VOL. 10, NO. 15, AUGUST 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

DYNAMIC STABILITY OF SHAFT INTERCONNECTED THROUGH JOINT: A REVIEW

Wahab A. M., Rasid Z. A., Mohd Rudin N. F. and Abu A.
Intelligent Dynamic System i-Khoza, Japan International Institute of Technology (MJIIT), Universiti Teknologi Malaysia Kuala Lumpur, Jalan Semarak, Kuala Lumpur, Malaysia
E-Mail: abdulmalekabdulwahab@gmail.com

ABSTRACT

Parametric instability is an important machine design consideration because it may cause failures such as fatigue and breakage in machine components. The problem of parametric instability in drive system is reviewed in this paper focusing on shafts that are interconnected through joint. The review covers aspects of modelling of the shafts, methods of solving parametric equation (Mathieu or Mathieu-Hill equation) in the drive system and sources of parametric instability in rotating shafts. Lumped-mass model has been used in modeling the drive shaft system during the early work. In order to obtain a realistic condition, continuous model has been established that considers the mass of the shaft. However these studies lacked in considering more degrees of freedom that can give better representation of the problem. The methods of solving Mathieu or Mathieu-Hill type equation in drive system are reviewed, analytical and numerical. Parametric instability in a rotating shaft system is due to asymmetric shaft, anisotropic bearing, cracked and the applied periodic axial compressive loading. In the case of the rotating shafts interconnected with joint, parametric instability may be a result of periodic variation of velocity ratio and the angular misalignment.

Keywords: drive trains, shaft, joint, dynamic stability, Mathieu-Hill equation.

INTRODUCTION

Drive trains have been used widely in vehicle system and industry. A drive train system may consist of mechanical components such as shafts, disks, bearings and joints. The important part of the drive train is the joint which interconnects the driver and driven shaft. Joints such as universal joint, U-joint, Cardan joint, Hardy-Spicer joint or Hooke's joint are used to transmit torque and rotatory motion at flexible connection throughout a change of drive angle in limited space. Drive train deals with great rotational energy as well as vibrational energy during operation. Thus, studies has been conducted to minimize the vibration energy to be as small as possible for better operation of the drive train (Ecker and Pumhossel, 2012).

In general, drive train may undergo vibration due to force or parametric excitations. Force excitation occurs in a dynamic system as periodic input (force) is applied onto the system. Dynamic resonance appears as the frequency of the external input or drive frequency is equal to the one of natural frequency of the system (Fossen and Nijmeijer, 2011).

On the other hand, parametric excitation is a result of having time-varying (periodic) parameters such as stiffness in a system. In this case, the system can experience dynamic parametric resonance if driving frequency is twice one of natural frequency of the system (Lacarbonara and Antman, 2008). As this phenomenon occurs, the small parametric excitation may amplify the oscillation of the system that can cause catastrophic failure to the system.

Bolotin (1960) suggested that parametric instability can bring failure if it continuously occurs to the system. It was reported that the existence of this instability can cause noticeable noise, severe mechanical shakes and premature fatigue failures in shafts, gear teeth etc. (Asokanthan and Meehan, 2000). In addition, as self-excited vibration (instability) aroused, the torque amplification factor of the drive train will be abruptly increased to a high value and consequently causes the instantaneous breakage of the entire drive train (Wang and Wang, 2000).

Numerous studies have been done to identify parametric excitation in a drive system. Numerical and analytical methods have been applied to estimate the dynamic stability chart as well as the dynamic response of drive system under parametric excitation.

Thus, this paper gives a summary of parametric excitation phenomenon that occurs in drive system. The main objective of this review is to show how parametric excitation may occur in the drive system. The review is structured as followed: System modeling is discussed in topic 2. The solutions of parametric excitation are treated in topic 3. Finally, sources of parametric excitation in drive system are presented in topic 4. It should be noted that some of the reviews are based on parametric excitations that occur in rotor system.

