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CHAOTIC BEHAVIOUR OF HOST-MONAD COMMENSAL SPECIES PAIR-A SPECIAL NUMERICAL CASE STUDY

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ABSTRACT

In this paper we present the numerical solutions of a week commensal of a two species commensalism interaction with Monad type-variable commensal coefficient with limited resources and with mortality rate for the commensal species. The commensal coefficient depends up on the host population size. Very interestingly we observed the chaotic behavior of the dynamical system for an arbitrary change in the commensal coefficient. Also we noted the bifurcation timings of the trajectories of the commensal species over the host species.

Keywords: commensalism interaction, commensal species, host species, trajectories of the species, bifurcation times of the trajectories.

1. INTRODUCTION

Mathematical modeling in mathematical biosciences is an attempt to identify and describe some instances of our routine in the language of mathematics. It is an endeavor as old as the first human being and as modern as tomorrow's newspaper. While widening and deepening the scope of mathematical modeling in life and medical sciences, one is not just restricted to the use of mathematical techniques already known. The mathematical constructions of biological phenomena's are commensal based on the systems of non linear differential equations. The formulation of the model and their simulations are very important to know the behavior of the biological system either qualitatively or quantitatively methods of the systems. Mathematical modeling of ecosystems was initiated by Lotka [4] and Volterra [10]. The general concepts of modeling have been presented in the treatises of Meyer [5], Cushing [1], Paul Colinvaux [6], Freedman [2], Kapur [3] and several others. Later Phanikumar, Seshagiri Rao and Pattabhi Ramacharyulu [7], Seshagiri Rao, Phanikumar and Pattabhi Ramacharyulu [8], Seshagiri Rao, Kalyani and Pattabhi Ramacharyulu [9] have discussed one of the ecological commensalism interaction between two and three species of course one of the species has a mortality rate.

This paper presents the numerical solutions of the growth rate equations of an ecological commensalism interaction between two species with the help of classical Runge-Kutta method where as the commensal species has a variable coefficient interms of the host population. Further both species are considered in the natural limited resources and the commensal species has mortality rate. This model is characterized by a couple of first order non linear ordinary differential equations. The trajectories of this model consist of the commensal (growing, balanced and mortal types) over the host species are drawn using DE discover software by changing the coefficient commensal and fixing the remaining parameters are constant in the model. Also observe the chaotic nature of the model in the small variations of the commensal coefficient and the bifurcation timings of the trajectories are highlighted. The dominance reversal time is found in all possible cases and the conclusions are given here under.

2. BASIC EQUATIONS

The model equations for a two species Monad system with limited resources and with mortality rate for the commensal are given by the following system of non-linear ordinary differential equations, employing the above terminology.

(i) Equation for the growth rate of the mortal commensal specie

$$\frac{dN_1}{dt} = N_1\left[-e_1 a_{11} - a_{11}N_1 + F(N_2)\right] \tag{1}$$

In the equation (1), the function $F(N_2)$ is the characteristic of the commensalism of N_1 with respect to the host N_2 with the conditions: $F(0)=0$ and $F(N_2)$ is bounded throughout.

A reasonably simple choice for $F(N_2)$ is the Monad type as given by Kapur [3].

$$F(N_2) = \frac{\alpha N_2}{\beta + N_2} \tag{1.1}$$

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$\alpha = lt_{N_2 \to \infty} F(N_2)$, α is a parameter characteristic of commensal. Further β is another parameter signifying the strength of the commensal? The commensalism is strong or weak according as $\beta > 0$ or $\beta < 0$ and interaction would be neutral when $\beta = 0$. The function F is widely known as Michaelis - Menten relationship and also known as Holling type -II.

Note: The growth rate equations for Balanced (i.e. birth rate of the commensal is equal to its death rate) and Growing species (i.e. birth rate of the commensal is greater than its death rate) can be obtained by taking $e_1 = 0$ and $e_1 = -g_1$ in equation (1).

(ii) Equation for the growth rate of the Host species

$$\frac{dN_2}{dt} = a_{22} N_2 [k_2 - N_2] \tag{2}$$

3. A NUMERICAL SOLUTIONS OF THE GROWTH RATE EQUATIONS

Numerical solution of the coupled non-linear basic differential equations (1) and (2) have been computed in the time interval [0, 10] in steps of one each employing Runge-Kutta system for a wide range of the model characterizing parameter: β (negative values) for the commensal species, fixing mortal commensal coefficient $e_1 = 0.4$, the parameter of the commensal $\alpha = 5$, the self inhibition coefficients $a_{11} = 0.5$, $a_{22} = 0.3$ and the host carrying capacity $k_2 = 10$ constants. The software DEDiscover has been used for this purpose and the graphical illustrations of the results obtained are shown in Figures 1 to 6.

