



# NUMERICAL ANALYSIS OF VEHICLE EXHAUST SYSTEM TO DETERMINE HANGER LOCATION USING ROOT MEAN SQUARE VALUE

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## ABSTRACT

Vibration of exhaust system due to the engine operation and the condition of the road surface which is transferable to the body through the hanger affects the noise, vibration and harshness performance of the vehicle. The exhaust manifold is mounted on the engine and the exhaust pipe is fitted to the exhaust manifold. Therefore, the fixed-free condition of exhaust system finite element model has been used to perform normal mode analysis and frequency response analysis using MSC Patran/Nastran software. According to the simulation results, the average excited load response at the different nodes is generated. The root mean square (RMS) values of these responses were calculated to select the suitable hanger positions which are confirmed at the point where the RMS value is comparatively small. This study can contribute to the design development and verifying the performance of an automobile exhaust system and can be broadened for the other exhaust system designs of various vehicles.

**Keywords:** hanger location, modal analysis, root mean square value, exhausts system, frequency response function.

## INTRODUCTION

An exhaust system is one of the important parts of automobile. The exhaust system of automobile is used to prevent exhaust gas from entering the passenger compartment, to carry and reduce releasing toxic gases into the environment and to silence noise that is occurred by the high pressure exhaust gas. It is mainly composed of exhaust manifold, exhaust pipe, flexible joint and muffler. The exhaust manifold is bolted to the engine and the others are attached to car body by hangers. Due to the operation of engine and the condition of road surface, vibration is transferred to the body structure via the hangers of the exhaust system. The hanger location influences on the noise, vibration and harshness issue. Previously, many works with respect to the hanger location of vehicle exhaust system have been studied (Noorazizi *et al.*, 2014; Noorazizi *et al.*, 2013; Shao *et al.*, 2008; Yansong *et al.*, 2010; Zhang and Song, 2012). Recently, the finite element method based simulation technique to determine hanger position of exhaust system using average driving DOF displacement (ADDOFD) method was presented by Noorazizi *et al.* (Noorazizi *et al.*, 2014). The addofd method was used in references (Noorazizi *et al.*, 2014; Noorazizi *et al.*, 2013; Shao *et al.*, 2008; Yansong *et al.*, 2010) but this method was more complex and they counted the times modal nodes of each modal shape appear at the same location (Zhang and Song, 2012). This method is less accurate than calculating root mean square (RMS) value of vibrating energy because the displacement of shapes is accumulated based on the visual motion of exhaust system in addofd method and the exact RMS value of the vibration energy can be calculated. This RMS value has previously been used in the study of dynamics characteristics of crankshaft system (Abu, 2006). Therefore, calculating RMS values is adopted in this study.

In order to reduce the vibration energy transferring to the body, understanding the dynamics characteristics of the exhaust system and the sensitivity locations of hangers are important. In this paper, the finite element based frequency response function (FRF) and modal analyses are performed utilizing MSC Patran/Nastran software. Based on the result of frequency response, RMS values at the each node are calculated. Then, the reasonable hanger locations where RMS value is relatively small are selected. This simulation technique can be used in the automobile development process to reduce noise, vibration and harshness problem of exhaust system.

## FEM MODELLING AND NUMERICAL ANALYSIS

Some simplifications need to be considered because the structure of exhaust system is complex to build a reasonable FEM model. In this paper, the exhaust system is idealized by a set of joined beams to detect the node point easily. In addition, the catalytic converter and muffler are assumed as the hollow model because the constructions inside these structures are complex. However, the actual dimension of exhaust pipe and material properties of sample exhaust system that impact on the vibration of system are considered in the simulation.

