COMPARISON OF FUZZY AHP AND TOPSIS EVALUATIONS OF LUGGAGE BAG DESIGN ALTERNATIVES

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ABSTRACT

Selection of design concepts is an area of design research that has been under considerable interest over the years. It has become a very critical activity to the performance of organizations and supply chains. Studies presented in the literature propose the use of the various Multi Criteria Decision Making (MCDM) approaches. This paper presents a comparative analysis of Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and Fuzzy Analytic Hierarchy Process (Fuzzy AHP) in the context of decision making for design selection. The comparison was made based on the factors: cost; quality; ergonomics and environmental factors. To illustrate the proposed model, a few design concepts of luggage bags are considered and the most appropriate one is determined. In addition, rank correlation is carried out to analyse the order of rankings obtained by TOPSIS and Fuzzy AHP techniques. This paper contributes to helping research practitioners as well as industrial decision makers to choose the more appropriate MCDM techniques for design selection.

Keywords: MCDM, fuzzy analytical hierarchy process, TOPSIS, luggage bags, rank correlation.

1. INTRODUCTION

The level of success of product designs achieved depends, significantly, on the conceptual design stage. Inappropriate decision making of effective design selection during the conceptual design stage. It may cause the product to be redesigned or remanufactured. To overcome such a problem, this paper proposes the use of multi criteria decision making (MCDM) approaches [10] [15]. These techniques can be helpful to assist in selecting the most appropriate design concepts and materials at the conceptual design stage. Multi-criteria decisionmaking or multi-criteria decision analysis (MCDA) [16] is a sub-discipline of operations research that explicitly multiple criteria decision-making considers in environments. In day to day life or in professional settings, there are typically multiple conflicting criteria that need to be evaluated in making decisions. Cost, quality, ergonomics and environmental factors are some of the important criteria which are considered in this paper.

Fuzzy set theory [7] [10] [11] combined with multi-criteria decision making (MCDM) methods has been extensively used to deal with uncertainty in the design selection decision process. It provides a suitable language to handle imprecise criteria, being able to integrate the analysis of qualitative and quantitative factors. This is the case of Fuzzy AHP and TOPSIS among others.

The Fuzzy AHP is a structured technique for organizing and analysing complex decisions, based on mathematics and psychology. It was developed by Thomas L. Saaty [2] during 1970s and has been extensively studied and refined since then. It has particular application in group decision making, and is used around the world in a wide variety of decision making areas such as government, business, industry, healthcare, education etc.

Whereas, TOPSIS was originally developed by Hwang and Yoon in 1981 with further developments by Yoon in 1987, and Hwang, Lai and Liu in 1993. TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution. It is a method of compensatory aggregation that compares a set of alternatives by identifying weights for each criterion. This is achieved by normalising scores for each criterion and calculating the geometric distance between each alternative and the ideal alternative.

In this paper, a descriptive quantitative approach is adopted as the research method. Algorithms of the methods TOPSIS [14] and Fuzzy AHP were developed in MATLAB and applied to the selection of design of luggage bags. Comparison of both methods was made based on the analysis of mathematical procedures considering the structure of the problem depicted by the illustrative application case.

The remainder of the work is organised into three sections. In the second section, methodology for fuzzy AHP and TOPSIS has been explained, which summarises multiple criteria decision making problems. The third section consists of the results and discussions, where all the data and results has been tabulated followed by a comparison graph. The last section presents the conclusions of this research followed by references.

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2. METHODOLOGY

2.1 Fuzzy Analytical Hierarchy Process (Fuzzy AHP)

A fuzzy set [13] is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function, which assigns to each object a grade of membership ranging between zero and one. A tilde " \sim " will be placed above a symbol if the symbol represents a fuzzy set [7] [11].

In applications it is often convenient to work with Triangular Fuzzy Numbers because of their computational simplicity and they are useful in promoting representation and information processing in a fuzzy environment.

