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FORCED TRANSVERSE VIBRATION ANALYSIS OF AN ELASTICALLY CONNECTED RECTANGULAR DOUBLE-PLATE SYSTEM WITH A PASTERNAK MIDDLE LAYER

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ABSTRACT

Forced transverse vibrations of an elastically connected rectangular double-plate system with a Pasternak layer in-between are considered in this paper. Based on the Kirchhoff-Love plate theory, the general solutions of forced vibrations of the plates subjected to arbitrarily distributed continuous loads are found. The forced vibration problem is generally solved by the application of the modal expansion method for the case of simply supported boundary conditions for the plates. The effects of Pasternak layer on the forced vibrations of the double-plate system are discussed for the case of particular excitation loading. The dynamic responses of the system caused by arbitrarily distributed continuous loads are obtained. Vibrations caused by the harmonic exciting forces are discussed, and conditions of resonance and dynamic vibration absorption are formulated. Shear foundation modulus of the Pasternak layer doesn’t have any effect on the first frequencies, but has an efficient effect on the second frequencies. Thus the plate-type dynamic absorber with a Pasternak layer can be used to more effectively suppress the excessive vibrations of corresponding plate systems with respect to those with a Winkler elastic layer in-between. The numerical results of the present method are verified once compared with those available in the literature.

Keywords: forced transverse vibration, rectangular double-plate system, Kirchhoff-love plate theory, pasternak layer, modal expansion method.

1. INTRODUCTION

Plate-type and beam-type structures are widely used in many branches of civil, mechanical and aerospace engineering. An important technological extension of the concept of the single plate is that of the elastically connected double-plate system. Various problems of double-plate systems occupy an important place in many fields of structural and foundation engineering. In many structure interaction problems, the elastic foundation has been modeled by a Winkler elastic layer. It is also known that a plate and an elastic layer of a double-plate system can be considered as a continuous dynamic absorber to suppress the vibration of another plate subjected to a dynamic force. A double-plate is a kind of composite plate structure bonded together to act as a whole plate. Most vibration monographs devoted to distributed systems contain fundamental theory concerning transverse vibrations of a single rectangular plate.

Oniszczuk (2003a) analyzed undamped free transverse vibrations of an elastically connected rectangular plate-membrane system. Solutions of the problem are formulated by using the Navier method and natural frequencies of the system are determined in the form of two infinite sequences. Normal mode shapes of vibration expressing two kinds of vibration, synchronous and asynchronous, are also presented. In a numerical example, the effect of membrane tension on the natural frequencies of the plate-membrane system is discussed. Also, Oniszczuk (2003b) analyzed undamped forced transverse vibrations of an elastically connected complex double-beam system with a Winkler elastic layer in-between. The problem is formulated and solved for the case of simply supported beams and the classical modal expansion method is applied to ascertain dynamic responses of beams due to arbitrarily distributed continuous loads. Several cases of particularly interesting excitation loadings are investigated in this paper and the action of stationary harmonic loads and moving forces is considered. Oniszczuk (2004) studied forced transverse vibrations of an elastically connected rectangular double-plate system with a Winkler elastic layer in-between. The forced vibration problem is solved generally by the application of the modal expansion method for the case of simply supported boundary conditions for plates. On the basis of general solutions obtained, three particular cases of the action of exciting stationary harmonic loads are considered. An analysis of harmonic responses of the system makes it possible to determine conditions of resonance and dynamic vibration absorption. Also a numerical example is given to illustrate the theory presented.

Zhang *et al.* (2008a) studied vibration and buckling of a double-beam system under compressive axial loading. In their paper, the properties of free transverse vibration and buckling of a double beam system under compressive axial loading are investigated on the basis of the Bernoulli-Euler beam theory. They assumed that the two beams of the system are simply supported and

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continuously joined by a Winkler elastic layer. They derived explicit expressions for the natural frequencies. Also, they obtained the associated amplitude ratios of the two beams, and the analytical solution of the critical buckling load. It was shown in their paper that the critical buckling load of the system is related to the axial compression ratio of the two beams and the Winkler elastic layer, and the properties of free transverse vibration of the system greatly depend on the axial compressions. They also studied Effect of compressive axial load on forced transverse vibrations of a double-beam (Zhang *et al.*, 2008b). The effects of compressive axial load on the forced vibrations of the double-beam system are discussed for two cases of particular excitation loadings and the properties of the forced transverse vibrations of the system are found to be significantly dependent on the compressive axial load.