**Table-1.** Specification of the exhaust system model.

Parameter	Dimension [m]
Diameter of front pipe	0.044
Diameter of rear pipe	0.0385
Thickness of pipes	0.002
Diameter of flexible bellow	0.0645
Diameter of muffler	0.102
Diameter of tail pipe	0.0755
Total length of exhaust system	3.5425

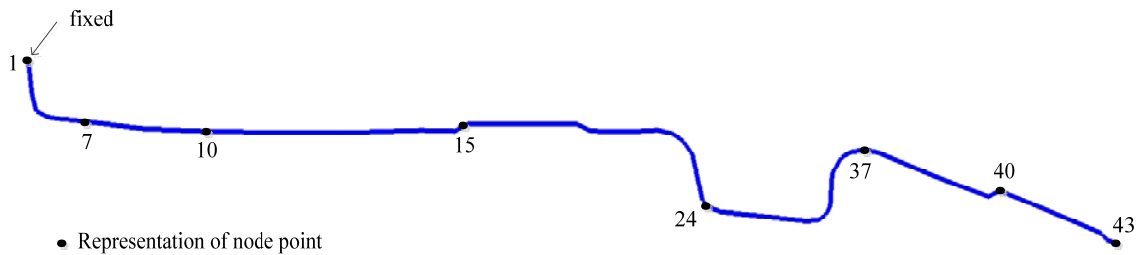


The specification of the exhaust system model is depicted in Table-1 and the material properties of exhaust model are presented in Table-2.

**Table-2.** Material properties of the exhaust system model.

Parameter	Value	unit
Young's modulus	103e9	N/m <sup>2</sup>
Poisson's ratio	0.3	
Density	7850	kg/m <sup>3</sup>

The node point one is fixed because in actual, the exhaust is mounted on the exhaust manifold but the effect of the original hangers, supports and flexible joint is ignored in the analysis procedures. Modal analysis is conducted by Lanczos modal extraction method in the MSC Nastran software which is provided by the solver processor. Using this method, the stiffness and mass matrices can be easily extracted. The FEM model shown in Figure-1 is solved by the SOL 103 module to know the dynamics characteristics of the exhaust system and the SOL 111 module to get the response vibration energy at different nodes in MSC Nastran. There are total 43 nodes of the model and the representative of node points are shown in Figure-1.



**Figure-1.** Model of vehicle exhaust.

**Governing Equation**

In this section, the calculation of frequency response function performs in the modal frequency response analysis of Nastran software is described briefly. After adding a bulk data file (\*.bdf), the mass and stiffness matrices of the system are collected to become a set of differential equations which have already been transformed in the frequency domain that is shown by equation (1). Here, the damping effect is ignored at first.

$$M\ddot{x} + Kx = f(\omega) \tag{1}$$

Secondly, the characteristic equation (equation (2)) is solved for eigenvalues and the corresponding Eigen modes are figured out.

$$\det(K - \omega^2 M) = 0 \tag{2}$$

A modal transformation is achieved, which diagonalizes the system from equation (1) dealing with equation (3) separately, utilizing the orthogonality properties of the eigenvectors in the modal matrix  $\Psi$  which is composed of the calculated eigenvectors (De Silva, 2006).

$$x = \Psi q(\omega) \tag{3}$$

With  $\Psi = [\psi_1, \psi_2, \psi_3, \dots, \psi_N]$

Hereby, the system with N degrees of freedom is separated to be a system in general coordinates  $q_i$  (equation (4))

$$m_i \ddot{q}_i(\omega) + k_i q_i(\omega) = p_i(\omega) \tag{4}$$

Here,  $m_i$  is the modal mass;  $k_i$  is the modal stiffness, and  $p_i = \psi_i^T \cdot f$  the modal force. With the eigen

frequencies  $\omega_i^2 = k_i/m_i$ , equation (5) can be transformed from equation (4). Here, damping can be introduced by a modal damping ratio  $\zeta_i$  (equation (6)).

$$\ddot{q}_i(\omega) + \omega_i^2 q_i(\omega) = \frac{p_i(\omega)}{m_i} \tag{5}$$

$$\ddot{q}_i(\omega) + 2\zeta_i \omega_i \dot{q}_i(\omega) + \omega_i^2 q_i(\omega) = \frac{p_i(\omega)}{m_i} \tag{6}$$

The solution of this differential equation is defined as equation (7).

$$q_i(\omega) = \frac{1}{\omega_i^2 - \Omega_i^2 + i2\zeta_i \omega_i \Omega_i} \cdot \frac{\hat{p}_i(\omega)}{m_i} \tag{7}$$

