, the values are normalized by dividing its value with the normalizing factor, max<sub>i</sub>, which is the maximum value of this criterion obtained from the solutions that have been explored so far and for a minimizing criterion value, it is normalized by dividing the normalizing factor, min<sub>i</sub>, with its value. The maximum and minimum value of the criterion will be updated whenever the proposed algorithm finds another feasible solution. In addition, to ensure the overall objective value to fall between 0 and 1, the weight of each criterion is also normalized. The normalized objective function is obtained as follows:

$$COF = \sum_{i=1}^n NW_i * N(X_i)$$

Where

COF = Combined objective function

$W_i$  = Pre normalized weight of criterion  $i$ .

$NW_i$  = Normalized weight of criterion  $i$ ,

$$\text{Where } NW_i = \frac{W_i}{\left( \sum_{i=1}^n W_i \right)} \quad (8)$$

$N(X_i)$  = normalized value of criterion  $i$  of solution  $X$ .

Where,  $N(X_i) = X_i / \max_i$  for maximizing criterion.

$N(X_i) = \min_i / X_i$  for minimizing criterion.

$X_i$  = Pre normalized value of criterion  $X$ .

max<sub>i</sub> = Pre normalized maximum value of criterion  $i$  among all solutions explored so far.

min<sub>i</sub> = Pre normalized minimum value of criterion  $i$  among all solutions explored so far.

$N$  = number of criteria.

Hence the COF for this problem is,

$$COF = \left[ \left( \frac{\text{power}}{\max \text{ .power}} \times xNW_1 \right) + \left( \frac{\min \text{ .weight}}{\text{weight}} \times xNW_2 \right) + \left( \frac{\text{efficiency}}{\max \text{ .efficiency}} \times xNW_3 \right) + \left( \frac{\min \text{ .cent .dist}}{\text{cent .dist}} \times xNW_4 \right) \right] \quad (9)$$

Where  $NW_1$ ,  $NW_2$ ,  $NW_3$  and  $NW_4 = 0.25$

## 3. SOLUTION METHODOLOGIES

Many researchers have shown enormous interests on non-traditional optimization techniques such as Genetic Algorithm, Simulated Annealing and applied the same in various engineering fields. In this paper, an attempt is made to bring out the best of this algorithm for the design of worm and worm wheel.

### a) LINGO software

LINGO is a software tool designed to efficiently build and solve linear, nonlinear, and integer optimization



models. In this work four models have been created in LINGO to solve the four objective functions. As these models have been solved separately, these values can not be taken as optimum values. Hence a fifth model has been created in LINGO which combines all the four objective functions. The minimum values of weight and centre distance and maximum values of power and efficiency obtained from the previous models have been given as inputs for the fifth model and COF value and corresponding optimal results have been found out.

#### b) Particle swarm optimization

PSO is an evolutionary computation technique inspired by social behavior of bird flocking or fish schooling. Similar to other nontraditional techniques, PSO is a population based optimization technique.

#### i) Adaptation of PSO to worm and worm wheel design

In PSO the initial population is randomly generated and COF value is computed for each particle. Then velocity of each particle is calculated according to the respective equation. The population best and global best particles are computed and the global best is updated continuously.

Then velocity of each particle is calculated according to the following equation.

$$v[i] = \omega * v[i] + C_1 * \text{rand}() * (pbest[i] - present[i]) + C_2 * \text{rand}() * (gbest[i] - present[i]) \quad (10)$$

Where,

$v[i]$  : The velocity for the  $i^{th}$  particle, represents the distance to be traveled by this particle from the current position.

$\omega$  inertia weights ranges from 0.8 to 0.9.

$\text{rand}()$  is a random number between (0,1)

$C_1$  and  $C_2$  are learning factors.  $C_1 = C_2 = 2$ .

$Present[i]$ : The location of the  $i^{th}$  particle i.e., particle position.

$Pbest[i]$ : The best previous position of the  $i^{th}$  particle is recorded and represented as  $pbest[i]$

$Gbest[i]$ : The index of the best particle among all the particles in the population is represented by  $gbest[i]$ .

The new particle for the next iteration is generated by adding the velocity with the Present Particle. This is given in the following equation.

$$Present[i] = present[i] + v[i] \quad (11)$$

The COF value is computed for the current particle and it is compared with present COF. If the COF value of current particle is better than the COF of the previous one, the current particle is set as new  $Pbest$ . The procedure is carried out for the required number of iterations to obtain the optimum value.

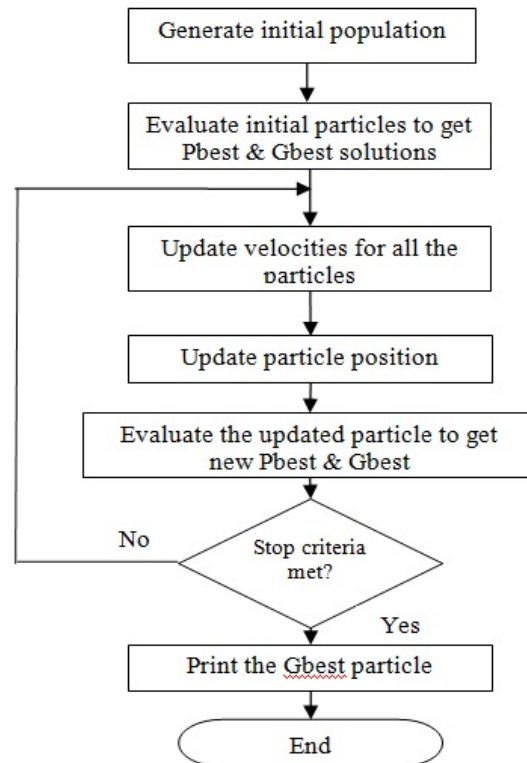


Figure-1. The algorithm for particle swarm optimization.

