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DESIGN OF SUBOPTIMAL AND OPTIMAL SU-MIMO SYSTEM WITH IMPROPER MODULATIONS USING PER ANTENNA POWER CONSTRAINT AND PERFECT CSI

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ABSTRACT

Achieving good bit error rate performance without considering practical constraint is a major concern of the industries in the design of MIMO transcevier system. In this paper two type of Single-User Multiple-Input, Multiple-Output (SU-MIMO) transceiver which employing one dimensional improper modulations are designed by considering the practical constraint and the perfect channel state information, Finally the performance of the both the design are analyzed in terms of Bit Error Rate(BER). From the result the system which providing low BER named as optimal SU-MIMO system and which providing high BER is named as suboptimal SU-MIMO System.

Keywords: MIMO, PAPC, CSI, MSE, SPC, BER.

INTRODUCTION

MIMO tansceiver is designed based on several design criteria and its output is verified by many measurements. Among the all available performance measurements, TMSE condition provided better bit error rate performance (Yang, J and Roy, S, 1994; Scaglione, A *et al*., 2002). Hence, TMSE is preferred over others. In general, the power is allocated based on the sum power constraint (SPC), at the transmitter, to maximize the receive signal to noise ratio (SNR). But the SPC does not take into account of power constraint of individual power amplifier (PA) at each transmit antenna (Lo, Titus. K. Y, 1999; Zhang, X *et al*., 2007). The problem is formulated into a non-convex optimization problem by using the newly proposed p-norm constraint which jointly meets both the per-antenna power constraint (PAPC) and the sum power constraint(SPC) to bound the dynamic range of the power amplifier at each transmit antenna (Feiten, A. *et al*., 2007). The minimum TMSE transceiver with SPC design is proposed (Ding, M and Blostein, S.D, 2009). Then the minimum TMSE precoding design for improper modulations was proposed and obtained Better BER performance than the conventional transceiver design (Xiao, P and Mathini, S, 2010). Transceiver system employing one dimensional improper modulation with SPC is proposed, it taking only real part of the output for decision making (Raja, M. *et al*., 2013). The minimum TMSE transceiver with per antenna power constraint is implemented. In that both real and imaginary part of the output is taken for decision making for the imperfect case. (Merline, A.and Thiruvengadam, S.J., 2013). Further to improve the performance, The optimal system with per antenna power constraint is proposed for the perfect case and its output is verified with suboptimal system.

PROBLEM FORMULATION

a) Suboptimal precoder and decoder design for MIMO system

In this methodology both real and imaginary part of its output is taken for the decision. The symbol estimation error is defined as follows,

$$e = \hat{s} - s \tag{1}$$

$$\hat{s} = GHFs + Gn \tag{2}$$

$$e = E[\|\hat{s} - s\|^2] = E[\|(GHFs + Gn) - s\|^2] \tag{3}$$

$$Tr\{E[[(GHFs + Gn) - s][(s^H F^H H^H G^H + n^H G^H) - s^H]]\} \tag{4}$$

Taking statistics of the channel, noise and data into consideration, we have

$$E[ss^H] = E[ss^T] = I_B$$
$$E[nn^H] = \sigma_n^2 I_{N_T} \; and \tag{5}$$
$$E[n] = E[nn^T] = E[n^* n^H] = 0$$

After expansion of equation (4) we get

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$$Tr\{E[GHFss^H F^H H^H G^H + GHFsn^H G^H - GHFss^H + Gns^H F^H H^H G^H + Gnn^H G^H - Gns^H - ss^H F^H H^H G^H - sn^H G^H + ss^H]]\} \quad (6)$$

Apply the conditions from equation (5)

$$Tr[GHFF^H H^H G^H - GHF + \sigma_n^2 GG^H - F^H H^H G^H + I_B] \quad (7)$$

The design target is to find a couple of matrices F and G to minimize $E[\|e\|^2]$ subject to p-norm. i.e. is

$$\min_{F,G} E[\|e\|^2] \quad subject \ to \quad [Tr(FF^H)^p]^{1/p} \le \alpha \quad (8)$$