A TFN can be defined by a triplet (l, m, u) whose membership function can be defined by the Equation used from [8].

$$\mu_{a} = \begin{cases} \frac{x-l}{m-l}, & l \le x \le m \\ \frac{u-x}{u-m}, & m \le x \le u \\ 0, & otherwise \end{cases}$$
(1)

The following steps may be incorporated to implement Fuzzy Analytical Hierarchy process (Fuzzy AHP) -

a) Triangular Fuzzy Numbers (TFN) [1]

Fuzzy pair-wise comparison is done on a defined linguistic scale. Table-1 gives us an idea about the range of TFN used in our comparison study.

Linguistic scales	TFNs	Reciprocals
Equally important	(1,1,3)	(1,1,1/3)
Weakly important	(1,3,5)	(1/5,1/3,1)
Essentially important	(3,5,7)	(1/7, 1/5, 1/3)
Very strong important	(5,7,9)	(1/9, 1/7, 1/5)
Absolutely important	(7,9,9)	(1/9,1/9,1/7)

Table-1. Linguistic scales for triangular fuzzy numbers.

b) Construction of the fuzzy pair-wise comparison matrix

The fuzzy judgment matrix $\tilde{A} = {\tilde{a}_{ij}}$ of n criteria or alternatives using pair-wise comparison is made by the use of TFNs as follows-

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \dots & \tilde{a}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{a}_{n1} & \dots & \dots & 1 \end{bmatrix}$$

where, \tilde{a}_{ij} is a fuzzy triangular number, $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, and $\tilde{a}_{ji} = 1/\tilde{a}_{ij}$. For each TFN, \tilde{a}_{ij} or M = (l, m, u), its membership function μ $\tilde{a}(x)$ or $\mu_M(x)$ is a continuous mapping from real number $-\infty \le x \le \infty$ to the closed interval [0, 1] and can be defined by equation (1).

The operations on TFNs can be addition, multiplication, and inverse. Suppose M_1 and M_2 are TFNs where $M_1=(l_1, m_1, u_1)$ and $M_2=(l_2, m_2, u_2)$, then from [6].

Addition:
$$M_1 \oplus M_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$$
(2)

Multiplication: $M_1 \otimes M_2 = (l_1 \cdot l_2, m_1 \cdot m_2, u_1 \cdot u_2)$ (3)

Inverse:
$$M_1^{-1} = (l_1, m_1, u_1)^{-1} \approx \left(\frac{1}{u_1}, \frac{1}{m_1}, \frac{1}{l_1}\right)$$
 (4)

c) Compute the value of fuzzy synthetic extent

Based on the aggregated pair-wise comparison matrix, $\tilde{U}=\{\tilde{u}_{ij}\}$, the value of fuzzy synthetic extent Si with respect to the ith criterion can be computed by making use of the algebraic operations discussed earlier

$$\mathbf{S_i} = \sum_{j=1}^m \tilde{u}_{ij} \otimes \left[\sum_{i=1}^n \sum_{j=1}^m \tilde{u}_{ij}\right]^{-1}$$

Where,

$$\sum_{j=1}^{m} \tilde{u}_{ij} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \quad \text{and}$$
$$\sum_{i=1}^{n} \sum_{j=1}^{m} \tilde{u}_{ij} = \left(\sum_{i=1}^{n} l_j, \sum_{i=1}^{n} m_j, \sum_{i=1}^{n} u_j\right)$$

d) Approximation of fuzzy priorities

On the basis of fuzzy synthetic extent values, the non-fuzzy values representing the relative preference or weight of one criterion over others are needed. Therefore, Chang's method [2] is used to find the degree of possibility that $S_b \ge S_a$ as follows:

$$V(S_{b} \geq S_{a}) = \begin{cases} 1 & , if \quad m_{b} \geq m_{a} \\ 0 & , if \quad l_{a} \geq u_{b} \\ \frac{(l_{a} - u_{b})}{(m_{b} - u_{b}) - (m_{a} - l_{a})}, otherwise \end{cases}$$
(6)

It is noted that both values of $V(S_a \ge S_b)$ and $V(S_{b} \ge S_{a})$ are required. The degree of possibility for a TFN S_i to be greater than the number of n TFNs S_k can be given by [3].

