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ANALYZING OF FREE TRANSVERSE VIBRATION OF AN ELASTICALLY CONNECTED RECTANGULAR PLATE- MEMBRANCE SYSTEM WITH A PASTERNAK LAYER IN-BETWEEN

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ABSTRACT

This paper theoretically analyzes free transverse vibrations of an elastically connected rectangular plate-membrane system with a Pasternak layer in-between. Solutions of the problem are formulated by using the Navier method. Also natural frequencies of the system are determined. The effect of Pasternak layer on the natural frequencies of this mixed system is discussed in a numerical example. Increasing shear foundation modulus of the Pasternak layer causes an increase in the value of natural frequency of the system (ω_{imn}); however this influence of the shear foundation modulus of the Pasternak layer is different at some particular frequencies. That is, this effect is stronger at ω_{2mn} frequencies rather than what is seen at ω_{1mn} frequencies. The mixed system has an interesting feature which allows each natural frequency to change as a function of shear foundation modulus of the Pasternak layer, whilst other constructional and physical parameters of the system can remain unchanged. The important result on which this paper puts an emphasize, is that the magnitudes of the frequencies become larger with increasing shear foundation modulus of the Pasternak layer and the Pasternak layer can increase the magnitude of frequencies more than Winkler elastic layer can. Thus the Pasternak layer can be used instead of Winkler elastic layer to more effectively suppress the excessive vibrations associated with plate-membrane systems. Numerical results of the present method are verified once compared with those available in the literature.

Keywords: free transverse vibration, rectangular plate-membrance system, membrane tension, Pasternak layer, Navier method.

1. INTRODUCTION

Structural members such as plate-type (beam-type) structures made of two parallel simply supported plates (beams) continuously joined by a linear, elastic Winkler type layer are increasingly used in aeronautical, mechanical and civil engineering applications. As a matter of fact, the phenomenon of transverse vibration and dynamical problems of such systems has a wide application in engineering practice.

In classical vibration plate theory, two basic and well known analytical methods are applied for analyzing free vibrations of a single rectangular plate, namely the Levy and Navier methods. Such structures have been extensively covered by many investigators.

Zhang *et al.* (2008a) studied vibration and buckling of a double-beam system under compressive axial loading. They investigated the properties of free transverse vibration and buckling of a double-beam system under compressive axial loading on the basis of the Bernoulli-Euler beam theory. They assumed that the two beams of the system are simply supported and continuously joined by a Winkler elastic layer. They derived explicit expressions for the natural frequencies. They also obtained the associated amplitude ratios of the two beams, and the analytical solution of the critical buckling load. They showed that the critical buckling load of the system is related to the axial compression ratio of the two beams and the Winkler elastic layer, and the properties of free transverse vibration of the system greatly depend on the axial compressions. They also studied effect of compressive axial load on forced transverse vibrations of a double-beam in another paper (Zhang *et al.*, 2008b). The effects of compressive axial load on the forced vibrations of the double-beam system are discussed for two cases of particular excitation loadings and the properties of the forced transverse vibrations of the system are found to be significantly dependent on the compressive axial load.

Stojanovic and Kozic (2012) considered forced transverse vibration and buckling of a Rayleigh and Timoshenko double-beam system continuously joined by a Winkler elastic layer under compressive axial loading. In their paper, deflections of the beams based on the Timoshenko beam theory are shown and general solutions of forced vibrations of beams subjected to arbitrarily distribute continuous loads are found. The analytical solution of forced vibration with associated amplitude ratios is determined. Also dynamic responses of the system caused by arbitrarily distributed continuous loads are obtained. Vibrations caused by harmonic exciting forces are discussed, and conditions of resonance and dynamic vibration absorption are formulated.

Oniszczuk (2003) theoretically analyzed undamped free transverse vibrations of an elastically connected rectangular plate-membrane system with Winkler elastic layer in-between. Solutions of the problem

VOL. 10, NO. 14, AUGUST 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

are formulated by using the Navier method and natural frequencies of the system are determined in the form of two infinite sequences. Also Normal mode shapes of vibration expressing two kinds of vibration, synchronous and asynchronous, are presented. In a numerical example, the effect of membrane tension on the natural frequencies of this mixed system is discussed.

Forced transverse vibrations of an elastically connected complex rectangular simply supported double-plate system with a Winkler elastic layer in-between was also studied by Oniszczuk (2004). Undamped motion of the system excited by arbitrarily distributed continuous loadings applied transversely to both plates was derived based on the Kirchhoff-Love plate theory. On the basis of general solutions obtained, three particular cases of the action of exciting stationary harmonic loads were considered.

The behavior of foundation materials in engineering practice cannot be represented by foundation model which consists of independent linear elastic springs. In order to find a physically close and mathematically simple foundation model, Pasternak proposed a so-called two-parameter foundation model with shear interactions. In an attempt to find a physically close and mathematically simple representation of an elastic foundation for these materials, Pasternak (1954) proposed a foundation model consisting of a Winkler foundation with shear interactions. This may be accomplished by connecting the ends of the vertical springs to a beam consisting of incompressible vertical elements, which deforms only by transverse shear.