Stojanovic *et al.* (2012) considered forced transverse vibration and buckling of a Rayleigh and Timoshenko double-beam system continuously joined by a Winkler elastic layer under compressive axial loading. In their paper, deflections of the beams are shown and general solutions of forced vibrations of beams subjected to arbitrarily distributed continuous loads are found based on the Timoshenko beam theory. The analytical solution of forced vibration with associated amplitude ratios is determined. Also dynamic responses of the system caused by arbitrarily distributed continuous loads are obtained. Vibrations caused by the harmonic exciting forces are discussed, and conditions of resonance and dynamic vibration absorption are formulated.

The behavior of foundation materials in engineering practice cannot be represented by foundation model which consists of independent linear elastic springs. In order to find a physically close and mathematically simple foundation model, Pasternak proposed a so-called two-parameter foundation model with shear interactions. In an attempt to find a physically close and mathematically simple representation of an elastic foundation for these materials, Pasternak (1954) proposed a foundation model consisting of a Winkler foundation with shear interactions. This may be accomplished by connecting the ends of the vertical springs to a beam consisting of incompressible vertical elements, which deforms only by transverse shear.
In Wang *et al.* (1977) and De Rosa (1995), the natural vibrations of a Timoshenko beam on a Pasternak-type foundation are studied. Frequency equations are derived for beams with different end restraints. A specific example is given to show the effects of rotary inertia, shear deformation, and foundation constants on the natural frequencies of the beam.

Forced transverse vibrations of an elastically connected complex rectangular simply supported double-plate system with a Pasternak layer in-between are studied in the present paper.

2. STRUCTURAL MODEL AND FORMULATION OF THE PROBLEM

Figure-1 shows the structural model of a layered-plate system composed of two parallel rectangular plates of uniform properties with a Pasternak layer in-between. The plates are subjected to arbitrarily distributed transverse continuous loads f_1 and f_2 which are distributed over the entire surface of both plates. The assumption is that the two beams have the same effective material constants and both plates are thin, uniform, homogeneous and isotropic. The plates are governed by simply supported boundary conditions. The vibrations of the system with no damping are investigated.

The Forced transverse vibrations of an elastically connected complex rectangular simply supported double-plate system with a Pasternak layer in-between are described by the following set of two coupled non-homogeneous partial differential equations based on the Kirchhoff-Love plate theory:

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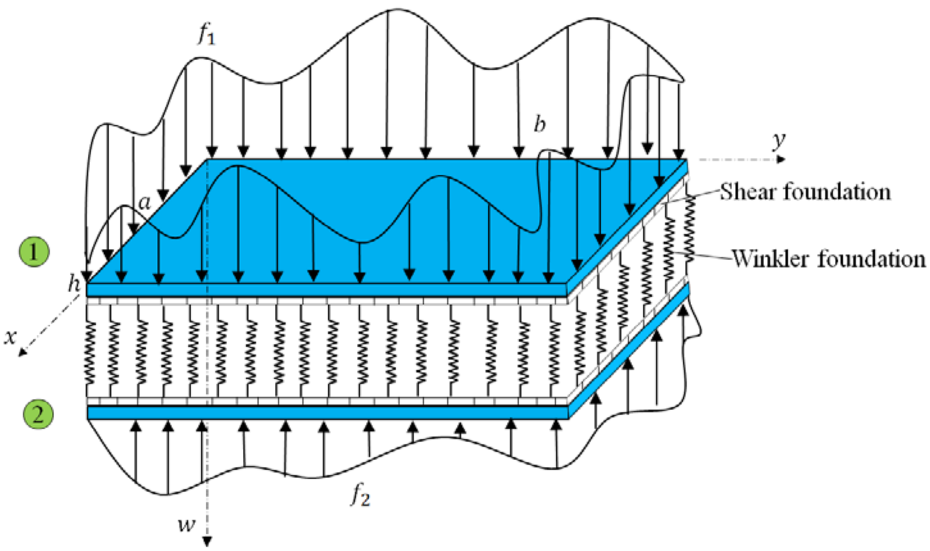

Figure-1. Double-plate system with a Pasternak middle layer.

$$D_1\Delta^2 w_1 + m_1\ddot{w}_1 + k(w_1 - w_2) - G_0\left[\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} - \left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right)\right] = f_1(x,y,t),$$
$$D_2\Delta^2 w_2 + m_2\ddot{w}_2 + k(w_2 - w_1) - G_0\left[\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} - \left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right)\right] = f_2(x,y,t) \quad (1)$$

where,

$$D_i = E_i\, h_i^3\,[12(1-v_i^2)]^{-1}\ ,\ \ m_i = \rho_i\, h_i\ ,\ \ \dot{w}_i = \frac{\partial w_i}{\partial t},$$

$$\Delta^2 w_i = \frac{\partial^4 w_i}{\partial x^4} + 2\frac{\partial^4 w_i}{\partial x^2 \partial y^2} + \frac{\partial^4 w_i}{\partial y^4}\ ,\ \ i = 1, 2$$