$$V(S_{i} \ge S_{1}, S_{2}, S_{3}, \dots S_{k}) = \min(S_{i} \ge S_{k}) = w'(S_{i}) \quad (7)$$

where k=1, 2, ..., n and k # i, and n is the number of criteria. Each w'(S) value represents the relative preference or weight, a non-fuzzy number, of one criterion over

others. However, these weights have to be. The normalized weight
$$w(S_i)$$
 will be formed in terms of a weight vector as follows:

$$W = \left(w(S_1), w(S_2), \dots, w(S_n)\right)^T$$
(8)

After this we need to calculate the scores of alternatives with respect to each criterion and then determine the composite weights of the decision alternatives by aggregating the weights through hierarchy.

e) Consistency test of the comparison matrix

The CR is defined as a ration between the consistency of a consistency index (CI) and the consistency of a random consistency index (RI) [4]. Its value should not exceed 0.1. The RI values for different number of respective inputs has been shown in the Table-2.

$$CR = \frac{CI}{RI} \tag{9}$$

Where,
$$CI = \frac{(\lambda_{\max} - n)}{(n-1)}$$

Table-2. Random Index values for 'n' number of inputs.

n	1	2	3	4	5	6	7	8	9	10
Random Index	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

2.2 Technique for Order of Preference by Similarity to **Ideal Solution (TOPSIS)**

A. Construction of the criteria comparison matrix for TOPSIS

This step transforms various attribute dimensions into non-dimensional attributes, which allows comparisons across criteria, M. Behzadian et al [9] -

$$X = \left[x_{ij} \right]$$

where the *i*th alternative (i = 1, ..., n) is evaluated with respect to *j*th criteria $(j = 1, \dots, m)$.

B. Normalization of the original criteria comparison matrix

To normalize the judgment matrix, the equation used to transform each element of [x_{ij}] by Deng et al [5] is as follows:

$$r_{ij} = \frac{x_{ij}}{\left(\sum_{i=1}^{n} x_{ij}^{2}\right)} \qquad i = 1, 2, \dots n$$
(10)

C. Computation of the weights of each comparison criterion

Computation of weight of each comparison criterion based on the calculation of entropy value and later on converting it into the weight is described in following two steps:

a. First to compute the entropy value (e_i) of each criterion C_1, C_2, \dots, C_n . Let e_i represents the entropy of the ith criterion,

$$e_{ij} = -\frac{1}{\ln n} \left(\sum_{i=1}^{n} r_{ij} \ln r_{ij} \right)$$
(11)

b. Computation of weights. The objective weight for each criterion is given by:

$$w_{j} = \frac{\left(1 - e_{j}\right)}{\left(\sum_{j=1}^{m} \left(1 - e_{j}\right)\right)}$$
(12)

where, j = 1, 2, 3, ..., m

D. Construct the weighted normalized decision matrix

Multiply each column of the normalized decision matrix by its associated weight. An element of the new matrix is:

$$v_{ij} = w_j \times r_{ij} \tag{13}$$

E. Determine the ideal and negative ideal solutions

Ideal solution: $A^* = \left\{ v_1^*, \dots, v_n^* \right\}$, where $v_j^* = \{ \max_i(v_{ij}) \text{ if } j \in J ; \min_i(v_{ij}) \text{ if } j \in J' \}$ Negative ideal solution: $A' = \left\{ v_1', \dots, v_n' \right\}$, where $v' = \{ \min_i(v_{ij}) \text{ if } j \in J ; \max_i(v_{ij}) \text{ if } j \in J' \}$

F. Calculate the separation measures for each alternative

The separation from the ideal alternative is

$$S_i^* = \left[\sum_{j} \left(v_j^* - v_{ij}\right)^2\right]^{\frac{1}{2}} \quad i = 1, ..., m$$
(14)

The separation from the negative ideal alternative is:

$$S'_{i} = \left[\sum_{j} \left(v'_{j} - v_{ij}\right)^{2}\right]^{\frac{1}{2}} i = 1, ..., m$$
(15)

G. Calculate the relative closeness to the ideal solution C_{i}^{\ast}

$$C_{i}^{*} = \frac{S_{i}'}{\left(S_{i}^{*} - S_{i}'\right)}, 0 < C_{i}^{*} < 1$$
(16)

The criterion having highest closeness to 1 is to be given priority.