In Wang *et al.* (1977) and De Rosa (1995), the natural vibration of a Timoshenko beam on a Pasternak-type foundation is studied. Frequency equations are derived for beams with different end restraints. A specific example is given to show the effects of rotary inertia, shear deformation, and foundation constants on the natural frequencies of the beam.

Free transverse vibrations of an elastically connected rectangular plate-membrane system with a Pasternak layer in-between are studied in the present paper. Free transverse vibrations of this mixed system are studied by using the Navier method.

2. FORMULATION OF THE PROBLEM

Figure-1 shows a complex continuous system which consists of a rectangular three-layered structure composed of isotropic plate and parallel membrane stretched uniformly by constant tensions applied at the edges and separated by the Pasternak layer. It is assumed that both plates and membrane are thin, homogeneous, uniform, and perfectly elastic. For the sake of simplicity of vibration analysis it is also assumed that the plates as well as the membrane are governed by simply supported boundary conditions. In the general case, the system is subjected to arbitrarily distributed transverse continuous loads.

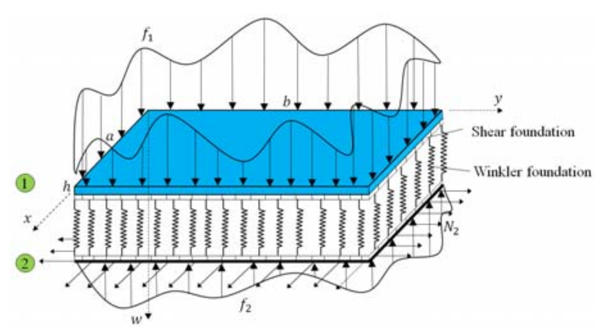

Figure-1. The physical model of an elastically connected rectangular plate-membrane complex system with a Pasternak layer in-between.

According to the Kirchhoff-Love plate theory, transverse vibrations of an elastically connected rectangular plate-membrane system with a Pasternak layer in-between are described by the following differential equations:

$$
\begin{aligned}
&m_1\ddot{w}_1 + D_1\Delta^2 w_1 + k(w_1 - w_2) - G_0\left[\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} - \left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right)\right] = f_1(x,y,t)\,, \\
&m_2\ddot{w}_2 - N_2\Delta w_2 + k(w_2 - w_1) - G_0\left[\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} - \left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right)\right] = f_2(x,y,t)
\end{aligned} \tag{1}
$$

where,

$$
\begin{aligned}
&D_1 = E_1 h_1^3 [12(1-v_1^2)]^{-1}, \quad m_i = \rho_i h_i, \quad \dot{w}_i = \partial w_i / \partial t, \quad i = 1, 2 \\
&\Delta^2 w_1 = \frac{\partial^4 w_1}{\partial x^4} + 2\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + \frac{\partial^4 w_1}{\partial y^4}\,, \quad \Delta w_2 = \frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\cdot
\end{aligned} \tag{2}
$$

The subscripts 1 and 2 refer to the plate and the membrane, respectively. The boundary conditions and initial conditions for the simply supported plate and membrane are as follows:

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ARPN Journal of Engineering and Applied Sciences

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$$
\begin{aligned}
&w_1(0,y,t)=w_1(a,y,t)=w_1(x,0,t)=w_1(x,b,t)=0\,,\\
&w_2(0,y,t)=w_2(a,y,t)=w_2(x,0,t)=w_2(x,b,t)=0\,,\\
&\left.\frac{\partial^2 w_1}{\partial x^2}\right|_{(0,y,t)}=\left.\frac{\partial^2 w_1}{\partial x^2}\right|_{(a,y,t)}=\left.\frac{\partial^2 w_1}{\partial y^2}\right|_{(x,0,t)}=\left.\frac{\partial^2 w_1}{\partial y^2}\right|_{(x,b,t)}=0
\end{aligned}
\tag{3}
$$

and

$$
w_i(x,y,0)=w_{i0}(x,y)\;,\;\dot{w}_i(x,y,0)=\dot{w}_{i0}(x,y)\;,\;i=1,2 \tag{4}
$$

3. SOLUTION OF THE FREE VIBRATION PROBLEM

Free vibrations of a rectangular plate-membrane system (see Figure-2) are governed by the following homogeneous partial differential equations:

$$
\begin{aligned}
&m_1\ddot{w}_1+D_1\Delta^2 w_1+k(w_1-w_2)-G_0\left[\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w_1}{\partial y^2}-\left(\frac{\partial^2 w_2}{\partial x^2}+\frac{\partial^2 w_2}{\partial y^2}\right)\right]=0\,,\\
&m_2\ddot{w}_2-N_2\Delta w_2+k(w_2-w_1)-G_0\left[\frac{\partial^2 w_2}{\partial x^2}+\frac{\partial^2 w_2}{\partial y^2}-\left(\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w_1}{\partial y^2}\right)\right]=0
\end{aligned}
\tag{5}
$$