The general form of boundary conditions and initial conditions for simply supported plates are assumed as follows:

$$w_i(0,y,t) = w_i(a,y,t) = w_i(x,0,t) = w_i(x,b,t) = 0,$$
$$\left.\frac{\partial^2 w_i}{\partial x^2}\right|_{(0,y,t)} = \left.\frac{\partial^2 w_i}{\partial x^2}\right|_{(a,y,t)} = \left.\frac{\partial^2 w_i}{\partial y^2}\right|_{(x,0,t)} = \left.\frac{\partial^2 w_i}{\partial y^2}\right|_{(x,b,t)} = 0\ ,\ \ i = 1, 2 \quad (2)$$

and

$$w_i(x,y,0) = 0\ ,\ \ \dot{w}_i(x,y,0) = 0\ ,\ \ i = 1, 2 \quad (3)$$

3. SOLUTION OF EQUATIONS

In order to solve the non-homogeneous partial differential equation (1) representing forced vibrations of a double-plate system, the natural frequencies and the corresponding mode shapes of the system should be obtained by solving the undamped free vibration with appropriate boundary conditions. In such a general case of loading, the most proper and useful method to solve the problem is the modal expansion method. Applying the mentioned method, the forced responses of a double-plate system can be assumed in the following form:

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$$
\begin{aligned}
w_1(x,y,t) &= \sum_{m,n=1}^{\infty}\sum_{i=1}^{2} W_{1imn}(x,y)\ p_{imn}(t) = \sum_{m,n=1}^{\infty} W_{mn}(x,y)\sum_{i=1}^{2} p_{imn}(t)\\
&= \sum_{m,n=1}^{\infty} \sin(a_m x)\sin(b_n y)\sum_{i=1}^{2} p_{imn}(t)\,,\\
w_2(x,y,t) &= \sum_{m,n=1}^{\infty}\sum_{i=1}^{2} W_{2imn}(x,y)\ p_{imn}(t) = \sum_{m,n=1}^{\infty} W_{mn}(x,y)\sum_{i=1}^{2} a_{imn}\ p_{imn}(t)\\
&= \sum_{m,n=1}^{\infty} \sin(a_m x)\sin(b_n y)\sum_{i=1}^{2} a_{imn}\ p_{imn}(t)
\end{aligned}
\tag{4}
$$

Where, $W_{mn}(x,y)$ satisfying the modal equation $\Delta^2 W_{mn} = k_{mn}^4 W_{mn}$, is defined as

$$
\begin{aligned}
&W_{1imn}(x,y) = W_{mn}(x,y) = \sin(a_m x)\sin(b_n y)\,,\\
&W_{2imn}(x,y) = a_{imn}\, W_{mn}(x,y) = a_{imn}\ \sin(a_m x)\sin(b_n y)\,,\\
&a_m = a^{-1} m\pi\ ,\quad b_n = b^{-1} n\pi\,,\\
&k_{mn}^4 = (a_m^2 + b_n^2)^2 = \pi^4[(a^{-1}m)^2 + (b^{-1}n)^2]^2
\end{aligned}
$$

Substitution of the general solutions (4) into equation (1), results in the following relationships:

$$
\begin{aligned}
&\sum_{m,n=1}^{\infty}\Big\{W_{mn}\sum_{i=1}^{2}\{\,m_1\ddot{P}_{imn} + [k(1-a_{imn}) + G_0(a_m^2+b_n^2)\,(1-a_{imn})]P_{imn}\}\\
&\qquad + D_1\,\Delta^2 W_{mn}\sum_{i=1}^{2} P_{imn}\Big\} = f_1(x,y,t)\,,\\
&\sum_{m,n=1}^{\infty}\Big\{W_{mn}\sum_{i=1}^{2} a_{imn}\,\{m_2\ddot{P}_{imn} + [k(1-a_{imn}^{-1}) + G_0(a_m^2+b_n^2)\,(1-a_{imn}^{-1})]P_{imn}\}\\
&\qquad + D_2\,\Delta^2 W_{mn}\sum_{i=1}^{2} a_{imn}\ P_{imn}\Big\} = f_2(x,y,t)
\end{aligned}
\tag{5}
$$