As the commensal parameter β decreases, the trajectories of the growing, balanced and mortal commensal (hitcher to very close to each other) get bifurcated at time t_b^* which decrease first and then increases with commensal parameter β. The bifurcation timings of the trajectories are given by the following table and conclusions are given cases wise.

Table.1.

S. No.	a_{11}	e_1	a_{22}	k_2	α	β	$t_{g_1}^* = t_{e_1}^* = t_0^* = t^*$	t_b^*	Fig. No.
1	0.5	0.4	0.3	10	5	-7.6	1.558	1.613	1
2	0.5	0.4	0.3	10	5	-7.65	1.287	1.307	2
3	0.5	0.4	0.3	10	5	-7.6544	3.045	3.05	3
4	0.5	0.4	0.3	10	5	-7.6548	7.038	7.073	4
5	0.5	0.4	0.3	10	5	-7.654807	8.35	8.419	5
6	0.5	0.4	0.3	10	5	-7.65481	-	0	6

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Case-1

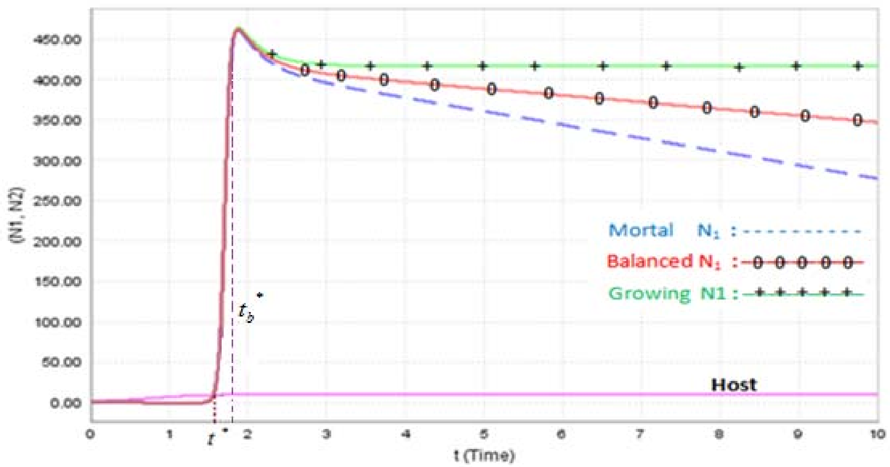

Figure-1. Variation of N_1, N_2 vs. t for $a_{11} = 0.05$, $e_1 = 0.4$, $g_1 = 0.4$, $\alpha = 5$, $\beta = -7.6$, $a_{22} = 0.3$, $k_2 = 10$, $N_{10} = 1$, $N_{20} = 1$.

The host dominates over the commensal irrespective of the commensal nature up to the time t^* =1.558 from then after the dominance is reversed. Further the commensal species decrease initially and then rise steeply at t^* =1.588 again decreases till that instant time (t^* =1.558) no appreciable departure in the trajectories of growing, balancing and mortal species is observed. After that instant, the trajectories get bifurcated Figure-1.

Case-2

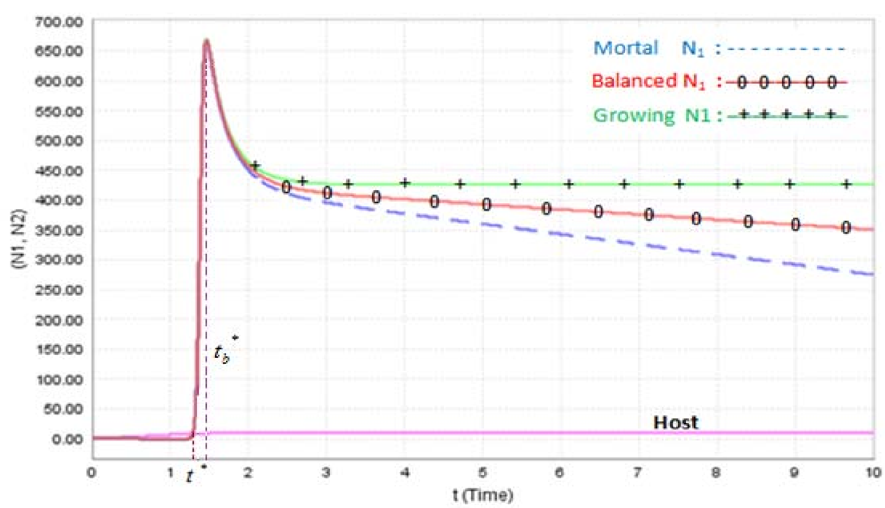

Figure-2. Variation of N_1, N_2 vs. t for $a_{11} = 0.05$, $e_1 = 0.4$, $g_1 = 0.4$, $\alpha = 5$, $\beta = -7.65$, $a_{22} = 0.3$, $k_2 = 10$, $N_{10} = 1$, $N_{20} = 1$.