3. RESULTS AND DISCUSSIONS

The above proposed methodologies have been implemented here for prioritization of luggage bags. This prioritization is carried out considering four criteria: Cost, quality, ergonomics and environment friendly. The two methods have been carried out separately and the respective final scores have been converted and rated on a scale from 0-1 for ease of comparison.

3.1 Fuzzy AHP

A MATLAB program has been coded which enables a user to define all the criteria, products and pairwise rating. The following Table-3 demonstrates the pairwise rating based on the linguistic scale (triangular fuzzy numbers) (Table-1) for criteria, followed by Table-4 which shows the rating of products for the first criteria, i.e. cost.

Table-3. Pair-wise rating for criteria.

Criteria	Cost	Quality	Ergonomics	Environmental
Cost	(1,1,1)	(1/7,1/5,1/3)	(1,3,5)	(1,3,5)
Quality	(3,5,7)	(1,1,1)	(7,9,9)	(5,7,9)
Ergonomics	(1/5,1/3,1)	(1/9,1/9,1/7)	(1,1,1)	(1,1,3)
Environmental	(1/5,1/3,1)	(1/9,1/7,1/5)	(1,1,3)	(1,1,1)

Criterion - Cost							
Product designs A B C							
А	(1,1,1)	(1,3,5)	(1,1,3)				
В	(1/5,1/3,1)	(1,1,1)	(1/7,1/5,1/3)				
С	(1,1,3)	(3,5,7)	(1,1,1)				

Table-4. Pair-wise rating of product designs for cost criterion.

These values are obtained as user-input. Thus, to verify its consistency we use the equation (9). Following are the consistency index values for pair-wise rating for all the four criteria:

Consistency Index for data of Cost criterion = 0.014532Consistency Index for data of Quality criterion = 0.014532Consistency Index for data of Ergonomics criterion = 0.04015

Consistency Index for data of Environmental criterion = 0.

Thereafter the steps mentioned in the methodology have been followed to obtain local weights (7) and overall weights (8) as shown in the Table-5 below. As we can see in the Table, many of the weights have negative signs. This is due to the fuzzy membership number which enables the possibility of a negative weight even for a set of data which is consistent. Due to which the overall weights might have a value out of the usual range of (0, 1) which is evident below. But their sum will always be equal to one.

Table-5. Scores and ranking of the product designs using fuzzy AHP.

		Local w	eight score				
Product designs	Cost	Quality	Ergonomi cs	Environme ntal	Overall weight	Priority ranking	Relative score on 0-1 scale
	(0.6599)	(2.6992)	(-0.77338)	(-1.5857)	score		
А	0.4306	0.6915	0.2201	0.2951	1.5125	2	0.3748
В	0.0617	-1.212	1.0185	0.4970	-4.8065	3	0.1525
С	0.5078	1.5205	-0.2386	0.2079	4.294	1	0.4727

In order to facilitate comparison between FAHP and TOPSIS we have introduced a new parameter "Relative Score". Relative score is a positive weight score and its sum is always equal to one. For this we have equated the lowest ranked product for both the techniques.

3.2 TOPSIS

The MATLAB program compiled for TOPSIS takes input from the preference vectors of Fuzzy AHP. This is done in order to achieve similar inputs. The following Table-6 shows the numerical scores of each product for the given set of criteria approximately to the closest whole number. The scores are given based on the linguistic scale of 1-9 as mentioned before.

Criteria Product designs	Cost	Quality	Ergonomics	Environmental
A	4	3	3	3
В	2	1	6	5
С	5	6	2	3

Table-6. Rating of product designs for each criterion.

The first step in the TOPSIS is normalization of the given values (10). The normalized values are used in the computation of weight ((11) - (12)). But in this

approach the weights have been taken to be proportionate in order to contribute to the similar nature of input data as shown in Table-7.

Criteria Product designs	Cost	Quality	Ergonomics	Environmental
А	0.3636	0.3	0.2727	0.2727
В	0.1818	0.1	0.5454	0.4545
С	0.4545	0.6	0.1818	0.2727

Table-7. Weights of product designs for each criterion.