Modeling parametric excitation

The discovery of parametric resonance dated back to 1831. Faraday (1831) was first to observe the phenomenon of parametric excitation, when he noticed that a fluid filled container vibrates vertically, fluid surface oscillates at half the frequency of the container. Melde (1860) reported parametric resonance in the case of lateral vibration of a string. Belyayev (1924) was first to provide a theoretical analysis of parameter resonance while dealing with the stability of prismatic rods. The Mathieu equation,

$$\ddot{x} - (\delta + \varepsilon \cos t)x = 0 \tag{1}$$

is the simple un-damped differential equation that model parametric excitation. Parameter δ is the natural frequency square of the homogenous system, while ε is the amplitude of the parametric excitation (McLachlan, 1951; Minorsky, 1951; Bolotin, 1960; Nayfeh and Mook, 2008). The results of the effect of several parameters on the stability of the system are shown in a form of stability chart or so the called Strut-Ince diagram.

Properties of the system are referred by the parameters that are involved in the system. Based on this properties, mathematical model are constructed as much as possible to truly represent the actual physical system. Early studies on the dynamic instability of interconnected shaft system considered the lumped-mass model and analytical method was applied to solve the parametric equation (Porter, 1961; Porter and Gregory, 1963; Chang, 2000; Asokanthan and Meehan, 2000).

Figure-1 shows an example of a two degree-of-freedom (dof) system incorporating a Hooke's joint. The angular misalignment of the system represents by   and is driven at a constant angular velocity and   represent the input and the output angles of the Hooke's joint, respectively, while   is the output angle of the system. The input and output shafts have been considered to be flexible and to have torsional stiffness   and. The moment of inertia for input and output represent by   and   respectively.

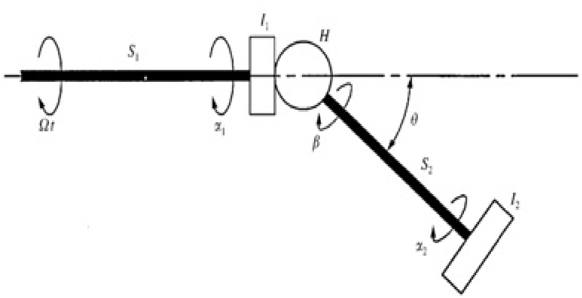

Figure-1. Model of shaft incorporates a Hooke's joint using two degree of freedom (Asokanthan and Meehan, 2000).

The equations of motion in terms of the twist variables x and y are derived by Lagrange's equation as in equation (2) and equation (3) where x equal to $\alpha_1 - \Omega t$ and y equal to $\alpha_2 - \beta$. Equation (2) and equation (3) represent driver and driven shaft respectively. The over dots denote the differentiation with respect to time.

$$\ddot{x} + \omega_{11}^2 x - \omega_{21}^2 y = \varepsilon\left\{\frac{\omega_{21}^2 y\cos(2x + 2\Omega t)}{2} - 2\mu_{11}\omega_{11}(\dot{x} + \Omega)\right\} + o(\varepsilon^2) \quad (2)$$

$$\ddot{x} + \ddot{y} + \omega_{22}^2 y = \varepsilon\left\{(\dot{x} + \Omega)^2\sin(2x + 2\Omega t) - \frac{\ddot{x}\cos(2x + 2\Omega t)}{2} - 2\mu_{22}\omega_{22}(\dot{x} + \dot{y} + \Omega)\right\} + o(\varepsilon^2) \quad (3)$$

Lumped-mass model is a simple way to predict the characteristics of dynamic instability in the system by considering the mass, stiffness (spring) and damper that are lumped at finite discrete points. These studies considered torsional stiffness and inertia load without considering the mass of the shaft. As such this model did not consider the flexural vibration behavior of the system that may greatly affect its dynamic instability behavior. Flexural vibration arises in a rotating system due to shear deformation and bending moment. By applying this simplified model, the input may not be enough to give precise and comprehensive prediction of the actual instability behavior of a shaft system especially in the case of two shafts system interconnected via joint.

On the other hand, Figure-2 shows the drive system that is modeled as continuous system. β is the Hooke's joint misalignment, ρ_i is shaft mass density, I_i is polar area moment of inertia, G_i is shear modulus and l_i is the length for discretizing element. The left end of shaft is rotated at constant speed Ω_0 as it is attached to flywheel. The system is assumed driving a mechanism with a constant inertia, which is represented by mass moment of inertia J_d.

