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HARVESTED HOST-MORTAL COMMENSAL ECO-SYSTEM-A NUMERICAL STUDY

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ABSTRACT

The present paper deals with the behaviour of the biological system comprising the commensal species and host species with a constant harvesting of the host species whereas the commensal has mortality rate. Further both the species are considered in the natural limited resources. The nature of the ecological system can be observed by increasing the harvesting values of the host species in the dynamical model. The corresponding trajectories of the commensal species (growth, balanced and mortal commensal species) over the host species have been drawn for wide range of the values of the parameters in the model. The dominance reversal time of the commensal species over the host species or vice versa and the sustainability of the commensalism interaction between the species are discussed.

Keywords: commensalism interaction, commensal species, host species, numerical solution, trajectories of the species, dominance reversal time.

1. INTRODUCTION

Ecology relates to the study of living beings (animals and plants) in relation to their habits and habitats. This area of knowledge explains how diverse kinds of plants and animals can live together in the same colony for generations together sharing common resources. Such a sharing may continue till one or other or some of the species may go extinct locally if not globally. However there are instances of several species of different nature coexist persistently in the same habitat indefinitely in spite of inevitable limitations on space and resources. As such, ecology may also be referred as the study of distribution and abundance of species in the same habitat availing the same resources. Most of the biological systems are modalized as a set of non-linear ordinary differential equations. The study of biological phenomena such as harvesting of species and availability of biological resources is relevant for the ecological life and for several species activities in the same area. First the mathematical modeling of ecosystems was initiated by Lotka [6] and Volterra [13]. After that many authors have been investigated the general concepts of modeling and presented in the treatises of Meyer [7], Cushing [1], Paul Colinvaux [8], Gause [3], Freedman [2], Pielou [9], Haberman [4], Kapur [5]. Phanikumar, Seshagiri Rao and Pattabhi Ramacharyuly [10] studied on the stability of a host- a flourishing commensal species pair with limited resources. Seshagiri Rao, Phanikumar and Pattabhi Ramacharyuly [11] investigated the stability of a host- a declaining commensal species pair with limited resources. The stability analysis of host-mortal commensal ecological model with host harvesting at a constant rate is carried out by Seshagiri Rao, Kalyani and Pattabhi Ramacharyuly [12].

The aim of this paper is to know how long the biological system is sustainable in the natural limited resources using numerical technique- Runge-Kutta method for the ecological commensalism between two species. The model comprises a host-commensal species pair with limited resources in nature where as the host species is harvested at constant rate. The investigated model is formed by a couple of first order non linear ordinary differential equations. The trajectories of this model are drawn by using DEDiscover software by increasing the harvest rates of the host species in the model and fixing the remaining parameters constants. If possible the dominance reversal times of the commensal over the host species or vice versa are noted and the conclusions are presented.

Nomenclature

$N_1(t)$, $N_2(t)$: The populations of the commensal and host at time t.

d_1: The mortal rate of the commensal species.

a_2: The rate of natural growth of the host species.

$a_{ii}(i=1,2)$: The rate of decrease of the commensal and host due to the limitations of its natural resources.

a_{12}: The rate of increase of the commensal due to the support given by the host.

$e_1(=d_1/a_{11})$: The mortality coefficient of commensal species.

$c(=a_{12}/a_{11})$: The coefficient of the commensal.

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$k_2 (= a_2 / a_{22})$: The carrying capacity of the host species.

$h_2 (= a_{22} H_2)$: The coefficient of harvesting /migration of the host species.

H_2: The harvesting/migration of host per unit time.

$t_{g_1}^*$, t_0^*, $t_{e_1}^*$: The dominance reversal time of the host over the growing-balanced-mortal commensal species

The defined above variables $N_1(t)$ and $N_2(t)$ as well as all the model parameters d_1, a_2, a_{11}, a_{12}, a_{22}, k_2, e_1, c, h_2, H_2 are assumed to be non-negative constants.

2. BASIC MODEL EQUATIONS

The basic growth rate equations of this ecological commensalism interaction between two species are given as follows in terms of the above terminology.

(i) Growth rate equation for the Mortal commensal species:

$$\frac{dN_1}{dt} = a_{11} N_1 \left[-e_1 - N_1 + cN_2 \right] \quad (1)$$

By putting $e_1 = 0$ and $e_1 = -g_1$, we can obtain the balanced and growing commensal species in the equation (1).