The above equation system with the boundary conditions (3) can be solved by the Navier method assuming solutions in the following form:

$$
\begin{aligned}
&w_1(x,y,t)=\sum_{m,n=1}^{\infty}W_{mn}(x,y)\,S_{1mn}(t)=\sum_{m,n=1}^{\infty}\sin(a_m x)\sin(b_n y)\,S_{1mn}(t)\,,\\
&w_2(x,y,t)=\sum_{m,n=1}^{\infty}W_{mn}(x,y)\,S_{2mn}(t)=\sum_{m,n=1}^{\infty}\sin(a_m x)\sin(b_n y)\,S_{2mn}(t)
\end{aligned}
\tag{6}
$$

Also,

$$
\begin{aligned}
&W_{mn}(x,y)=X_m(x)\,Y_n(y)=\sin(a_m x)\sin(b_m y)\\
&X_m(x)=\sin(a_m x)\;,\quad Y_m(y)=\sin(b_n y),\quad m,n=1,2,3,...\\
&a_m=a^{-1}m\pi\;,\quad b_m=b^{-1}n\pi
\end{aligned}
\tag{7}
$$

Where, $W_{mn}(x,y)$ satisfies the corresponding boundary conditions (3) for the simply supported plate and membrane as well as the homogeneous differential equations (5).

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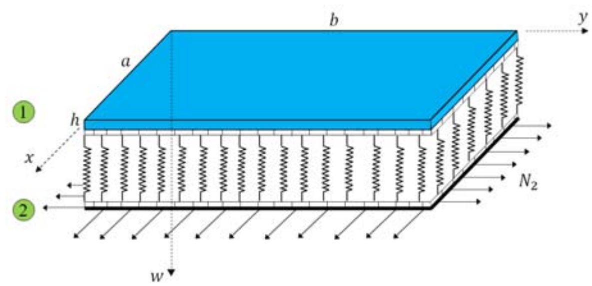

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Figure-2. The physical model of an elastically connected rectangular plate-membrane system with a Pasternak layer in-between analyzed for free vibrations.

Substituting solutions equations (6) into equations (5) gives the following expressions:

$$
\begin{aligned}
&\sum_{m,n=1}^{\infty}[\ddot{S}_{1mn}+(D_1k_{mn}^4+k+G_0k_{mn}^2)\,m_1^{-1}S_{1mn}\;-(k+G_0)m_1^{-1}S_{2mn}]\,W_{mn}=0\,,\\
&\sum_{m,n=1}^{\infty}[\ddot{S}_{2mn}+(N_2k_{mn}^2+k+G_0k_{mn}^2)m_2^{-1}S_{2mn}-(k+G_0)m_2^{-1}S_{1mn}]\;W_{mn}=0
\end{aligned}
\tag{8}
$$

Where, $k_{mn}^2=a_m^2+b_n^2=\pi^2[(a^{-1}m)^2+(b^{-1}n)^2]$. From which a set of ordinary differential equations for the unknown time functions is obtained

$$
\begin{aligned}
&\ddot{S}_{1mn}+\Omega_{11mn}^2 S_{1mn}-\Omega_{10}^2 S_{2mn}=0\\
&\ddot{S}_{2mn}+\Omega_{22mn}^2 S_{2mn}-\Omega_{20}^2 S_{1mn}=0
\end{aligned}
\tag{9}
$$

Where,

$$
\begin{aligned}
&\Omega_{11mn}^2=(D_1k_{mn}^4+k+G_0k_{mn}^2)\,m_1^{-1}\\
&\Omega_{22mn}^2=(N_2k_{mn}^2+k+G_0k_{mn}^2)\,m_2^{-1}\\
&\Omega_{i0}^2=(k+G_0)m_i^{-1},\quad \Omega_{120}^4=\Omega_{10}^2\,\Omega_{20}^2=(k+G_0)^2(m_1m_2)^{-1}\,,\;i=1,2
\end{aligned}
$$

The solutions of equations (9) are as follows:

$$
S_{1mn}(t)=C_{mn}\,\mathrm{e}^{\mathrm{i}\omega_{mn}t},\quad S_{2mn}(t)=D_{mn}\,\mathrm{e}^{\mathrm{i}\omega_{mn}t} \tag{10}
$$

Introducing them into equations (9) results in a system of algebraic equations with unknown constants of C_{mn} and D_{mn}:

$$
\begin{aligned}
&(\Omega_{11mn}^2-\omega_{mn}^2)C_{mn}-\Omega_{10}^2\,D_{mn}=0\,,\\
&(\Omega_{22mn}^2-\omega_{mn}^2)D_{mn}-\Omega_{20}^2\,C_{mn}=0
\end{aligned}
\tag{11}
$$