Where,

$$
\begin{aligned}
a_{imn} &= [k + G_0(a_m^2+b_n^2)]^{-1}\,(D_1 k_{mn}^4 + k + G_0(a_m^2+b_n^2) - m_1\,\omega_{imn}^2)\\
&= [k + G_0(a_m^2+b_n^2)]\,(D_2 k_{mn}^4 + k + G_0(a_m^2+b_n^2) - m_2\,\omega_{imn}^2)^{-1}\\
&= \omega_{10}^{-2}\,(\omega_{11mn}^2 - \omega_{imn}^2) = \omega_{20}^{-2}\,(\omega_{22mn}^2 - \omega_{imn}^2)^{-1}
\end{aligned}
\tag{6}
$$

By using modal equation, the relations (5) will take the following form

$$
\begin{aligned}
&\sum_{m,n=1}^{\infty} W_{mn}\sum_{i=1}^{2}[\ddot{P}_{imn} + (\omega_{11mn}^2 - \omega_{10}^2\ a_{imn})P_{imn}] = m_1^{-1} f_1(x,y,t)\,,\\
&\sum_{m,n=1}^{\infty} W_{mn}\sum_{i=1}^{2}[\ddot{P}_{imn} + (\omega_{22mn}^2 - \omega_{20}^2\ a_{imn}^{-1})P_{imn}]\,a_{imn} = m_2^{-1} f_2(x,y,t)
\end{aligned}
\tag{7}
$$

Where,

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$$\omega_{i0}^2 = [k + G_0(a_m^2 + b_n^2)]\, m_i^{-1} = K\, M_i^{-1},$$
$$\omega_{iimn}^2 = [D_i\, k_{mn}^4 + k + G_0(a_m^2 + b_n^2)]\, m_i^{-1} = (abD_i\, k_{mn}^4 + K) M_i^{-1},$$
$$K = ab\,[k + G_0(a_m^2 + b_n^2)],$$
$$M_i = abm_i = abh_i\, \rho_i \;\;,\quad i = 1, 2 \;\;,\quad m, n = 1, 2, 3, ...$$

Substitution of expression (6) into equation (7) gives

$$\sum_{m,n=1}^{\infty} W_{mn} \sum_{i=1}^{2} (\ddot{P}_{imn} + \omega_{imn}^2\, P_{imn}) = m_1^{-1} f_1 \;\;,\;\; \sum_{m,n=1}^{\infty} W_{mn} \sum_{i=1}^{2} (\ddot{P}_{imn} + \omega_{imn}^2\, P_{imn})\, a_{imn} = m_2^{-1} f_2 \tag{8}$$

Where

$$\omega_{1,2mn}^2 = \frac{1}{2}\{(\omega_{11mn}^2 + \omega_{22mn}^2) \mp [(\omega_{11mn}^2 - \omega_{22mn}^2)^2 + 4\omega_{120}^4]^{1/2}\} \;\;,\quad \omega_{1mn} < \omega_{2mn} \tag{9}$$

Or

$$\omega_{1,2mn}^2 = \frac{1}{2}\left\{\begin{matrix} [D_1\, k_{mn}^4 + k + G_0(a_m^2 + b_n^2)]\, m_1^{-1} + [D_2 k_{mn}^4 + k + G_0(a_m^2 + b_n^2)]\, m_2^{-1} \\ \mp\{\{[D_1\, k_{mn}^4 + k + G_0(a_m^2 + b_n^2)]\, m_1^{-1} + [D_2 k_{mn}^4 + k + G_0(a_m^2 + b_n^2)]\, m_2^{-1}\}^2 \\ - 4k_{mn}^4\, [D_1\, D_2\, k_{mn}^4 + k + (D_1 + D_2) + G_0(a_m^2 + b_n^2)\,(D_1 + D_2)](m_1 m_2)^{-1}\}^{1/2} \end{matrix}\right\},$$
$$\omega_{120}^4 = \omega_{10}^2\, \omega_{20}^2 = [k + G_0(a_m^2 + b_n^2)]^2\, (m_1 m_2)^{-1}$$