(ii) Growth rate equation for the Host species:

$$\frac{dN_2}{dt} = a_{22} \left[k_2 N_2 - N_2{}^2 - H_2 \right] \quad (2)$$

3. NUMERICAL SOLUTIONS AND THE BEHAVIOUR OF THE SYSTEM

The trajectories (Figures- 1 to 7) of this system of equations have been drawn by using the well-known DEDiscover software and the conclusions are given under. Here the approximated solutions of the non-linear coupled differential equations (1) and (2) can be obtained in the time interval [0, 10] in one step each using Runge-Kutta technique taking increasing values to the harvesting rate H_2 for the host species, the remaining parameters keeping as constant. The following is the table which shows the values of parameters in the dynamic model and also noted the dominance reversal times of the host over the commensal species.

Table-1.

S.No.	a_{11}	e_1	c	a_{22}	k_2	N_{10}	N_{20}	H_2	$t_{g_1}^*$	t_0^*	$t_{e_1}^*$	Fig. No.
1	0.13	0.22	1	0.11	5	1	1	1	-	-	-	Fig. No. 1
2	0.13	0.22	1	0.11	5	1	1	2.5	-	-	-	Fig. No. 2
3	0.13	0.22	1	0.11	5	1	1	3.5	-	-	-	Fig. No. 3
4	0.13	0.22	1	0.11	5	1	1	3.9	5.143	-	-	Fig. No. 4
5	0.13	0.22	1	0.11	5	1	1	3.975	-	-	-	Fig. No. 5
6	0.13	0.22	1	0.11	5	1	1	4.022	-	9.928	-	Fig. No. 6
7	0.13	0.22	1	0.11	5	1	1	4.1	-	-	7.661	Fig. No. 7

The corresponding graphs of this biological model consist of the host and commensal (growing, balance and mortal types) are drawn and the conclusions are discussed case wise as follows.

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Case-1

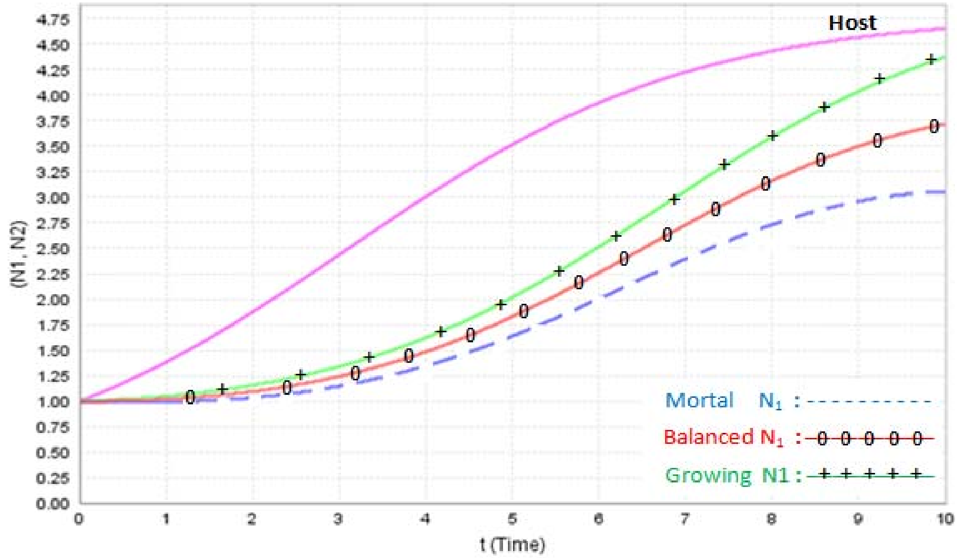

Figure-1.

In this case the host species always dominates over that of the commensal irrespective of the nature of the commensal species. Also we notice that the host species steeply rises and then reaches its asymptotic value whereas the commensal species has initially low growth rate that would rise later.