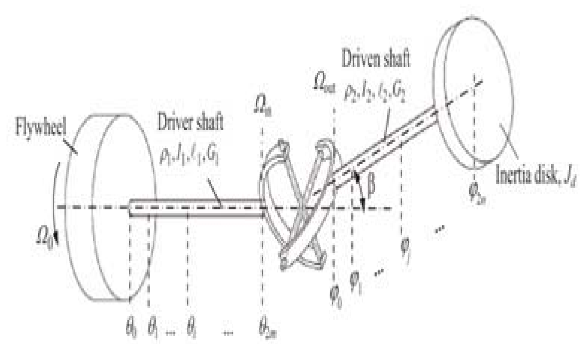

Figure-2. Model of shaft incorporates a Hooke's joint using continuous system (Bulut, 2014).

Equation (4) and equation (5) show the equations of motion for the driven system modeled as a continuous system. In comparison, the mass distribution of shaft represents by density   is considered in continuous model while in the lump-mass model mass of shaft is neglected. Bulut (2014) reported that, by considering mass of shaft, stability analysis of the system may be done in higher mode condition.

$$\rho_1 I_1 l_1 . M_1 . \ddot{\theta} + \left(a_1 . \rho_1 I_1 l_1 . M_1 + b_1 . \frac{G_1 I_1}{l_1} . K_1\right) . \dot{\theta} + \frac{G_1 I_1}{l_1} . K_1 = T . e_2 m \quad (4)$$

$$[\rho_2 I_2 l_2 . M_2 + J_d . f_{2n+1} . (f_1^T + f_{2n+1}^T)] . \ddot{\varphi} + \left(a_2 . \rho_2 I_2 l_2 . M_2 + b_2 . \frac{G_2 I_2}{l_2} . K_2\right) . \dot{\varphi} + \frac{G_2 I_2}{l_2} . K_2 . \varphi = -T^{*} . f^{*} \quad (5)$$

Continuous model can be expected to better predict the dynamic stability of a shaft system. The parameters of the system are distributed continuously in an infinite degree of freedom. The interconnected joint shaft can be modeled as continuous by mean of beam theories. Han *et al.* (1999) suggested that the selection of the best theory to model a drive shaft system as a simple beam is subjected to the slenderness ratio of the shaft. This result is in line with the study by Sabuncu and Evran (2006) that showed that the dissimilarity between beam theories could become deceptive as slenderness ratio changes.

Slenderness ratio can be defined as the ratio of the length of a beam to the radius of gyration of the cross section of the beam. The first order effect, bending moment and lateral displacement can be shown by a slender beam. On the other hand, second order effect, shear deformation and rotary inertia is important to be considered for shafts modeled as non-slender beams.

Recent study on the dynamic stability in drive shaft system has been done by considering the rotating shaft as a slender beam (Bulut, 2014). Therefore, Rayleigh beam theory has been used. As this theory neglects the transverse shear deformation, it can only predict a limited numbers of critical speeds. Several other works applied the Rayleigh beam theories to model their rotating shafts (Hosseini and Khadem, 2009, 2009; Khadem, Shahgholi and Hosseini, 2010; Shahgholi and Khadem, 2012; Hosseini and Zamanian, 2013).

The neglecting of shear deformation may affect the characteristics of the stability region. Specifically, it was reported that shear coefficient has an effect towards the stability region (Sabuncu and Evran, 2005). The regions of the dynamic instability for the bending mode were shifted as the shear coefficient was decreased. Figure 3 shows the studies of dynamic stability of a hinged-hinged static shaft subjected to axial periodic forces applying to both Euler-Bernoulli's and Timoshenko's beam (Chen and Ku, 1990). It can be seen that the dynamic instability region of the system shifted and increased in Timoshenko's beam theory compare to Euler-Bernoulli's beam theory. This result is in line with the work of Han, Benaroya and Wei (1999) which stated that shear deformation has more dominant effect for non-slender beam model.

Therefore, in the case of modeling the rotating shaft interconnecting through joint as non-slender beam, Timoshenko beam should be considered. Timoshenko beam theory deliberates all four assumptions in modeling the beam, bending moment, lateral displacement, shear deformation and rotary inertia. Table-1 shows a comparison of assumptions made by four beam models: the Euler-Bernoulli, Rayleigh, Shear and Timoshenko.