Then, equation (8) shows a linear combination of the solutions for all oscillators which is the solution of the system.

$$x(\omega) = \sum_{i=1}^N \psi_i q_i(\omega) \tag{8}$$

The ratio of the response of a system  $x(\omega)$ ; displacement, velocity and acceleration to its excitation force  $F(\omega)$  can be specified as the frequency response function. Equation (9) describes the frequency response function for the displacement of the system.

$$H(\omega) = \frac{x(\omega)}{F(\omega)} = \sum_{i=1}^N \psi_i \cdot \frac{1}{\omega_i^2 - \Omega_i^2 + i2\zeta_i \omega_i \Omega_i} \cdot \frac{\psi_i^T}{m_i} \tag{9}$$



Thus, the frequency response function at the  $k$ th node due to a single excitation force at  $j$ th degree of freedom can be calculated by equation (10).

$$H_k(\omega) = \sum_{i=1}^N \frac{\Psi_{ik} \cdot \Psi_{ij}}{m_i(\omega_i^2 - \Omega_i^2 + i2\zeta_i\omega_i\Omega_i)} \quad (10)$$

**Root Mean Square**

A square root of the mean value of quantity  $f(t)$  at a proper average time  $T$  is termed the root mean square value for a signal  $f(t)$ . This value is described by equation (11).

$$\psi_s = \sqrt{\lim_{T \rightarrow 0} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f^2(t) dt} \quad (11)$$

The information related with characteristics of random variables in a time domain is provided by auto-correlation function  $R_f$ . The similar information in a frequency domain is given by the power density function  $S_f$ .

$$\begin{aligned} R_f(\tau) &= \lim_{T \rightarrow 0} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)f(t+\tau) dt \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_f(\omega) e^{i\omega\tau} d\omega \end{aligned} \quad (12)$$

When  $\tau = 0$  in time domain;

$$\begin{aligned} R_f(0) &= \lim_{T \rightarrow 0} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f^2(t) dt = \psi_s^2 \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_f(\omega) d\omega \end{aligned} \quad (13)$$

The root mean square value at the time domain is equivalent to the area under the frequency response curve in the frequency domain. So hanger locations of the exhaust system can be selected by the lowest RMS value.

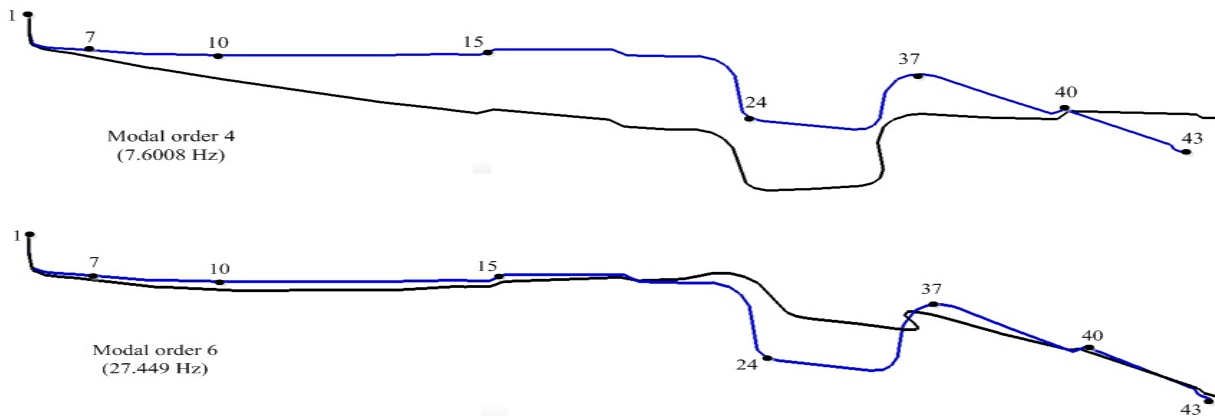
**RESULTS AND DISCUSSIONS**

The fixed-free mode of exhaust system is analyzed in MSC Patran /Nastran. In this simulation, the frequency range from 1 Hz to 100 Hz is considered. The natural frequencies and mode shape of the exhaust system depicted in Table-3 are calculated by modal analysis. Among the twelve numbers of modes, the modal orders one to four are the bending modes and the rest are complex mode shapes.