By multiplying equation (8) by W_{kl} and then integrating over the plate surface and using orthogonality condition

$$\int_0^a\!\!\int_0^b W_{kl}\, W_{mn}\, \mathrm{d}x\, \mathrm{d}y = \int_0^a \sin(a_k x)\sin(a_m x)\;\mathrm{d}x \int_0^b \sin(b_l y)\sin(b_n y)\,\mathrm{d}y = c\,\delta_{klmn}\,,$$
$$c = c_{mn}^2 = \int_0^a\!\!\int_0^b W_{mn}^2 \mathrm{d}x\, \mathrm{d}y = \int_0^a \sin^2(a_m x)\,\mathrm{d}x \int_0^b \sin^2(b_m y)\,\mathrm{d}y = \frac{ab}{4} \tag{10}$$

We have

$$\sum_{i=1}^{2} (\ddot{P}_{imn} + \omega_{imn}^2\, P_{imn}) = (cm_1)^{-1} \int_0^a\!\!\int_0^b f_1\, W_{mn}\, \mathrm{d}x\, \mathrm{d}y\,,$$
$$\sum_{i=1}^{2} (\ddot{P}_{imn} + \omega_{imn}^2\, P_{imn})\, a_{imn} = (cm_2)^{-1} \int_0^a\!\!\int_0^b f_2\, W_{mn}\, \mathrm{d}x\, \mathrm{d}y\,. \tag{11}$$

After some algebra we obtain

$$\ddot{P}_{1mn} + \omega_{imn}^2\, P_{1mn} = K_{imn}(t)\;,\;\; i = 1, 2\;,\;\; m, n = 1, 2, 3, ... \tag{12}$$

Where,

$$K_{1mn}(t) = d_{1mn} \int_0^a\!\!\int_0^b [a_{2mn} M_1^{-1} f_1(x, y, t) - M_2^{-1} f_2(x, y, t)] \sin(a_m y)\sin(b_n y)\,\mathrm{d}x\, \mathrm{d}y\,,$$
$$K_{2mn}(t) = d_{2mn} \int_0^a\!\!\int_0^b [a_{1mn} M_1^{-1} f_1(x, y, t) - M_2^{-1} f_2(x, y, t)] \sin(a_m y)\sin(b_n y)\,\mathrm{d}x\, \mathrm{d}y\,, \tag{13}$$
$$d_{1mn} = -d_{2mn} = 4(a_{2mn} - a_{1mn})^{-1} = 4\omega_{10}^2 (\omega_{1mn}^2 - \omega_{2mn}^2)^{-1}$$

Their particular solutions satisfying homogeneous initial conditions (3) are as follows

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$$P_{imn}(t) = \omega_{imn}^{-1} \int_0^t K_{imn}(s) \sin[\omega_{imn}(t-s)]\,\mathrm{d}s \ , \ i = 1, 2 \ , \ m, n = 1, 2, 3, ... \tag{14}$$

The forced transverse vibrations of an elastically connected rectangular simply supported double-plate system with a Pasternak layer in-between can be described by

$$\begin{aligned} w_1(x,y,t) &= \sum_{m,n=1}^{\infty} \sin(a_m x)\sin(b_n y) \sum_{i=1}^{2} \omega_{imn}^{-1} \int_0^t K_{imn}(s)\sin[\omega_{imn}(t-s)]\,\mathrm{d}s \ , \\ w_2(x,y,t) &= \sum_{m,n=1}^{\infty} \sin(a_m x)\sin(b_n y) \sum_{i=1}^{2} a_{imn}\, \omega_{imn}^{-1} \int_0^t K_{imn}(s)\sin[\omega_{imn}(t-s)]\,\mathrm{d}s \end{aligned} \tag{15}$$

Now these general solutions (15) are used to find the vibrations of the double-plate system.

In the following, we conduct an analysis of forced vibrations for the case of harmonic concentrated force. For the sake of simplicity in further analysis, it is assumed that only one of the two plates is subjected to the exciting load, whilst the other one is not loaded (see Figure-2). The first plate is loaded by the concentrated harmonic force applied transversely at the point which position is described by the corresponding rectangular coordinates $x = x_0$ and $y = y_0$. Without loss of generality, we suppose:

$$f_1(x,t) = F\sin(pt)\,\delta(x-x_0)\,\delta(y-y_0) \ , \ f_2(x,t) = 0 \tag{16}$$