VOL. 10, NO. 14, AUGUST 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

For non-trivial solutions of the above equations, the determinant of the system coefficient matrix is set equal to zero, yielding the following frequency equation:

$$\omega_{mn}^4 - (\Omega_{11mn}^2 + \Omega_{22mn}^2)\omega_{mn}^2 + (\Omega_{11mn}^2\Omega_{22mn}^2 - \Omega_{120}^4) = 0 \quad (12)$$

or

$$\begin{aligned}&\omega_{mn}^4 - [(D_1k_{mn}^4 + k + G_{01}k_{mn}^2)m_1^{-1} + (N_2k_{mn}^2 + k + G_{01}k_{mn}^2)m_2^{-1}]\,\omega_{mn}^2\\ &+ \{k_{mn}^2[D_1N_2k_{mn}^4 + (D_1k_{mn}^2 + N_2 + 2G_0)k + k_{mn}^2G_0(D_1k_{mn}^2 + N_2 + G_0)] - G_0^2 - 2kG_0\}(m_1m_2)^{-1} = 0\end{aligned} \quad (13)$$

and thus the frequency equation (12) has two roots:

$$\omega_{(1,2)mn}^2 = \frac{1}{2}\{(\Omega_{11mn}^2 + \Omega_{22mn}^2) \mp [(\Omega_{11mn}^2 + \Omega_{22mn}^2) + 4\Omega_{120}^2]^{1/2}\} \;,\quad \omega_{1mn} < \omega_{2mn} \quad (14)$$

The time functions (10) can be assumed to have the following forms

$$\begin{aligned}S_{1mn}(t) &= C_{1mn}\mathrm{e}^{\mathrm{i}\omega_{1mn}t} + C_{2mn}\mathrm{e}^{-\mathrm{i}\omega_{1mn}t} + C_{3mn}\mathrm{e}^{\mathrm{i}\omega_{2mn}t} + C_{4mn}\mathrm{e}^{-\mathrm{i}\omega_{2mn}t},\\ S_{2mn}(t) &= D_{1mn}\mathrm{e}^{\mathrm{i}\omega_{1mn}t} + D_{2mn}\mathrm{e}^{-\mathrm{i}\omega_{1mn}t} + D_{3mn}\mathrm{e}^{\mathrm{i}\omega_{2mn}t} + D_{4mn}\mathrm{e}^{-\mathrm{i}\omega_{2mn}t}\end{aligned} \quad (15)$$

or in trigonometric form

$$\begin{aligned}S_{1mn}(t) &= \sum_{i=1}^{2}T_{imn}(t) = \sum_{i=1}^{2}[A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)],\\ S_{2mn}(t) &= \sum_{i=1}^{2}a_{imn}T_{imn}(t) = \sum_{i=1}^{2}[A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)]a_{imn}\end{aligned} \quad (16)$$

Where

$$\begin{aligned}a_{imn} &= \frac{D_{mn}}{C_{mn}} = (D_1k_{mn}^4 + k + G_0k_{mn}^2 - m_1\omega_{imn}^2)(k + G_0)^{-1}\\ &= (k + G_0)(N_2k_{mn}^2 + k + G_0k_{mn}^2 - m_2\omega_{imn}^2)^{-1}\\ &= \Omega_{10}^{-2}(\Omega_{11mn}^2 - \omega_{imn}^2) = \Omega_{20}^2(\Omega_{22mn}^2 - \omega_{imn}^2)^{-1}\end{aligned}$$

and

$$T_{imn}(t) = A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t), \quad m,n = 12,3... \quad (17)$$

Finally, the forced vibrations of an elastically connected rectangular plate–membrane system with a Pasternak layer in-between can be described by

$$\begin{aligned}w_1(x,y,t) &= \sum_{m,n=1}^{\infty}W_{mn}(x,y)\sum_{i=1}^{2}T_{imn}(t) = \sum_{m,n=1}^{\infty}\sum_{i=1}^{2}W_{1imn}(x,y)T_{imn}(t)\\ &= \sum_{m,n=1}^{\infty}\sin(a_mx)\sin(b_ny)\sum_{i=1}^{2}[A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)],\end{aligned} \quad (18)$$

And

$$\begin{aligned}w_2(x,y,t) &= \sum_{m,n=1}^{\infty}W_{mn}(x,y)\sum_{i=1}^{2}a_{imn}T_{imn}(t) = \sum_{m,n=1}^{\infty}\sum_{i=1}^{2}W_{2imn}(x,y)T_{imn}(t)\\ &= \sum_{m,n=1}^{\infty}\sin(a_mx)\sin(b_ny)\sum_{i=1}^{2}[A_{imn}\sin(\omega_{imn}t) + B_{imn}\cos(\omega_{imn}t)]a_{imn}\end{aligned} \quad (19)$$

ARPN Journal of Engineering and Applied Sciences

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The unknown constants A_{imn} and B_{imn} in expressions (18) and (19) are calculated by solving the initial-value problem. By multiplying the relations (18) and (19) by the eigen-function W_{kl}, then integrating them and using orthogonality condition

$$\int_0^a\int_0^b W_{kl}W_{mn}dxdy = \int_0^a \sin(a_k x)\sin(a_m x)dx\int_0^a \sin(b_l y)\sin(b_n y)dy = c\delta_{klmn} \tag{20}$$

Where,

$$c = c_{mn}^2\int_0^a\int_0^b W_{mn}^2 dxdy = \int_0^a \sin^2(a_m x)\,dx\int_0^a \sin^2(b_n y)\,dy = \frac{ab}{4}$$