Here in the above equation α is a constant and p is also a constant. The value of p is calculated based on the SPC and PAPC. Mathematically p is defined as follows,

$$\frac{\alpha B}{\beta^{1/p}} = \beta$$

$$\beta^{1/p} = \frac{\alpha B}{\beta}$$

$$\ln \beta^{1/p} = \ln \frac{\alpha B}{\beta}$$

$$p = \frac{\ln \beta}{\ln \frac{\alpha B}{\beta}} \quad (9)$$

If p=1 then the constant α will be equal to the SPC, β. when $p = \infty$ then it corresponds to an EPA scheme with the per antenna constraint α=β/B, where B is the number of bit streams. For p in the interval $1 < p < \infty$, p-norm constraint sufficiently meets both the SPC and PAPC. The formulation in the equation can be referred to as minimum TMSE design for SU-MIMO systems with per antenna power constraint.

To obtain the solution of the above issue, form the Lagrangian.

$$\eta = E\left\|e^2\right\| + \mu[Tr(FF^H)^p]^{1/p}) - \alpha \quad (10)$$

μ is the Lagrange multiplier.

Taking the derivatives of η concerning F and G, the associated Karush-Kuhn-Tucker (KKT) conditions can be determined by utilizing the cyclic property of the following function.

$$\frac{\partial \eta}{\partial G} = 0$$
$$= [\frac{\partial Tr(GHFF^H H^H G^H)}{\partial G} - \frac{\partial Tr(GHF)}{\partial G} + \frac{\partial Tr(\sigma_n^2 GG^H)}{\partial G} - \frac{\partial Tr(F^H H^H G^H)}{\partial G} + \frac{\partial Tr(I_B)}{\partial G}] + [\frac{\partial Tr(\mu[Tr(FF^H)^p]^{1/p} - \alpha)}{\partial G}] \quad (11)$$

$$G^* H^* F^* F^T H^T - F^T H^T + \sigma_n^2 G^* = 0 \quad (12)$$

Applying Complex conjugate on both sides, we get

$$GHFF^H H^H - F^H H^H + \sigma_n^2 G = 0 \quad (13)$$

$$GHFF^H H^H + \sigma_n^2 G = F^H H^H \quad (14)$$

Similarly

$$\frac{\partial \eta}{\partial F} = 0 = [\frac{\partial Tr(GHFF^H H^H G^H)}{\partial F} - \frac{\partial Tr(GHF)}{\partial F} + \frac{\partial Tr(\sigma_n^2 GG^H)}{\partial F} - \frac{\partial Tr(F^H H^H G^H)}{\partial F} + \frac{\partial Tr(I_B)}{\partial F}] + [\frac{\partial Tr(\mu[Tr(FF^H)^p]^{1/p} - \alpha)}{\partial F}] \quad (15)$$

$$G^* H^* F^* H^T G^T - H^T G^T + \mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1} F^*] = 0 \quad (16)$$

$$G^* H^* F^* H^T G^T + \mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1} F^*] = H^T G^T \quad (17)$$

where the partial derivative of $[Tr(FF^H)^p]^{1/p}$ with respect to F is obtained using the Chain rule of Matrix differentiation and the following properties (Hjorungers, A *et al*., 2007).

$$\frac{\partial g(U)}{\partial F} = Tr[(\frac{\partial g(U)}{\partial U})^T \frac{\partial g(U)}{\partial F}]$$

$$Tr(\frac{\partial g(U)}{\partial F}) = \frac{\partial Tr(g(U))}{\partial F}$$

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$$\frac{\partial Tr(F^p)}{\partial F} = p(F^T)^{p-1}$$

Applying Complex conjugate on both sides, we get

$$GHFH^H G^H + \mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1}F^*]^* = H^H G^H \quad (18)$$

Post multiplying equation (14) by G^H we have,

$$GHFF^H H^H G^H + \sigma_n^2 GG^H = F^H H^H G^H \quad (19)$$

Pre multiplying equation (18) by F^H we have,

$$GHFF^H H^H G^H + \mu F^H[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1}F^*]^* = F^H H^H G^H \quad (20)$$