#### 4. COMPUTATIONAL RESULTS

The efficiency of the proposed algorithms are validated through the test problem. The proposed algorithms have been executed in MATLAB. The Input parameters considered for the worm and worm wheel design is given the Table-1 below.

Table-1. Input parameters.

Parameter/Constraint	Values for Worm and Worm Wheel
Material of worm	Hardened steel
Material of worm wheel	Bronze
Density of hardened steel ( $\rho_1$ )	$7.8 \times 10^{-6}$ kg/mm <sup>3</sup>
Density of bronze ( $\rho_2$ )	$8.9 \times 10^{-6}$ kg/mm <sup>3</sup>
Gear ratio ( $i$ )	24
Allowable bending stress of bronze	50N/mm <sup>2</sup>
Allowable crushing stress of bronze	149N/mm <sup>2</sup>
Input speed	1440 rpm
Normal pressure Angle of worm wheel ( $\Phi_n$ )	20°
Lead angle of the worm ( $\lambda$ )	$Z_2/q$
Diameter factor ( $q$ )	11
Co-efficient of friction ( $f$ )	0.08
Product of Load Concentration Factor ( $k$ ) & Dynamic Load Factor ( $k_d$ )	1

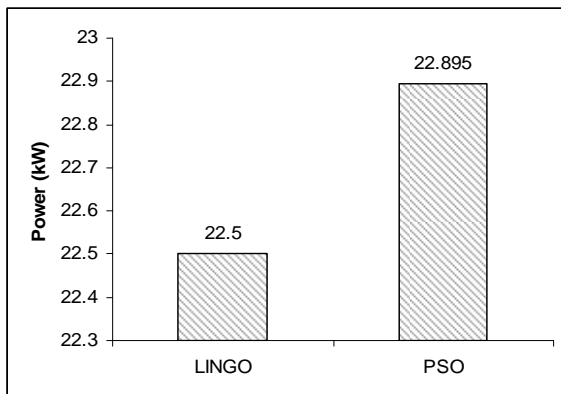


The Mathematical models for the test problem have been formulated in terms of design variable  $m$ ,  $z$  and  $P$ . Initially, the input values are generated randomly with their variable bounds.

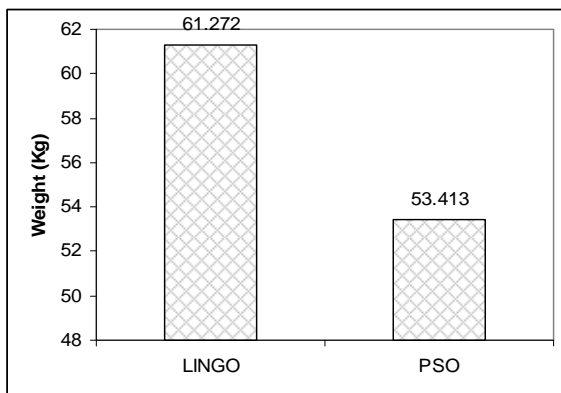
If the generated values satisfy the design constraints, then the values of objective functions  $f_1$ ,  $f_2$ ,  $f_3$  and  $f_4$  are computed along with  $COF$ . The optimum values of objective function and design variables corresponding to the minimum  $COF$  value obtained by the proposed algorithms for the test problem are shown in a Table-2.

**Table-2.** Optimum results for worm and worm wheel for LINGO& PSO (Module range: 4 to 5).

Parameters	LINGO	PSO
Axial module (mm)	5.000	4.823
No. of teeth in worm wheel	96	96
Power (kW)	22.500	22.895
Weight (kg)	61.272	53.413
Efficiency (%)	78.519	78.519
Center distance (mm)	267.500	258.030



**Figure-2.** Comparison of power transmitted.



**Figure-3.** Comparison of weight reduction.

Table-2 shows the comparative results of the proposed algorithms for the test problem. The PSO algorithm performed better when compared with LINGO in terms of Power transmitted, Weight reduction and center distance.

## 5. CONCLUSIONS

In this paper an attempt has been made to obtain optimal solution of Worm and Worm Wheel design problem. Within the various design variables available for a gear pair design, the power, weight, efficiency and center distance have been considered as objective functions and bending stress, crushing stress as vital constraints to get an efficient compact and high power transmitting drive.

Among the proposed algorithm PSO has produced convincing results for the test problems when compared with the LINGO. As a future work, minimization of vibration, maximization of life and minimization of noise can also be included in the objective function to obtain a more reliable gear pair design.

## Notations

$\rho_1$  = density of the material of worm in  $\text{kg/mm}^3$   
 $\rho_2$  = density of the material of worm wheel in  $\text{kg/mm}^3$   
 $q$  = diameter factor  
 $\lambda$  = lead angle of the worm  
 $i$  = Gear (or) transmission ratio  
 $z_1, z_2$  = number of starts in worm, number of teeth in worm wheel  
 $d_1, d_2$  = Pitch circle diameter of worm, worm wheel in mm  
 $E$  = Young's modulus in  $\text{N/mm}^2$   
 $a$  = Centre distance between shafts in mm  
 $\eta$  = Percentage efficiency  
 $y$  = Form factor  
 $f$  = Average coefficient of friction  
 $\Phi_n$  = Normal pressure angle in degrees  
 $m_x$  = Axial module in mm

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