Case-2

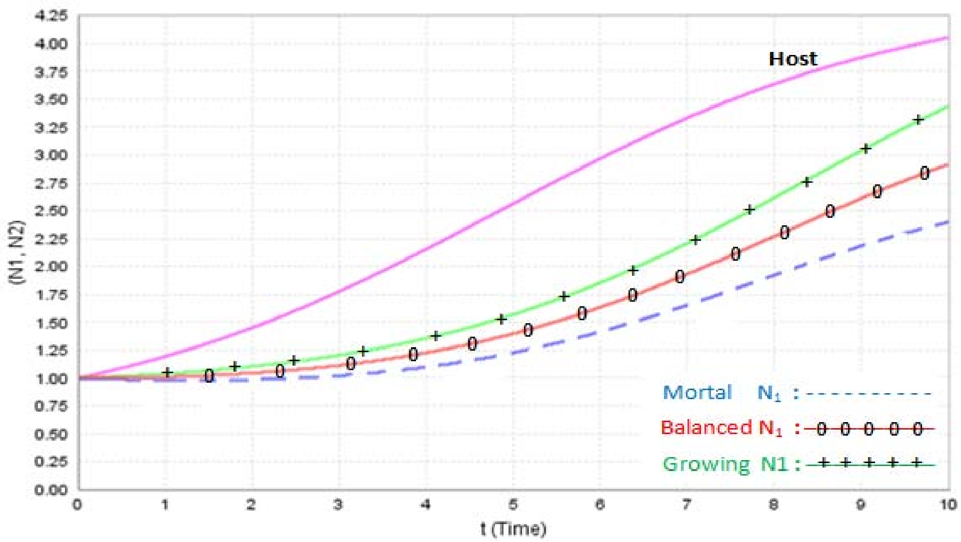

Figure-2.

The host species would always out numbers the commensal irrespective of the nature of the commensal (i.e., growing, balanced and mortal). Further we observe that there is a steep rise in the host species whereas the commensal species have initially every low growth rate for some time and then increases gradually.

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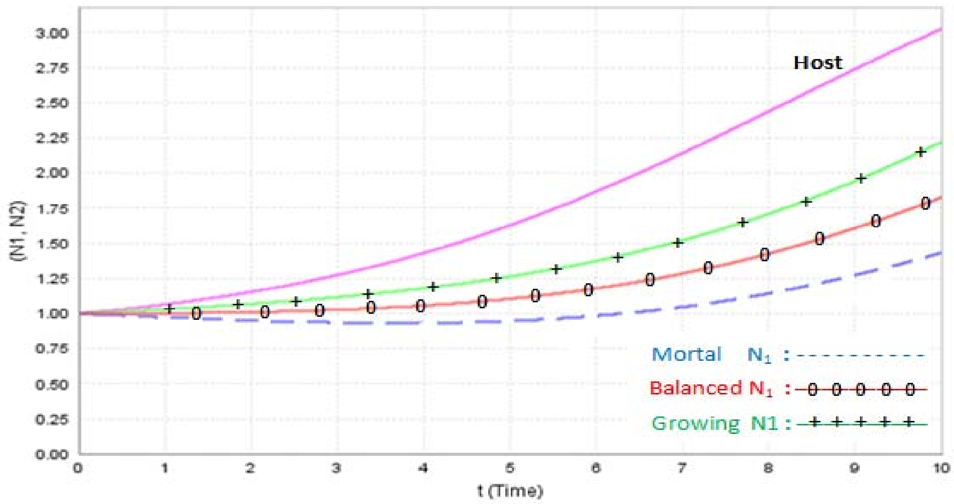
Case-3



Fiure-3.

In this case the host dominates over that of the commensal irrespective of the nature of the commensal and continues all the time. Further we see that the host species slowly increases and the commensal species has negligible growth rate.