Equating equations (19) and (20)

$$\sigma_n^2 GG^H = \mu F^H[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1}F^*]^*$$

$$= \mu F^H\left[\frac{[Tr(FF^H)^p]^{1/p}}{Tr(FF^H)^p}((FF^H)^T)^{p-1}F^*\right]^*$$

$$= \mu F^H\left[\frac{[Tr(FF^H)^p]^{1/p}}{Tr(FF^H)^p}\frac{(F^T F^*)^p}{(F^T F^*)}F^*\right]^*$$

$$= \mu\left[\frac{[Tr(FF^H)^p]^{1/p}}{Tr(FF^H)^p}\frac{(F^T F^*)^p}{(F^T F^*)}F^T F^*\right]^*$$

Considering α= $Tr(FF^H)^p]^{1/p}$ we get,

$$\sigma_n^2 GG^H == \mu\left[\frac{\alpha}{Tr(FF^H)^p}\frac{(F^T F^*)^p}{(F^T F^*)}F^T F^*\right]^*$$

Applying trace on both sides, we get

$$\mu = \frac{\sigma_n^2 Tr(GG^H)}{\alpha} \quad (21)$$

$$where,\ \alpha = [Tr(FF^H)^p]^{1/p}$$

Using equation (14), we derive

$$GHFF^H H^H + \sigma_n^2 G = F^H H^H$$

$$G(HFF^H H^H + \sigma_n^2) = F^H H^H$$

$$G = F^H H^H\left[(HFF^H H^H + \sigma_n^2)\right]^{-1} \quad (22)$$

Using equation (18), we derive

$$GHFH^H G^H + \mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1}F^*]^* = H^H G^H$$

$$GHFH^H G^H + \mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1}]^* F = H^H G^H$$

$$F[GHH^H G^H + \mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1}]^* = H^H G^H$$

$$F = H^H G^H\left[\begin{matrix} GHH^H G^H \\ + \mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1}]^* \end{matrix}\right]^{-1} \quad (23)$$

b) Optimal precoder and decoder design for MIMO system

In this methodology only the real part of its output is taken for the decision making. It will bring an enhanced BER performance

The TMSE can be calculated as follows,

$$E[\|e^2\|] = E[\|\hat{s} - s^2\|] \quad (24)$$

$$= E[\|\mathrm{R}(GHFs + Gn) - s^2\|] \quad (25)$$

$$= E[\|(GHFs + G^* H^* F^* s^*)/2 + (Gn + G^* n^*)/2 - s\|^2] \quad (26)$$

$$\begin{aligned} Tr\{E[[0.5(GHFs + G^*H^*F^*s^*) + 0.5(Gn + G^*n^*) \\ - s][0.5(S^H F^H H^H G^H + S^T F^T H^T G^T) \\ + 0.5(n^H G^H + n^T G^T) - s^H]]\} \end{aligned} \quad (27)$$

$$\begin{aligned} = Tr\{0.25(GHFF^H H^H G^H + GHFF^T H^T G^T \\ + G^* H^* F^* F^H H^H G^H + G^* H^* F^* F^T H^T G^T) \\ - 0.5(GHF + G^* H^* F^* + F^H H^H G^H \\ + F^T H^T G^T) + I_B + 0.25\sigma_n^2(GG^H + G^* G^T)\} \end{aligned} \quad (28)$$

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The design goal is to find optimum F and G which minimize the mean square error subject to SPC and per antenna power constraint. Mathematically it can be defined as

$$\min_{F,G} E[\|e\|^2] \quad subject\ to \quad [Tr(FF^H)^p]^{1/p} \le \alpha \tag{29}$$

To obtain the solution of the above matter, form the Lagrangian.

$$\eta = E\left\|e^2\right\| + \mu[Tr(FF^H)^p]^{1/p}) - \alpha$$

Then taking the derivatives of η with respect to F and G, the related Karush-Kuhn-Tucker (KKT) conditions can be acquired and given in the following.

$$\frac{\partial \eta}{\partial G} = 0$$

$$\begin{aligned} &0.25[G^*(HFF^H H^H)^T + GHFF^T H^T + GHFF^T H^T \\ &+ G^* H^* F^* F^T H^T] - 0.5[F^T H^T + F^T H^T] \\ &+ 0.25\sigma_n^2 (G^* + G^*) = 0 \end{aligned} \tag{30}$$

$$\begin{aligned} &0.25[G^* H^* F^* F^T H^T + 2GHFF^T H^T + \\ &G^* H^* F^* F^T H^T] - 0.5[2F^T H^T] + 0.5[\sigma_n^2 G^*] = 0 \end{aligned} \tag{31}$$

$$\begin{aligned} &G^* H^* F^* F^T H^T + GHFF^T H^T - 2F^T H^T \\ &+ \sigma_n^2 G^* = 0 \end{aligned} \tag{32}$$