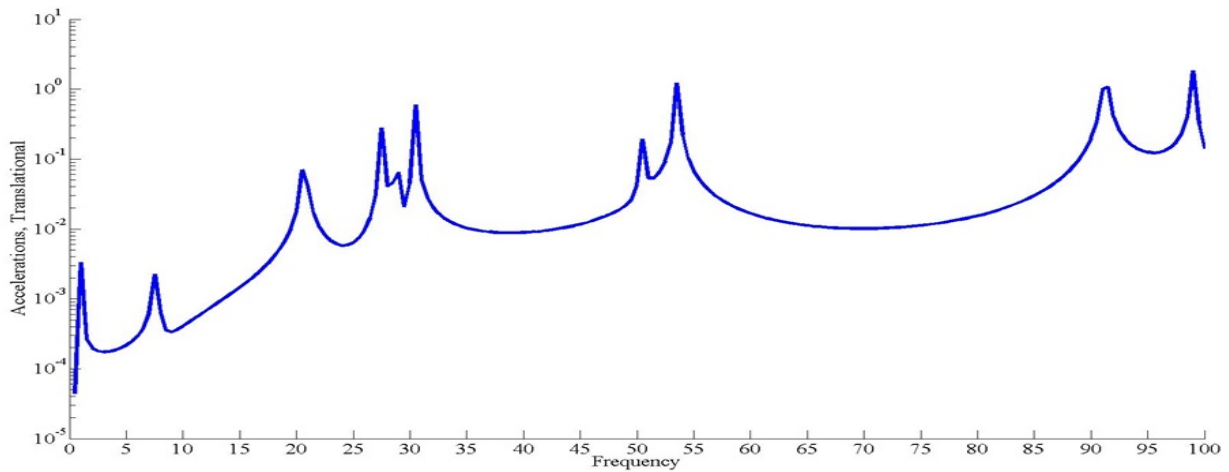
**Table-3:** The calculated natural frequencies of the exhaust system.

Modal order	Frequency (Hz)	Description
1	1.0191	Bending mode
2	1.037	Bending mode
3	7.4008	Bending mode
4	7.6008	Bending mode
5	20.677	Bending at the front pipe and compression at middle & tail pipe
6	27.449	Bending at the front pipe and torsion at the tail pipe
7	28.854	Twisting at the front pipe and bending at the rear pipe
8	30.474	Twisting at the front pipe and bending at the rear pipe
9	50.423	Bending at the front and tail; and torsion at the middle
10	53.558	Torsion at the front and bending at the tail
11	91.258	Bending mode at the front pipe and twisting at the middle
12	99.001	Bending at the front pipe and compression at the middle

The natural frequency of the modal order one and two from Table-3 are very small which can be defined as fundamental frequency of the exhaust system.