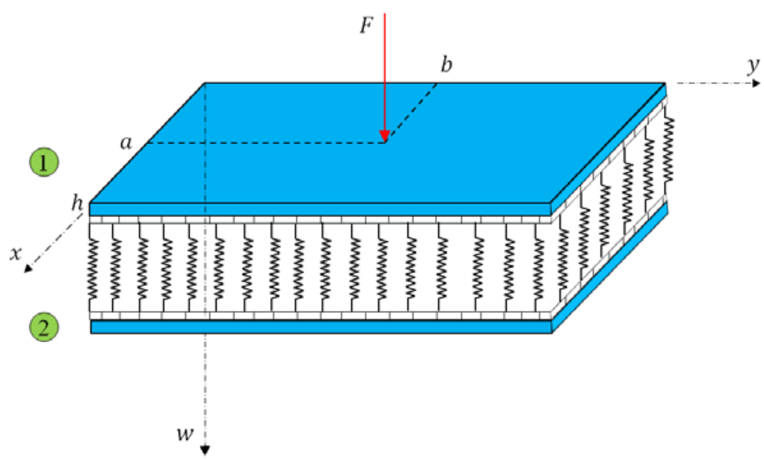

Figure-2. Double-plate system with a Pasternak layer in-between subjected to harmonic concentrated force.

Substituting equation (16) into equation (13), we can obtain

$$\begin{aligned} K_{imn}(t) &= c_{imn} \int_0^a\!\!\int_0^b F\sin(pt)\,\delta(x-x_0)\,\delta(y-y_0)\sin(a_m y)\sin(b_n y)\,\mathrm{d}x\,\mathrm{d}y \\ &= c_{imn}\, F \int_0^a \sin(a_m x)\,\delta(x-x_0)\,\mathrm{d}x \int_0^b \sin(b_n y)\,\delta(y-y_0)\,\mathrm{d}y = F\, c_{mn} \ , \ i = 1, 2 \end{aligned} \tag{17}$$

Where,

$$c_{mn} = \sin(a_m x_0)\sin(b_n x_0) = \sin(a^{-1} m\pi\, x_0)\sin(b^{-1} n\pi\, x_0) \ , \ m, n = 1, 2, 3, ...$$

Substituting equation (17) into equation (15) gives

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$$w_1(x,t)=\sum_{m,n=1}^{\infty}\sin(a_m x)\sin(b_n x)\left[A_{1mn}\sin(pt)+\sum_{i=I}^{II}B_{imn}\sin(\omega_{imn}t)\right],$$
$$w_2(x,t)=\sum_{m,n=1}^{\infty}\sin(a_m x)\sin(b_n x)\left[A_{2mn}\sin(pt)+\sum_{i=I}^{II}a_{imn}\,B_{imn}\sin(\omega_{imn}t)\right] \quad (18)$$

Where,

$$A_{1mn}=4F\,c_{mn}\,M_1^{-1}(\omega_{22mn}^2-p^2)\,[(\omega_{1mn}^2-p^2)\,(\omega_{2mn}^2-p^2)]^{-1},$$
$$A_{2mn}=4F\,c_{mn}\,M_1^{-1}\,\omega_{20}^2\,[(\omega_{1mn}^2-p^2)\,(\omega_{2mn}^2-p^2)]^{-1},$$
$$B_{1mn}=4F\,a_{2mn}\,c_{mn}\,M_1^{-1}\,p\,[(a_{1mn}-a_{2mn})\,\omega_{1mn}(\omega_{1mn}^2-p^2)]^{-1},$$
$$B_{2mn}=4F\,a_{1mn}\,c_{mn}\,M_1^{-1}\,p\,[(a_{1mn}-a_{2mn})\,\omega_{2mn}\,(\omega_{1mn}^2-p^2)]^{-1}$$

Ignoring the free response, the forced vibrations of the double-plate system can be obtained by

$$w_1(x,y,t)=\sin(pt)\sum_{m,n=1}^{\infty}A_{1mn}\sin(a_m x)\sin(b_n y),$$
$$w_2(x,y,t)=\sin(pt)\sum_{m,n=1}^{\infty}A_{2mn}\sin(a_m x)\sin(b_n y) \quad (19)$$

The following fundamental conditions of resonance and dynamic vibration absorption have practical significance:

1- Resonance

$$p=\omega_{imn}\ ,\ \ i=1,2\ ,\ \ m,n=1,2,3,...$$

2- Dynamic vibration absorption

$$p^2=p_{mn}^2=\omega_{22mn}^2=[Dk_{mn}^4+k+G_0(a_m^2+b_n^2)]\,m_2^{-1}=(abDk_{mn}^4+K)M_2^{-1}$$
$$A_{1mn}=0\ ,\ \ A_{2mn}=-4F_{mn}\,K^{-1}\ ,\ \ m,n=1,2,3,...$$

4. NUMERICAL RESULTS AND DISCUSSIONS

The values of the parameters characterizing properties of the system are shown in Table-1.