Taking complex conjugates on both sides, we get

$$GHFF^H H^H + G^* H^* F^* F^H H^H + \sigma_n^2 G = 2F^H H^H \tag{33}$$

Now setting,

$$\frac{\partial \eta}{\partial F} = 0$$

$$\begin{aligned} &0.25[(H^H G^H GHF)^* + 2H^T G^T GHF + H^T G^T G^* H^* F^*] \\ &- 0.5[H^T G^T + H^T G^T] \\ &+ \mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1} F^*] = 0 \end{aligned}$$

Again, taking the complex conjugates of both sides, we get

$$\begin{aligned} &H^H G^H GHF + H^H G^H G^* H^* F^* \\ &+ 2\mu[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1} F^*]^* = 2H^H G^H \end{aligned} \tag{35}$$

Next, by post multiplying (33) by G^H

$$\begin{aligned} &GHFF^H H^H G^H + G^* H^* F^* F^H H^H G^H \\ &+ \sigma_n^2 GG^H = 2F^H H^H G^H \end{aligned} \tag{36}$$

Next, by pre multiplying (35) by F^H

$$\begin{aligned} &F^H H^H G^H GHF + F^H H^H G^H G^* H^* F^* \\ &+ 2\mu F^H [[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1} F^*]^* \\ &= 2F^H H^H G^H \end{aligned} \tag{37}$$

Equating the equations (36) and (37), we get

$$\sigma_n^2 GG^H = 2\mu F^H [[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1} F^*]^*$$

Applying trace on both sides, we get

$$\mu = \frac{\sigma_n^2 Tr(GG^H)}{2\alpha} \quad where, \alpha = [Tr(FF^H)^p]^{1/p} \tag{38}$$

An iterative procedure is developed to find the solutions; we define from Equation (33)

$$G = G_{Re} + jG_{Im} \quad and \quad G^* = G_{Re} - jG_{Im} \tag{39}$$

$$HFF^H H^H = A_{Re} + jA_{Im} \tag{40}$$

$$H^* F^* F^H H^H = B_{Re} + jB_{Im} \tag{41}$$

$$2F^H H^H = C_{Re} + C_{Im} \tag{42}$$

A, B, C represents the terms in the equation

$$\begin{aligned} &(C_{Re} \quad C_{Im}) = \\ &(G_{Re} \quad G_{Im}) \begin{pmatrix} A_{Re} + B_{Re} + \sigma_n^2 I_{NR} & A_{Im} + B_{Im} \\ B_{Im} - A_{Im} & A_{Re} - B_{Re} + \sigma_n^2 I_{NR} \end{pmatrix} \end{aligned} \tag{43}$$

The above equation can be rewritten as,

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$$(G_{Re} \quad G_{Im}) = (C_{Re} \quad C_{Im}) \begin{pmatrix} A_{Re}+B_{Re}+\sigma_n^2 I_{NR} & A_{Im}+B_{Im} \\ B_{Im}-A_{Im} & A_{Re}-B_{Re}+\sigma_n^2 I_{NR} \end{pmatrix}^{-1} \quad (44)$$

Similarly, we define from Equation (35)

$$F = F_{Re} + jF_{Im} \quad and \quad F^* = F_{Re} - jF_{Im} \quad (45)$$

$$H^H G^H G H = P_{Re} + jP_{Im} \quad (46)$$

$$H^H G^H G^* H^* = Q_{Re} + jQ_{Im} \quad (47)$$

$$2H^H G^H = R_{Re} + R_{Im} \quad (48)$$

Assuming k= $[[Tr(FF^H)^p]^{(1/p)-1}((FF^H)^T)^{p-1}]^*$
P, Q, R represents the terms in the equation.

$$\begin{pmatrix} R_{Re} \\ R_{Im} \end{pmatrix} = \begin{pmatrix} P_{Re}+Q_{Re}+2\mu k I_{NT} & Q_{Im}-P_{Im} \\ P_{Im}+Q_{Im} & P_{Re}-Q_{Re}+2\mu k I_{NT} \end{pmatrix} \begin{pmatrix} F_{Re} \\ F_{Im} \end{pmatrix} \quad (49)$$