Then substituting solutions (18) and (19) into the initial conditions (4), and then performing the known usual transformation procedure and applying the orthogonality condition (20), the following formulae for evaluating A_{imn} and B_{imn} are obtained:

$$A_{1mn} = (\omega_{1mn}z_{1mn})^{-1}\int_0^a\int_0^b (a_{2mn}v_{10} - v_{20})\sin(a_m x)\sin(b_n y)dxdy,$$

$$A_{2mn} = (\omega_{2mn}z_{2mn})^{-1}\int_0^a\int_0^b (a_{1mn}v_{10} - v_{20})\sin(a_m x)\sin(b_n y)dxdy,$$

$$B_{1mn} = z_{1mn}^{-1}\int_0^a\int_0^b (a_{2mn}w_{10} - w_{20})\sin(a_m x)\sin(b_n y)dxdy,$$

$$B_{2mn} = z_{2mn}^{-1}\int_0^a\int_0^b (a_{1mn}w_{10} - w_{20})\sin(a_m x)\sin(b_n y)dxdy,$$

Where,

$$z_{2mn} = -z_{1mn} = (a_{1mn} - a_{2mn})c$$
$$= 0.25ab(a_{1mn} - a_{2mn}) = 0.25ab\Omega_{10}^{-2}(\omega_{2mn}^2 - \omega_{1mn}^2)$$

4. NUMERICAL RESULTS AND DISCUSSION

In the following the effects of shear foundation modulus of the Pasternak layer (G_0) on the natural frequencies of the system are considered. Values of the parameters characterizing properties of the system are shown in Table-1.

Table-1. Values of the parameters characterizing properties of the system.

a	b	E_1	h_1	h_2	k
1 m	2 m	1×10^8 N/m^2	1×10^{-2} m	4×10^{-3} m	1×10^4 N/m^3
v_1	m_1	m_2	ρ_1	ρ_2	N_2
0.3	50kg/m^2	1kg/m^2	5×10^3 kg/m^3	2.5×10^2 kg/m^3	0,100,200,300,400,500 N/m

Results of the calculations for $i = 1,2$ and $m,n = 1,2$ are presented in Tables-2 to 7 and in Figures-3 to 8. In any case, increasing N_2 causes an increase in ω_{imn}. An evident influence of shear foundation modulus of the Pasternak layer on the frequencies of the system is observed and it can be seen that with increasing G_0 the frequencies increase as well. However this influence of the membrane tension and shear foundation modulus of the Pasternak layer is different on some particular frequencies, being stronger on ω_{2mn} frequencies while weaker on the ω_{1mn} frequencies. If we introduce $G_0 = 0$ in the whole equations of this paper we can obtain the vibration equations of the plate-membrane system with a Winkler elastic layer in-between, the results of which are verified by comparing with those available in Oniszczuk (2003).

VOL. 10, NO. 14, AUGUST 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

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Table-2. Natural frequencies of the rectangular plate-membrane system $\omega_{imn}(s^{-1})$ for $G_0 = 0$.

$G_0 = 0$	N_2					
	0	**100**	**200**	**300**	**400**	**500**
ω_{111}	5.2276	7.0042	8.1606	9.0019	9.6499	10.1677
ω_{211}	100.9978	106.8294	112.3774	117.6773	122.7581	127.644
ω_{112}	8.3638	10.1432	11.2542	12.0248	12.5938	13.0322
ω_{212}	101.002	110.1926	118.712	126.6828	134.1955	141.3192
ω_{121}	17.7686	19.4292	20.2731	20.7836	21.1256	21.3707
ω_{221}	101.0273	119.7469	136.0162	150.579	163.8728	176.1773
ω_{122}	20.9016	22.5253	23.2923	23.7383	24.0298	24.2351
ω_{222}	101.0403	122.7739	141.3262	157.7559	172.6493	186.3671

Table-3. Natural frequencies of the rectangular plate-membrane system $\omega_{imn}(s^{-1})$ for $G_0 = 100$.

$G_0 = 100$	N_2					
	0	**100**	**200**	**300**	**400**	**500**
ω_{111}	8.3546	9.3673	10.1252	10.7184	11.1973	11.5931
ω_{211}	106.8478	112.3931	117.6908	122.77	127.6545	132.3645
ω_{112}	11.7669	12.7608	13.462	13.9848	14.3903	14.7141
ω_{212}	110.2103	118.7259	126.694	134.2049	141.3272	148.1144
ω_{121}	21.392	22.1842	22.6648	22.9872	23.2185	23.3924
ω_{221}	119.7627	136.0263	150.5861	163.8782	176.1816	187.687
ω_{122}	24.543	25.2709	25.6945	25.9714	26.1665	26.3113
ω_{222}	122.789	141.3354	157.7622	172.654	186.3707	199.1513

Table-4. Natural frequencies of the rectangular plate-membrane system $\omega_{imn}(s^{-1})$ for $G_0 = 200$.