As mentioned in the methodology, the distances are obtained ((14) - (15)) which indicates the position of the different products for each criterion from ideal solution and negative ideal solution, shown in Table-8. These distances, thereafter, are used in the calculation closeness to the ideal solution, i.e. the product having score closest to 1 will be considered to be the best.

	Table-o. Distance from negative and positive ideal solution.								
Product	C	Cost		Quality		Ergonomics		Environmental	
designs	d _{i1} ⁺	d _{i1} -	d _{i2} ⁺	d _{i2}	d _{i3} +	d _{i3}	d _{i4} ⁺	d _{i4}	
А	0.0016	0.0066	0.0588	0.0261	0.0064	0.0007	0.0020	0	
В	0.0148	0	0.1634	0	0	0.0114	0	0.0020	
С	0	0.0148	0	0.1634	0.0114	0	0.0020	0	

Table-8 Distance from negative and positive ideal solution

As seen, product C has the value closest to one and therefore is ranked one and so on. The relative score is a weight score and is used for comparison with Fuzzy AHP results.

Table-9. Final scores and ranking of product designs by TOPSIS.

Product designs	ns Final score Ranking		Relative score on 0-1 scale
А	0.4106	2	0.2910
В	0.2152	3	0.1525
С	0.7848	1	0.5563

3.3 Comparison of FAHP and TOPSIS scores

Following Figure-1 depicts a graph which illustrates the relation between the relative scores of products through FAHP and TOPSIS methods. Both of the methods have produced the same priority order. Although,

there is significant difference between the respective relative scores and pattern. The graph also shows the quadratic relation between the relative scores of each method which is of the order 2.



Figure-1. Graphical Comparison of scores obtained from Fuzzy AHP and TOPSIS evaluations.

3.4 Spearman's Rank-Order Correlation

It assesses how well the relationship between two variables can be described using a monotonic function. If there are no repeated data values, a perfect Spearman correlation of +1 or -1 occurs when each of the variables is a perfect monotone function of the other. The Spearman correlation coefficient, ρ , can take values from +1 to -1. A ρ of +1 indicates a perfect association of ranks, ρ of zero indicates no association between ranks and ρ of -1 indicates a perfect negative association of ranks. The closer ρ is to zero, the weaker the association between the ranks.

Here rank correlation between fuzzy AHP and TOPSIS has been tabulated in Table-10.

Product	Fuzzy AHP	TOPSIS	$\begin{array}{c} \textbf{Rank fuzzy} \\ \textbf{AHP} (r_1) \end{array}$	Rank TOPSIS (r ₂)	$\mathbf{d}_{i} = \mathbf{r}_{1} \mathbf{-} \mathbf{r}_{2}$	d_i^2
А	0.3748	0.291	2	2	0	0
В	0.1525	0.1525	3	3	0	0
С	0.4727	0.5563	1	1	0	0

Table-10. Difference and squared differences between the scores of Fuzzy AHP and TOPSIS.

Spearman's Rank co relation

$$(\rho) = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

Using the above equation spearman's rank order correlation coefficient is determined as 1. This indicates a perfect association of ranks.

4. CONCLUSIONS

This paper presents a comparative study of Fuzzy AHP and TOPSIS evaluations in regard to four factors that are particularly relevant to the problem of design selection. The comparison was based on qualitative analysis of the algorithms of both methods. The two different multi criteria decision making approaches used here serve the same purpose. While Fuzzy AHP method measures the performance in a hierarchical structure by use of pair wise comparison matrix, TOPSIS method measures the performance by use of the distance to negative and positive ideal principle. These two methods are commonly used in the literature relative to the other methodologies.

As a result, it is observed that the priority order is same for both the methods but the relative weight scores vary. Even though fuzzy brings in the subjective factor in AHP, it can produce negative weights which lead to values out of range as observed. The input requirements of fuzzy AHP are significantly more than TOPSIS which can make it tedious for the user. Computational time of the MATLAB program is also higher for fuzzy AHP, therefore, when used for a more complex application might be time consuming. It is difficult to rank out one method over the other. Depending upon the degree of fuzziness, subjective nature, complexity and criteria independency and consistency, the selection of implementation of one of these two might vary.

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