The above equation can be rewritten as

$$\begin{pmatrix} F_{Re} \\ F_{Im} \end{pmatrix} = \begin{pmatrix} P_{Re}+Q_{Re}+2\mu k I_{NT} & Q_{Im}-P_{Im} \\ P_{Im}+Q_{Im} & P_{Re}-Q_{Re}+2\mu k I_{NT} \end{pmatrix}^{-1} \begin{pmatrix} R_{Re} \\ R_{Im} \end{pmatrix} \quad (50)$$

Based on the above expressions, an iterative approach is obtained for updating the precoder matrix F and decoder matrix G using per antenna power allocation.

ITERATIVE ALGORITHM

Step-1: Initialize $F = F_0$ the upper matrix of F_0 is choosen to be scaled identity, while the remaining entries of F_0 are set to zero.

Step-2: Update G using (equation 22 and 44 for suboptimal and optimal design).

Step-3: Update μ using (equation 21 or 38 for suboptimal and optimal design).

Step-4: Update F using (equation 23 and 50 for suboptimal and optimal design), if $[Tr(FF^H)^p]^{1/p} > \alpha$. Scale such that $[Tr(FF^H)^p]^{1/p} = \alpha$.

Step-5: If $[Tr((F_i - F_{i-1})(F_i - F_{i-1})^H)^p]^{1/p}$ is sufficiently less than 10^{-4} stop.otherwise go back to Step-2. Here $F_i, (F_{i-1})$ denotes F in the i-th and (i-1)-th iteration.

NUMERICAL RESULTS

The above problem formulation is simulated in MATLAB and the results are as follows.Here, N_T = Number of transmit antennas , N_R = Number of receive antennas and B=Number of bit streams.In all the below figures, SNR is defined as $P_T / \sigma_n{}^2$ For the purpose of simulation, SNR is taken to be 26.016 dB.The value of 'p' is calculated as follows, $p = \frac{\ln \beta}{\ln \frac{\alpha B}{\beta}}$

Considering, B=4, PAPC $\alpha = 1.1W$ and SPC $\beta = 3.16\,W$ we get,

$$p = \frac{ln(3.16)}{ln(\frac{(1.1)(4)}{3.16})}$$

p = 4.12.

First, Figure-1 compares the performance of optimal transceiver design employing one dimensional improper modulation using per antenna constraint with the sub optimal transceiver design for the values of N_T = 4, N_R =4,B =4 and p=4.12. It is clearly observed that, optimal design has superior Bit error rate performance compared to the suboptimal design.

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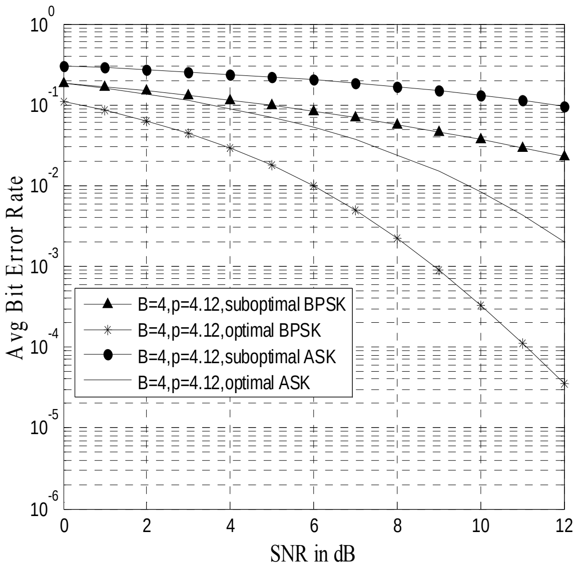

Figure-1. Performance comparison of sub optimal and optimal transceiver system when $N_T = 4$, $N_R = 4$, B =4 and p=4.12.

Figure-2 compares the optimal design by varing B value as B=3 and 4. It is observed that the B=3 provided better BER performance than B=4. It adds the worth of spatial diversity in MIMO systems.