$G_0 = 200$	N_2					
	0	**100**	**200**	**300**	**400**	**500**
ω_{111}	10.4333	11.134	11.6881	12.1386	12.5127	12.8286
ω_{211}	112.4089	117.7045	122.782	127.6651	132.3739	136.9254
ω_{112}	14.1059	14.7586	15.2484	15.6299	15.9355	16.1859
ω_{212}	118.74	126.7054	134.2143	141.3352	148.1213	154.6149
ω_{121}	23.9423	24.4007	24.7085	24.9293	25.0954	25.2249
ω_{221}	136.0366	150.5934	163.8836	176.1859	187.6905	198.5341
ω_{122}	27.1046	27.5114	27.7772	27.9644	28.1034	28.2107
ω_{222}	141.3448	157.7686	172.6587	186.3744	199.1543	211.1662

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Table-5. Natural frequencies of the rectangular plate-membrane system $\omega_{imn}(s^{-1})$ for $G_0 = 300$.

$G_0 = 300$	N_2					
	0	**100**	**200**	**300**	**400**	**500**
ω_{111}	12.0571	12.5819	13.0107	13.3681	13.6707	13.9305
ω_{211}	117.7184	122.7941	127.6758	132.3835	136.934	141.3416
ω_{112}	15.949	16.414	16.7773	17.0689	17.3083	17.5083
ω_{212}	126.717	134.2239	141.3433	148.1282	154.621	160.8559
ω_{121}	26.0204	26.3169	26.5296	26.6896	26.8144	26.9143
ω_{221}	150.6008	163.8892	176.1903	187.6941	198.5371	208.821
ω_{122}	29.2149	29.4721	29.6533	29.7876	29.8913	29.9737
ω_{222}	157.7751	172.6636	186.3781	199.1572	211.1687	222.5356

Table-6. Natural frequencies of the rectangular plate-membrane system $\omega_{imn}(s^{-1})$ for $G_0 = 400$.

$G_0 = 400$	N_2					
	0	**100**	**200**	**300**	**400**	**500**
ω_{111}	13.415	13.8268	14.1709	14.463	14.714	14.9322
ω_{211}	122.8064	127.6867	132.3932	136.9427	141.3495	145.6259
ω_{112}	17.5012	17.8503	18.1309	18.3616	18.5545	18.7182
ω_{212}	134.2336	141.3515	148.1353	154.6272	160.8614	166.8658
ω_{121}	27.832	28.0384	28.1936	28.3145	28.4114	28.4908
ω_{221}	163.8948	176.1947	187.6977	198.5401	208.8235	218.626
ω_{122}	31.0744	31.2506	31.3814	31.4822	31.5623	31.6275
ω_{222}	172.6684	186.3819	199.1603	211.1712	222.5377	233.3532

Table-7. Natural frequencies of the rectangular plate-membrane system $\omega_{imn}(s^{-1})$ for $G_0 = 500$.

$G_0 = 500$	N_2					
	0	**100**	**200**	**300**	**400**	**500**
ω_{111}	14.5962	14.9296	15.2131	15.4571	15.6694	15.8558
ω_{211}	127.6977	132.403	136.9515	141.3574	145.6332	149.7893
ω_{112}	18.8615	19.1334	19.357	19.5442	19.7031	19.8397
ω_{212}	141.3598	148.1425	154.6335	160.8669	166.8707	172.6683
ω_{121}	29.4696	29.6209	29.7388	29.8332	29.9106	29.9751
ω_{221}	176.1992	187.7013	198.5431	208.8261	218.6282	228.011
ω_{122}	32.7699	32.8977	32.9962	33.0745	33.1381	33.1909
ω_{222}	186.3857	199.1634	211.1738	222.5399	233.355	243.6922

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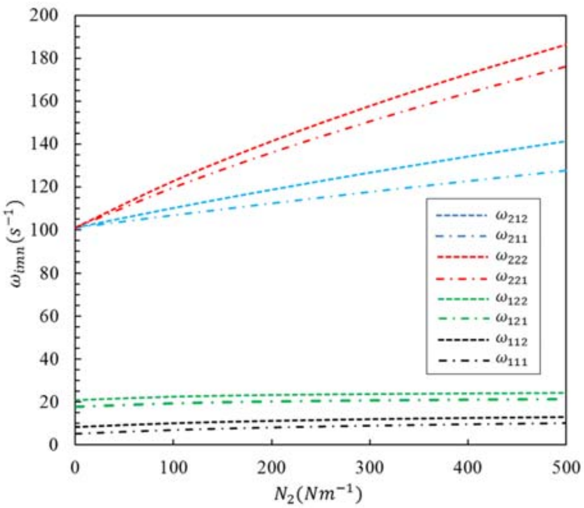

Figure-3. The natural frequencies of the plate-membrane system ω_{imn} $(i=1,2, m,n=1,2)$ as a function of membrane tension N_2 and $G_0=0$.