Table-1. Values of the parameters characterizing properties of the system $(i=1,2)$.

a	b	$E=E_i$	$h=h_i$	k
1 m	2 m	1×10^{10} Nm^{-2}	1×10^{-2} m	0.6×10^5 Nm^{-3}
$K=abk$	$\rho=\rho_i$	$m_0=m_i=\rho h$	$M=M_i=abm_i$	$\nu=\nu_i$
1.2×10^5 Nm^{-1}	5×10^3 kgm^{-3}	0.5×10^2 kgm^{-2}	1×10^2 kg	0.3

In Table-2 the effects of the Pasternak layer on the frequencies are shown. An evident influence of the shear foundation modulus of the Pasternak layer on the frequencies of the system is observed and it can be seen that by increasing G_0 the frequencies increase. However, this influence of the shear foundation modulus of the Pasternak layer is different on some particular frequencies, being stronger on the frequencies ω_{2mn} while weaker on ω_{1mn} frequencies. If we set $G_0=0$ in all equations through this paper, we can obtain the vibration equations of the double-plate system with a Winkler elastic layer in-between, the results of which are verified by comparing with those available in Oniszczuk (2004).

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Table-2. Natural frequencies of rectangular double-plate system ω_{imn} for different shear modulus of the Pasternak layer.

	G_0		G_0			
	0	**100**	**200**	**300**	**400**	**500**
ω_{111}	52.7975	52.7975	52.7975	52.7975	52.7975	52.7975
ω_{211}	72.0248	72.3666	72.7068	73.0453	73.3823	73.7178
ω_{112}	84.476	84.476	84.476	84.476	84.476	84.476
ω_{212}	97.6535	98.0569	98.4587	98.8588	99.2574	99.6543
ω_{113}	137.274	137.274	137.274	137.274	137.274	137.274
ω_{213}	145.753	146.193	146.631	147.068	147.503	147.938
ω_{121}	179.512	179.512	179.512	179.512	179.512	179.512
ω_{221}	186.076	186.527	186.976	187.424	187.871	188.317
ω_{122}	211.19	211.19	211.19	211.19	211.19	211.19
ω_{222}	216.798	217.252	217.706	218.159	218.611	219.062
ω_{123}	263.988	263.988	263.988	263.988	263.988	263.988
ω_{223}	268.495	268.954	269.412	269.87	270.326	270.782
ω_{131}	390.702	390.702	390.702	390.702	390.702	390.702
ω_{231}	393.761	394.224	394.687	395.15	395.611	396.073
ω_{132}	422.38	422.38	422.38	422.38	422.38	422.38
ω_{232}	425.212	425.676	426.139	426.602	427.064	427.526
ω_{133}	475.178	475.178	475.178	475.178	475.178	475.178
ω_{233}	477.696	478.161	478.625	479.089	479.552	480.015

The resonant diagram of steady state harmonic response of the system is presented for component amplitudes $A_{imn}(i=1,2, \quad m,n=1,3)$ in Figures-3 and 4 for $G_0 = 500$, respectively.

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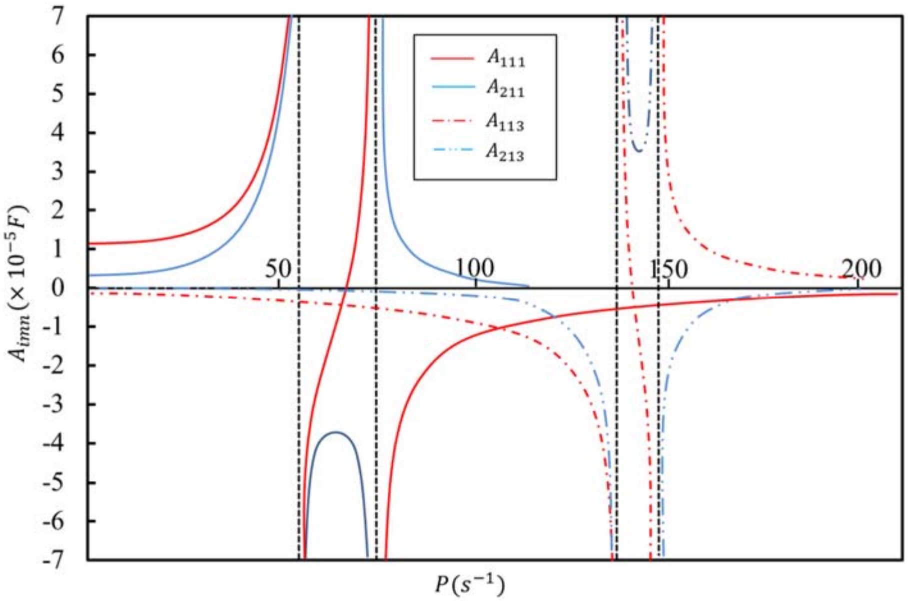

Figure-3. The resonant diagram of the steady state forced harmonic vibrations for component amplitudes $A_{111}, A_{211}, A_{113}, A_{213}$ of a double-plate system with Pasternak layer in-between for $G_0 = 500$.