Table-1. Four aspects of consideration in four beam theories i.e. the Euler-Bernoulli (EB), Raleigh (RL), Shear (SH) and Timoshenko (TI) (Han, Benaroya and Wei, 1999).

Beam Theory	Bending moment	Lateral Displace.	Shear Deformation	Rotary inertia
EB	Yes	Yes	No	No
RL	Yes	Yes	No	Yes
SH	Yes	Yes	Yes	No
TI	Yes	Yes	Yes	Yes

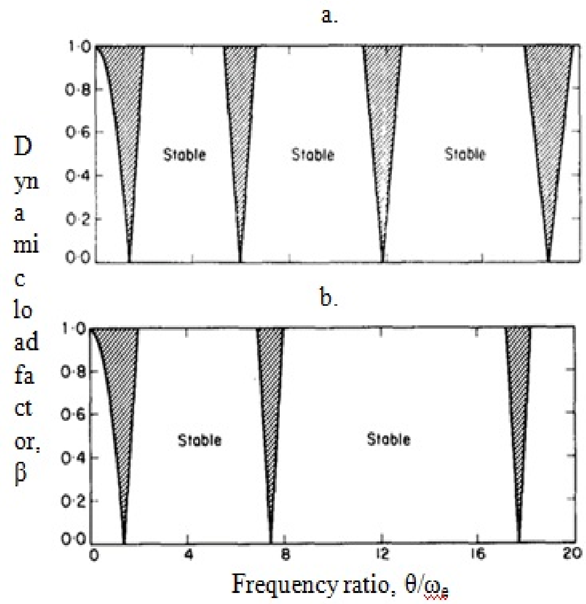

Figure-3. Stability chart for static beam subjected to periodic axial loading modelled by (a) Timoshenko's theory and (b) Euler-Bernoulli's.

Studies to investigate the dynamic instability of rotating shaft using the Timoshenko beam theory have been conducted by several researchers (Abbas and Thomas, 1978; Shastry and Rao, 1984, 1986; Abbas, 1986; Shastry and Venkateswara Rao, 1987; Chen and Ku, 1990; Lee, 1995, 1997; Chen, Chen and L.w.Chen, 1995; Chen and Peng, 1997; Mazzei Jr, Argento and Scott, 1999; Adewusi and Al-Bedoor, 2001; Sabuncu and Evran, 2005, 2006; Mandal and Mahapatra, 2007; Ouyang and Wang, 2007; Hosseini and Khadem, 2009; Boru, 2010; Rao, 2011; Shahgholi and Khadem, 2012; Hosseini and Zamanian, 2013; Miranda and Faria, 2014; Bazehhour, Mousavi and Farshidianfar, 2014; Chen and Chen, 2014; Dakel, Baguet and Dufour, 2014).

However the similar study Chen and Ku (1990) reported that gyroscopic moment is seen to be less sensitive to the dynamic instability problem of the rotating Timoshenko shaft. Figure-4 (b) shows that the region of dynamic instability based on the Euler-Bernoulli's beam grows as the rotating speed is increased. In comparison, the region of dynamic stability does not change for

VOL. 10, NO. 15, AUGUST 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

rotating Timoshenko shaft such as shown in Figure-4 (a). This condition was interpreted as the effect of the gyroscopic moment that destabilizes the rotating shaft. Thus, finding shows that more accurate Timoshenko beam theory needs to be considered.

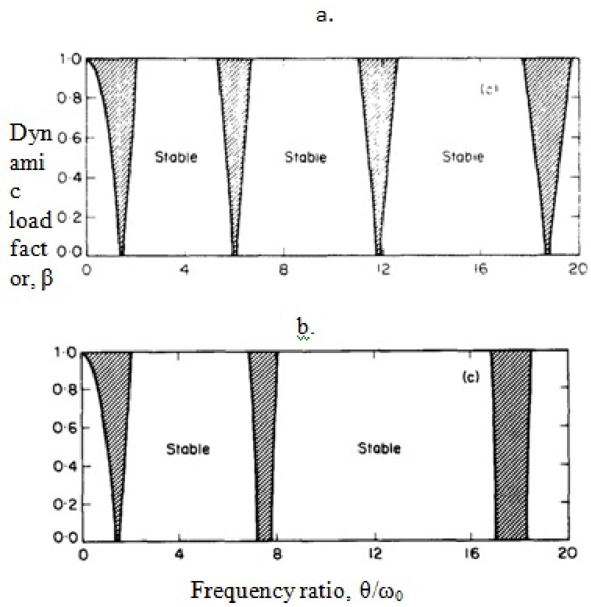

Figure-4. Stability chart for rotating beam at 20000 rpm subjected to periodic axial loading modelled by (a) Timoshenko's theory and (b) Euler-Bernoulli's.