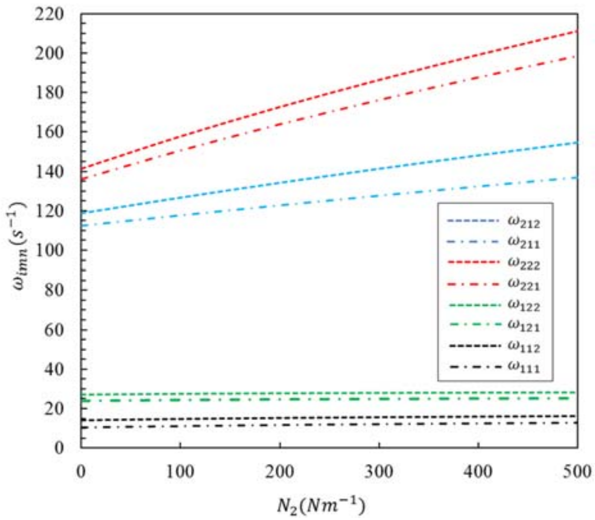

Figure-5. The natural frequencies of the plate-membrane system ω_{imn} $(i=1,2, m,n=1,2)$ as a function of membrane tension N_2 and $G_0=200$.

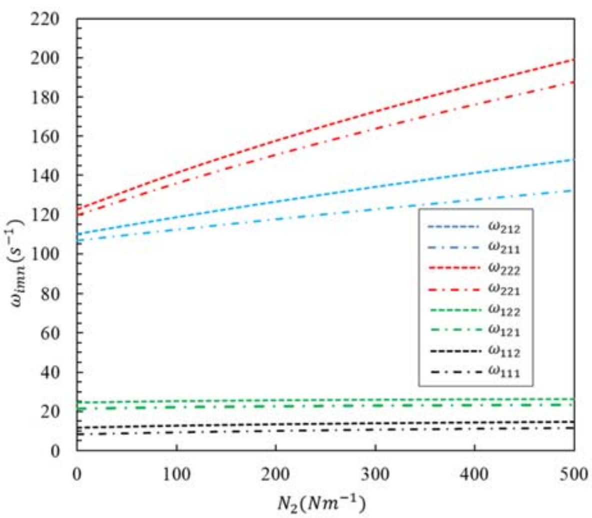

Figure-4. The natural frequencies of the plate-membrane system ω_{imn} $(i=1,2, m,n=1,2)$ as a function of membrane tension N_2 and $G_0=100$.

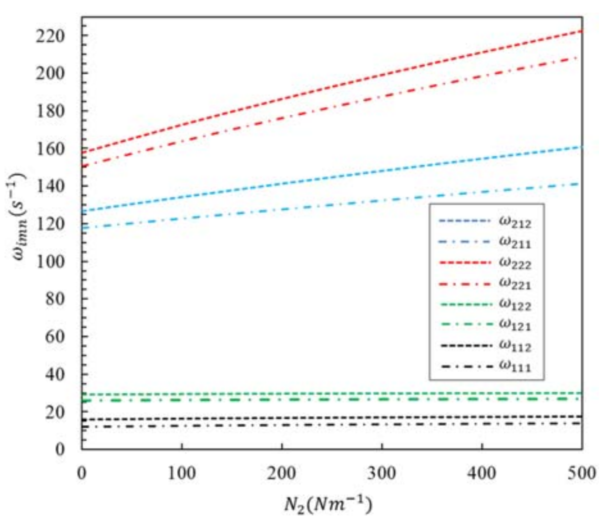

Figure-6. The natural frequencies of the plate-membrane system ω_{imn} $(i=1,2, m,n=1,2)$ as a function of membrane tension N_2 and $G_0=300$.

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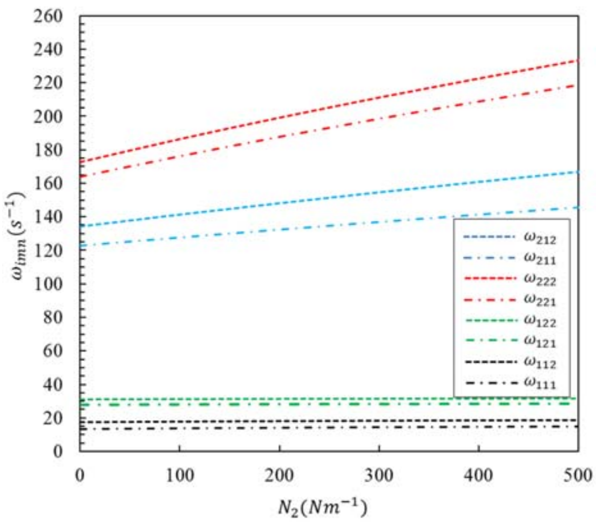

Figure-7. The natural frequencies of the plate-membrane system ω_{imn} $(i=1,2, m,n=1,2)$ as a function of membrane tension N_2 and $G_0 = 400$.