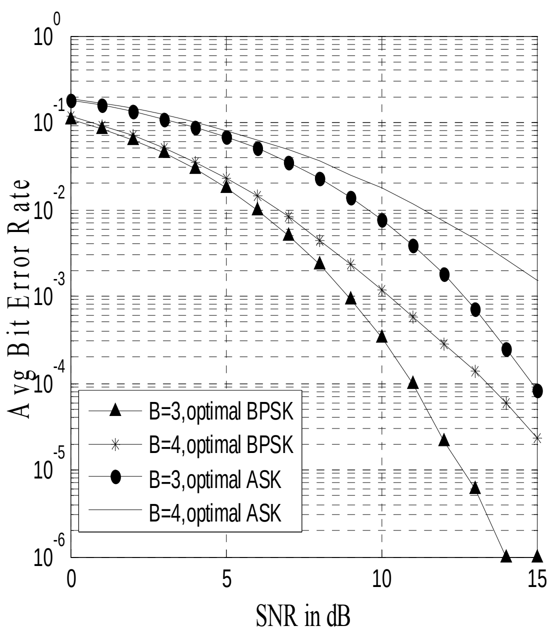

Figure-2. Performance Comparison optimal transceiver system when $N_T = 4$, $N_R = 4$, p=4.12, B=4 and B=3.

Figure-3 and Figure-4 explains the importance of MIMO by showing the Bit error rate performances for different values of $N_T \times N_R$. It can be seen that, the graph follows a rising pattern of bit-error rate performances from 1 to 4 in the cases of both BPSK and 4-ASK.

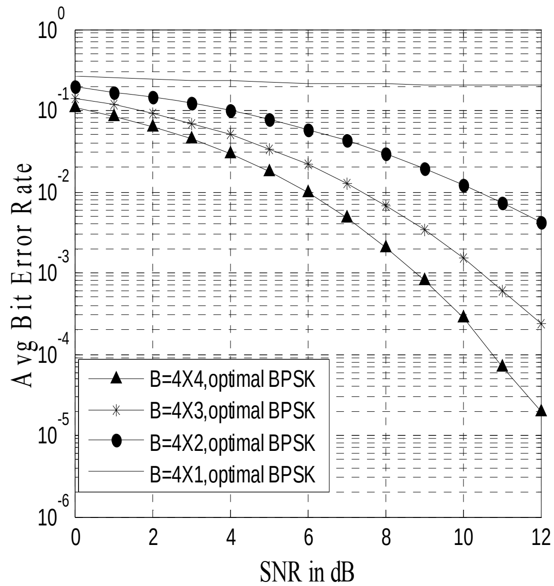

Figure-3. Performance Comparison optimal transceiver system with BPSK Modulator for B=4, p=4.12, $N_T = 4$ and N_R =1, 2, 3, 4.

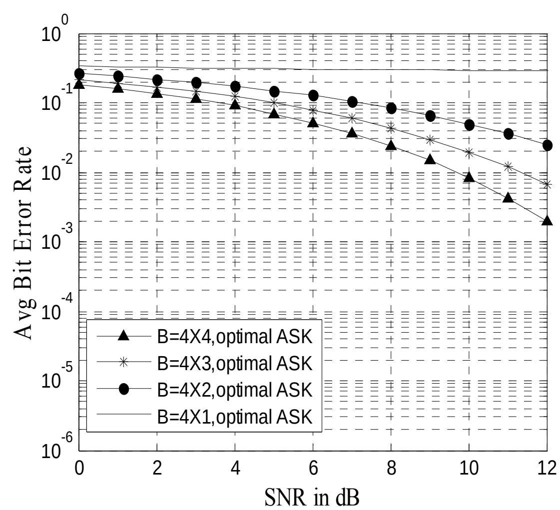

Figure-4. Performance Comparison optimal transceiver system with ASK Modulator for B=4, p=4.12, $N_T = 4$ and N_R =1, 2, 3, 4.

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CONCLUSIONS

In this paper, performance analysis have been done for both optimal and suboptimal transceiver designs with Improper modulations using p-norm constraint and perfect CSI. The results shows that the optimal design with per antenna conatraint has a superior BER performance than suboptimal design. At last, it is pointed out that the work proposed in this paper can be stretched out to imperfect case of Single-User Multiple-Input Multiple-Output (SU-MIMO) system.

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