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Initially the host out-numbers the commensal species (i.e., growing, balanced and mortal) up to the time instant $t^{*} = 1.287$ there after the commensal dominates. Also we notice that the commensal species decreases initially and then rise steeply at t^{*} =1.287 again decrease steeply and then the trajectories are bifurcated whereas there is no appreciable growth rate in the host species. (Figure-2)

Case-3

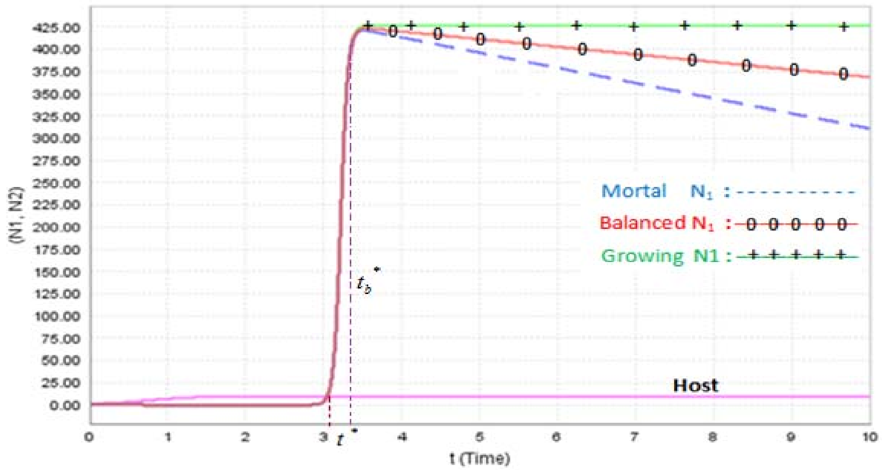

Figure-3. Variation of N_1, N_2 vs. t for $a_{11} = 0.05$, $e_1 = 0.4$, $g_1 = 0.4$, $\alpha = 5$, $\beta = -7.6544$, $a_{22} = 0.3$, $k_2 = 10$, $N_{10} = 1$, $N_{20} = 1$.

Case-4

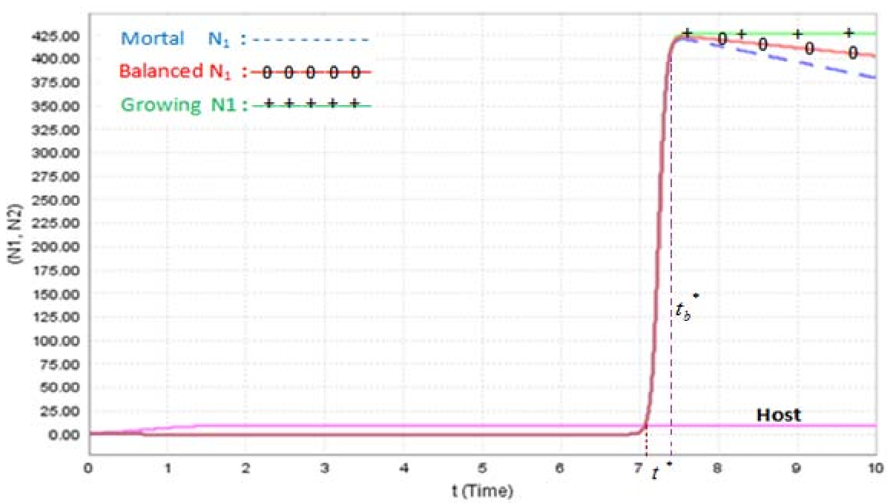

Figure-4. Variation of N_1, N_2 vs. t for $a_{11} = 0.05$, $e_1 = 0.4$, $g_1 = 0.4$, $\alpha = 5$, $\beta = -7.6548$, $a_{22} = 0.3$, $k_2 = 10$, $N_{10} = 1$, $N_{20} = 1$.