Case-4

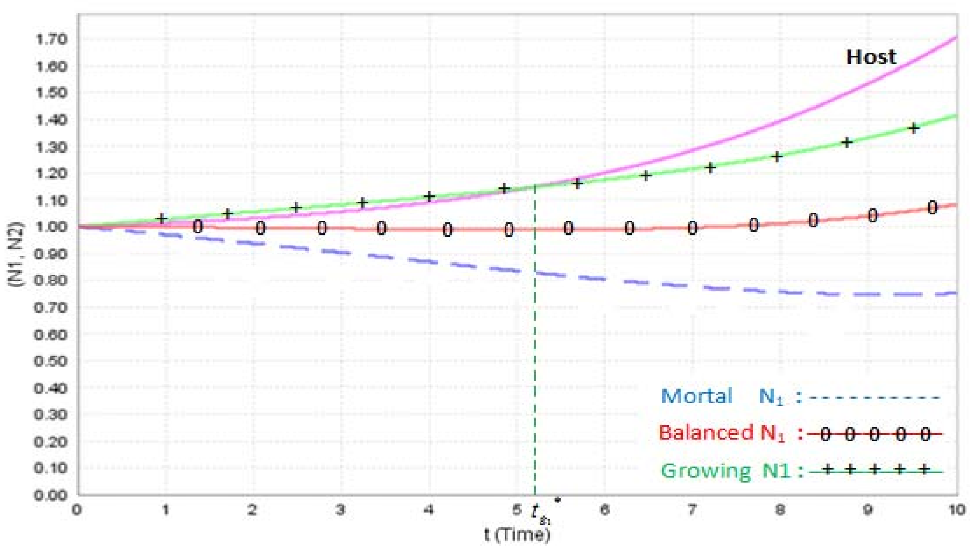


Figure-4.

In this case the growing commensal out-numbers the host species till the time instant $t_{g_1}{}^* = 5.143$ after which the out numbering is reversed. Here both the growing, balanced commensal has initially very low growth rate for some time and then increase later whereas the mortal commensal decreases.

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Case-5

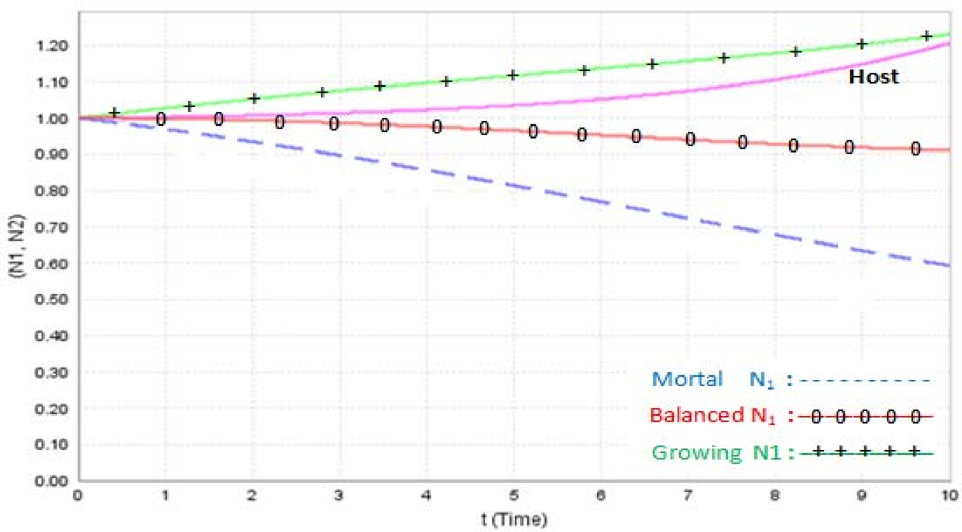

Figure-5.

In this case the growing commensal dominates over that of the host throughout the interval [1, 10] and both the balanced, mortal commensal declining further.