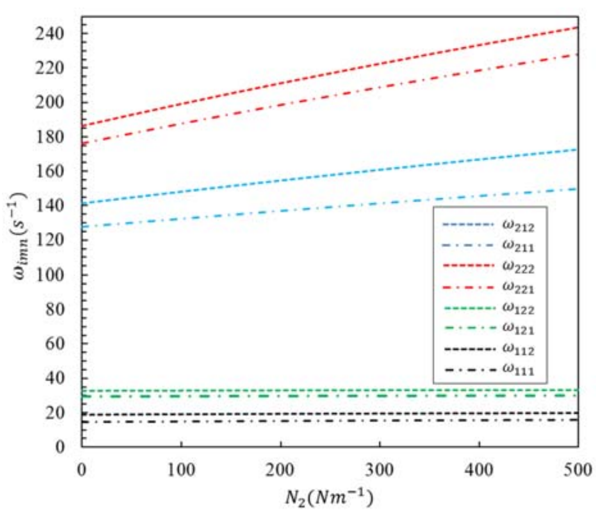

Figure-8. The natural frequencies of the plate-membrane system ω_{imn} $(i=1,2, m,n=1,2)$ as a function of membrane tension N_2 and $G_0 = 500$.

It can be seen that the mixed system discussed has an interesting feature, which allows each natural frequency to change as a function of membrane tension and shear foundation modulus of the Pasternak layer whilst the other constructional and physical parameters of the system can remain unchanged. Selecting suitable tension of the membrane and shear foundation modulus of the Pasternak layer gives desirable values of the system frequencies in certain limited domains, so that it is possible, for instance, to avoid resonance phenomena or to generate a dynamic vibration absorption phenomenon. As is well known, vibration absorption can be used to suppress excessive forced vibration amplitudes.

5. CONCLUDING REMARKS

In this study, free transverse vibrations of an elastically connected rectangular plate–membrane system with a Pasternak layer in-between are theoretically analyzed. The vibratory system model considered comprises a three-layered structure which is composed of a thin plate, a Pasternak layer in-between, and a parallel membrane stretched uniformly by suitable constant tensions applied at the edges. The problem is solved by using the Navier method. It should be noted that the natural frequencies of the system may be varied by changing shear foundation modulus of the Pasternak layer without the necessity to vary parameters characterizing physical and geometrical properties of the system. This possibility is of great practical importance. Magnitudes of the frequencies become larger when the shear foundation modulus of the Pasternak increases and Pasternak layer can increase the magnitudes of frequencies more than Winkler elastic layer can. Thus in order to suppress the excessive vibrations of corresponding plate-membrane systems, one can use a Pasternak layer instead of Winkler elastic layer. The considered system has an interesting feature which enables to change each natural frequency within a certain limited interval as a function of the membrane tension and shear foundation modulus of the Pasternak layer only. With proper control of the membrane tension and shear foundation modulus of the Pasternak layer, it is possible to avoid a resonance phenomenon or to generate a dynamic vibration absorption phenomenon for the system subjected to harmonic loadings. This can have significant impacts in practical applications.

Nomenclature

w_i	**Transverse plate (membrane) displacement**
f_i	Exciting distributed load
x, y	Space co-ordinates
t	Time
D_1	Flexural rigidity of the plate
E_1	Young's modulus of elasticity of the plate
N_2	Uniform constant tension per unit length of the membrane
k	Winkler foundation modulus
G_0	Shear foundation modulus

a, b, h_i	Plate (membrane) dimensions
v_1	Poisson's ratio
ρ_i	Mass density
$S_{imn}(t), i = 1,2$	Unknown time functions
$W_{mn}(x, y)$	Known mode shape functions
$\Omega_{iimn}(i = 1,2)$	Partial frequency of the system
Ω_{120}	Coupling frequency of the system
ω_{mn}	Natural frequency of the system
δ_{klmn}	Kronecker delta function

REFERENCES

De Rosa M.A. 1995. Free vibrations of Timoshenko beams on two-parameter elastic foundation, Computers & Structures. 57: 151-156.

Oniszczuk Z. 2003. Free transverse vibrations of an elastically connected rectangular plate-membrane complex system. Journal of Sound and Vibration. 264: 37-47.

Oniszczuk Z. 2004. Forced transverse vibrations of an elastically connected complex rectangular simply supported double-plate system. Journal of Sound and Vibration. 270, 997-1011.

Pasternak P.L. 1954. On a new method of analysis of an elastic foundation by means of two foundation constants. (In Russian) Gosudarstvennoe Izdatelstvo Linteraturi Po Stroitelstvu, IArkhitektura, Moscow.

Stojanovic V., Kozic P. 2012. Forced transverse vibration of Rayleigh and Timoshenko double-beam system with effect of compressive axial load. International Journal of Mechanical Sciences. 60: 59-71.

Wang T.W., Stephens J.E. 1977. Natural frequencies of Timoshenko beams on Pasternak foundations. Journal of Sound and Vibration. 51: 149-155.

Zhang Y.Q., Lu Y., Wang S.L., Liu X. 2008a. Vibration and buckling of a double-beam system under compressive axial loading. Journal of Sound and Vibration. 318: 341-352.

Zhang Y.Q., Lu Y., Ma G.W. 2008b. Effect of compressive axial load on forced transverse vibrations of a double-beam system. International Journal of Mechanical Sciences. 50: 299-305.