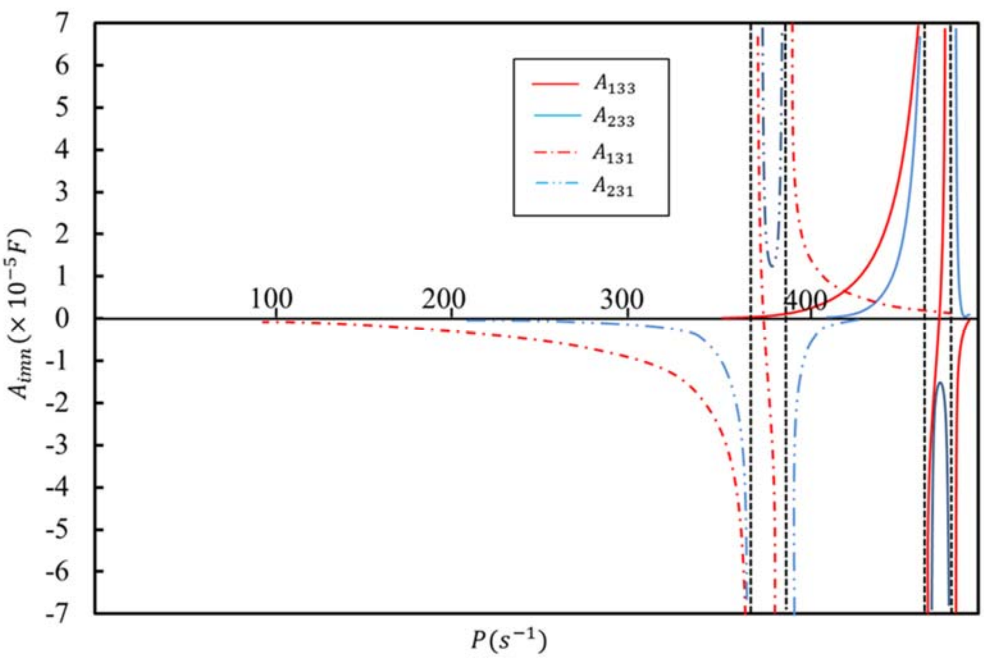

Figure-4. The resonant diagram of the steady state forced harmonic vibrations for component amplitudes $A_{131}, A_{231}, A_{133}, A_{233}$ of a double-plate system with Pasternak layer in-between for $G_0 = 500$.

It shows the progress of component amplitudes A_{imn} as a function of the excitation frequency p. It can be clearly seen from the figures when the resonance occurs and when the system acts as a dynamic absorber. This can help to prevent resonance conditions and the resulting consequences. The dynamic absorption phenomenon can be used to reduce excessive forced harmonic vibrations of elastically connected double-plate systems.

5. CONCLUSIONS

Based on Kirchhoff-Love plate theory, forced transverse vibrations of an elastically connected rectangular simply supported double-plate system with a Pasternak layer in-between are studied. The dynamic response of the system caused by arbitrarily distributed continuous loads is obtained. General solutions of the problem formulated for isotropic, thin plates subjected to arbitrarily distribute continuous loads are found by applying the classical modal expansion method. The effects of the shear foundation modulus of the Pasternak layer on the forced vibrations of the double-plate system are discussed for the case of particular excitation loadings. As is seen, in an elastically connected double-plate system, when the first plate (main plate) is subjected to an exciting harmonic load, the second plate can act like a dynamic vibration absorber in relation to the first one. Shear foundation modulus of the Pasternak layer doesn't have any effect on the first frequencies, but has an efficient effect on the second ones. Thus the plate-type dynamic absorber with a Pasternak layer can be used to more effectively suppress the excessive vibrations of corresponding plates systems rather than those with a Winkler elastic layer in-between. However, it should be noted that the continuous absorber only reduces the forced vibrations of the first plate but never liquidates them absolutely. The plate-type dynamic absorber is an accepted concept for a Continuous Dynamic Vibration Absorber (CDVA).

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