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Case-5

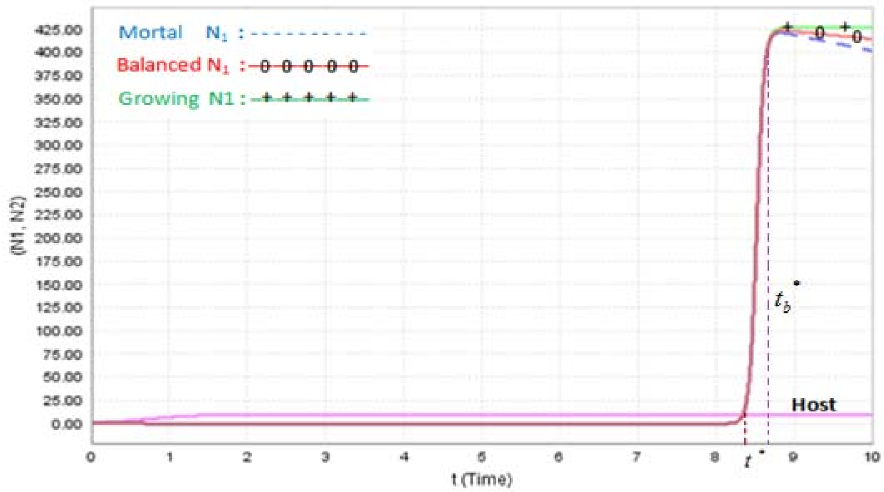

Figure-5. Variation of N_1, N_2 vs. t for $a_{11} = 0.05$, $e_1 = 0.4$, $g_1 = 0.4$, $\alpha = 5$, $\beta = -7.654807$, $a_{22} = 0.3$, $k_2 = 10$, $N_{10} = 1$, $N_{20} = 1$.

In Figures 3.30 to 3.32, the trajectories correspond to small values of the parameter β. In these cases 3, 4 and 5 the host dominates the commensal till the time instants t^* = 3.045, t^* = 7.038 and t^* = 8.35 respectively after which the dominance is reversed. Further we observe that the commensal species steeply rise at above times and then the trajectories get bifurcated.

Case-6

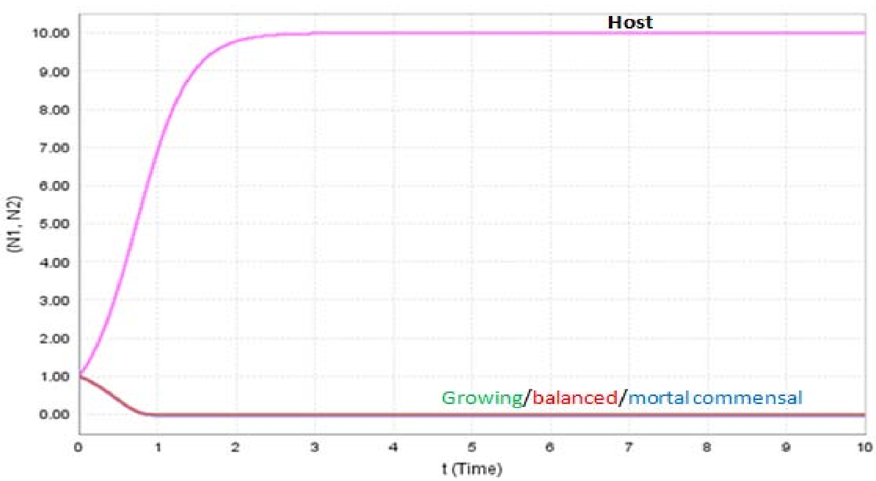

Figure-6. Variation of N_1, N_2 vs. t for $a_{11} = 0.5$, $e_1 = 0.4$, $g_1 = 0.4$, $\alpha = 5$, $\beta = -7.65481$, $a_{22} = 0.3$, $k_2 = 10$, $N_{10} = 1$, $N_{20} = 1$.

In this case the host species out-numbers the commensal species throughout. Further, we see that the host species rises initially and attains an asymptotic level with no appreciable growth rate, whereas the commensal species decreases and in course of time it is very low level irrespective of the commensal nature. (Figure-6)

An interesting observation

(i). For $0 < \beta < -7.6$ a chaotic variation in the trajectories of the commensal.

(ii). For $-7.6 < \beta < -7.6581$ commensal split-trajectories after time instant t^*.

(iii). For $-7.6581 < \beta$ the growing/balancing/mortal commensal have a common trajectory.

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Nomenclature

$N_1(t)$, $N_2(t)$: The populations of the commensal and host at time t.

d_1 : The mortal rate of the commensal species.

a_2 : The rate of natural growth of the host species.

$a_{ii}(i = 1,2)$: The rate of decrease of both the commensal and host due to the limitations of its natural resources.

$e_1(= d_1 / a_{11})$: The mortal coefficient of the commensal species.

$k_2(= a_2 / a_{22})$: The carrying capacity of the host.

$t_{g_1}^*$: The dominance reversal time of the host over the commensal when birth rate is greater than the death rate.

t_0^* : The dominance reversal time of the host over the commensal when death rate is equal to the birth rate.

$t_{e_1}^*$: The dominance reversal time of the host over the commensal when death rate is greater than the birth rate.

t_b^* : Bifurcation time.

α, β : Commensal constants

The state variables N_1(t) and N_2(t) as well as all the model parameters d_1, a_2, a_{11}, a_{22}, a_{12}, k_1, e_1, k_2 are assumed to be non-negative constants.