Case-6

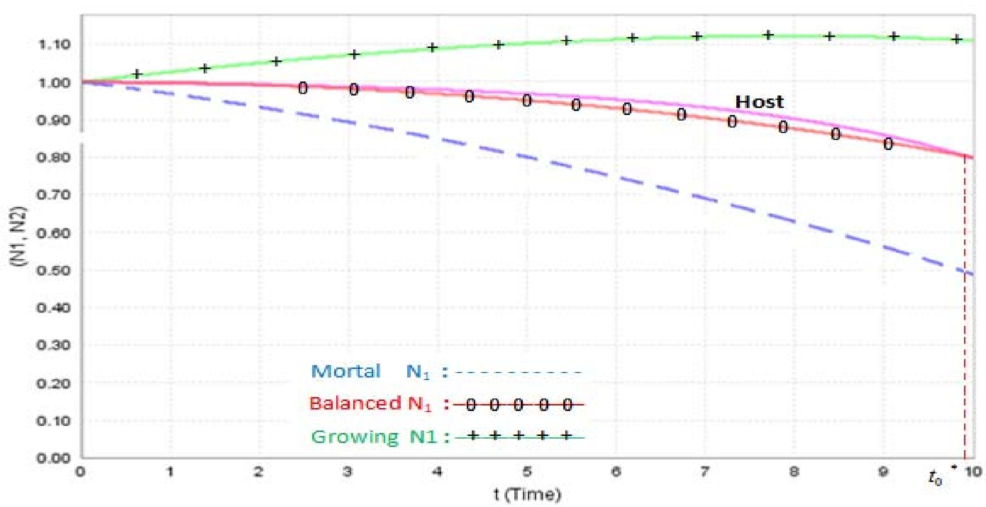

Figure-6.

In this case the growing commensal always dominates over that of the host. Also the host out-numbers the balance commensal till the time instant $t_0^* = 9.928$ there after the out numbering is reversed. Further, we see that both the species are decreasing gradually.

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Case-7

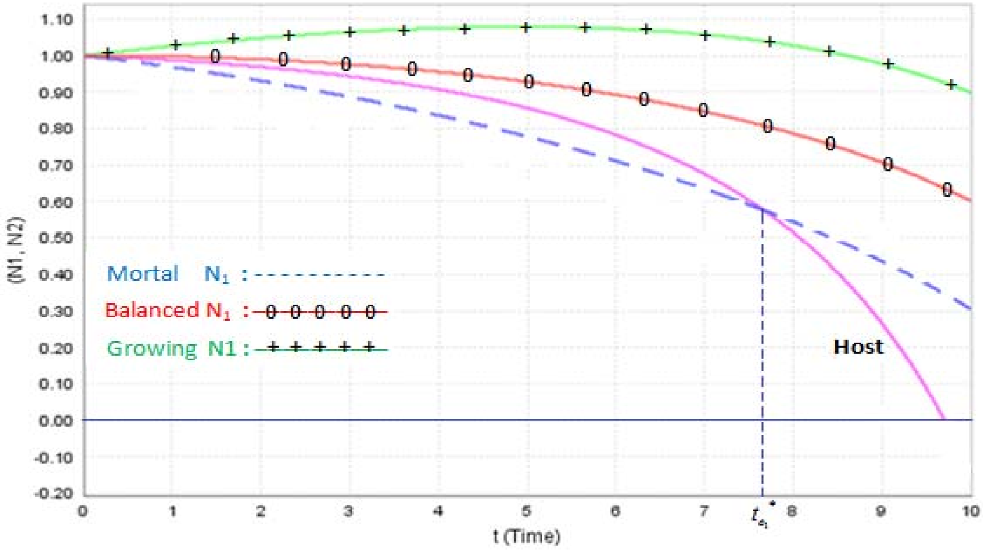

Figure-7.

In this case both growing and the balanced commensal out-number the host throughout. Here the host species dominates over that of the mortal commensal till the time instant $t_{e_1}^* = 7.661$ there after the dominance is reversed. Further we see that the host species extinct earlier than the commensal species. The extinct times of the host, commensal species (mortal, balanced, growing) are t = 9.701, 11.648, 12.895, 13.869 respectively.

Note: For higher harvesting rates of the host species of this mortal commensal ecological model with constant harvesting of the host there is no existence for the commensal species because the host species existents earlier than the commensal.

CONCLUSIONS

i) For higher harvesting rates H_2 of the host species, initially the host dominates over the commensal irrespective of the nature of the commensal species and in the course of time the commensal decreases gradually and would become extinct after the host species extinct.

ii) The higher values H_2 of the host species there is no existence for the mortal commensal ecosystem because the host species existents earlier than the commensal.

iii) For the higher values H_2 of the host species the growing and balanced commensal species are dominating the host species but the mortal commensal existent after the host.

iv) The commensal species extinct in the order, first the mortal commensal, next the balanced commensal and then the growing commensal for higher harvesting values of the host species.

Open problems

The following are the interesting problems for further investigation in the basic model equations of the ecological commensalism interaction between the host and commensal species.

a) Situations involving delayed commensalism as they occur are of interest in nature at times.

b) Migration at variable rate of the host.

c) Migration of the commensal and Immigration of the host at (a). Constant (b). Variable rates.

d) Constant migration and immigration of both the species in the model equations.

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