**Figure-2.** Modes shapes of exhaust system.



**Figure-3.** The response of the exhaust system.

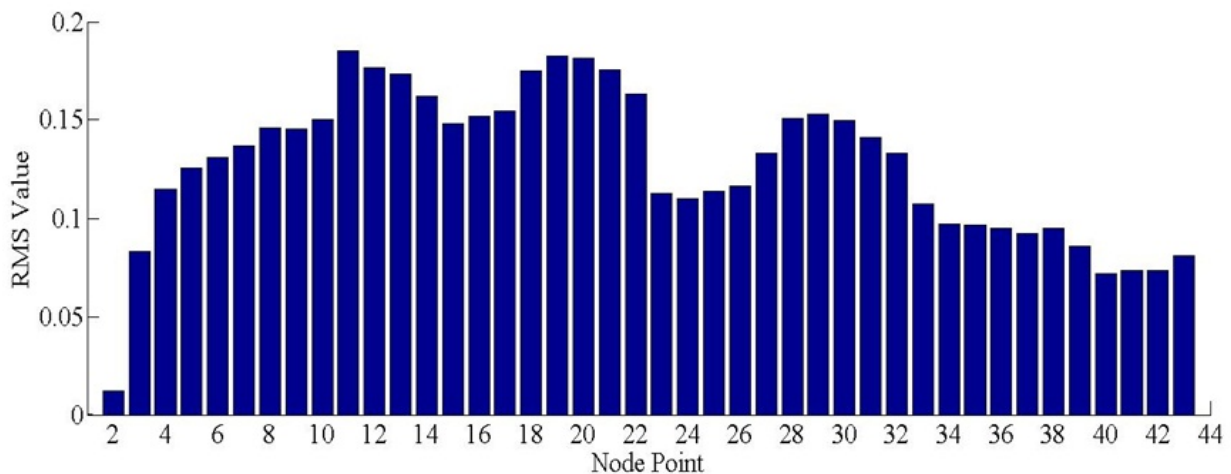
For instance, two mode shapes of exhaust system of modal order four at 7.6008 Hz frequency and six at 27.449 Hz frequency are shown in Figure-2. In Figure-2, the blue colour shape is the original shape of the exhaust system, and the black colour shape is the deformed shape at the different natural frequency. The calculated natural frequencies of exhaust system in Table-3 can be concluded that the higher natural frequency, the more complex mode shape occurs and then the displacement distribution of each mode is different meaning that vibration energy at each node is different.

Moreover, the vibration energy distribution should be considered when excitation force is subjected to the exhaust system according to actual condition. Therefore, frequency response function analysis is performed to know the different energy distribution at different nodes of exhaust system in this study. Solving SOL 111 module in Nastran, the load response at the various node points is generated.

The response at node 40 of the exhaust system for an instant is shown in Figure-3 describing acceleration translational magnitude with respect to the frequency. The area under the graph represents the vibrating energy of the given node.

Based on the results, the average excited load response at the different nodes is identified utilizing RMS value that is the transferable energy to the body. The RMS value is extremely simple and makes it easier to recognize the average transferred energy at various node points. The highest RMS values give the highest energy.

However, the high energy occurs is advantage for the hangers' positions of the exhaust system because the energy can be transferred through the hangers to the car body and it may then be happened noise, vibration and harshness problem of the automobile. Therefore, the small RMS value should be selected for the hanger location. The level of RMS value at the different nodes is shown in Figure-4.



**Figure-4.** Level of RMS value at the different.



Originally, there are three hanger locations; the first one is at node 10 which is near the flexible joint, the second is at 1.5 cm away from node 37 which is between node 37 and 38, and the third is at 8.5 cm away from node 40 between node 40 and 41 or at the rear part of the muffler. The some original positions are at high vibrating energy that transfers more energy to the body. Therefore, changing the hanger locations is required to reduce the vibration of exhaust system and the body.

The first original hanger at node 10 is unnecessary to fix there because it is too near to the fixed end. Therefore, node 15 which is near catalytic converter is selected as the first new hanger location because its RMS value is smaller than other surrounding node. The node 24 is chose as the second hanger location which is at the bend of the exhaust pipe considering its weight distribution of bended pipe and RMS value. The third new hanger location is at the node 40 which is on the front part of the muffler. It is smaller RMS value comparing with the third original location and it is suitable location to hang the rear part of the exhaust system.

To sum up, the new selected hanger locations are at node 15, 24 and 40. These are the lowest RMS value which is recommended as the best hanger locations of the sample exhaust system.

## CONCLUSIONS

In this paper, the modal analysis and frequency response analysis were performed in MSC Patran/Nastran to determine the hanger positions of the sample exhaust system using RMS value. These positions were selected at the smaller RMS value. As a result, the hanger locations can be investigated easily and quickly to reduce the transferable vibrating energy from the exhaust system to the car body that occur the noise, vibration and harshness problem. Thus, this procedure can be used in the automobile development process to find the suitable hanger location easily and quickly, and can be broadened to other exhaust system designs of various vehicles.

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