Solving parametric equation

Researchers for a long time have been interested to explore different solution methods for the Mathieu-Hill type parametric equation. Numerical and analytical methods have been used to estimate the stability chart. For one dof, Porter (1961) investigated the instability speed range of linearized drive shaft model using the Floquet's theory. However, due to linearize procedure of the equation this method did not provide the realistic prediction of oscillation amplitude during instabilities. Then, method of Kryloff and Bogoliuboff was applied to non-linear equation of similar problem to estimate the amplitude of oscillation during instability (Porter and Gregory, 1963) . The studies continue as Vol der Pol's method was introduced to investigate non-linear torsional vibration in Cardan shaft (Zahradka, 1973). Furthermore, strained parameters method was applied for linearized higher order of dynamic stability chart and higher order averaging for non-linear system to determine the basin of attraction (Chang, 2000).

Besides that, Lyapunov exponent method (Asokanthan and Wang, 1996) and averaging method (Asokanthan and Meehan, 2000) has been applied for linear and non–linear conditions respectively in the case of 2 dof. Bulut and Parlar (2011) conducted a parametric stability analysis of shafts interconnected through a Hooke's joint using a simplified 2-dof-lumped-mass model. The monodromy matrix method was used to solve the Mathieu-Hill's equation in order to estimate the instability chart. The finding shows that unstable region can occur at below the fundamental torsional frequency, harmonic and sub-harmonic parametric resonances. Other than that, both sum and difference type combination resonances can be encountered in the considered system.

Nayfeh and Mook (2008) summarized three distinct groups of analytical methods to solve parametric equation: the method of infinite determinants, the method of Liaopunov and the method of averaging. Each of the method has its strength and weakness in assessing the dynamic stability of the system, which is explained well in their book.

On the other hand, numerical method such as the finite element method (FEM) has been used to estimate dynamic stability region of more complicated systems such as that of the Timoshenko beam model. Equation (6) shows the un-damped Mathieu's equation in general in the FEM form where M is the mass matrix, K is the stiffness matrix for the system ,Kg is the geometry stiffness matrix and P(t) = αP*+βP*cosΩt is the periodical axial load. P* is the fundamental static buckling load while α and β are called the static and dynamic load factors respectively.

$$[M]\{\ddot{q}\} + [K]\{q\} - P(t)[K_g]\{q\} = 0 \qquad (6)$$

Early work by Brown, Hutt and Salama (1968) showed the application of the FEM in solving the Mathieu-Hill equation of bars with various set of boundary conditions. It was found that the stability charts were similar regardless of the boundary conditions probably because the vibration and buckling mode shapes were similar. This work was continued by Abbas and Thomas (1978) who studied the instability behavior of a Timoshenko beam under the effect of elastic foundation and periodic axial load. It was found that the increase of the elastic foundation constant as well as the effect of the periodic axial load has decreased the region of dynamic instabilities. Besides that, study showed that the instability regions for Timoshenko beam were unaffected by boundary conditions (Shastry and Rao, 1984).

The FEM has also been used in solving dynamic instability of rotating shaft by a number of researchers (Dakel, Baguet and Dufour, 2014; Raffa and Vatta, 2007; Chen and Ku, 1990; Chen, Chen and L.w.Chen, 1995; Chen and Peng, 1997; Sabuncu and Evran, 2006; Abbas, 1986; Li *et al.* 2015; Pei, Lu and Chatwin, 2010.

Abbas (1986) conducted a study on the effects of rotational speed and root flexibilities on buckling load and on the regions of dynamic instability of rotating Timoshenko beams. It was reported that the beam became less sensitive to periodic excitation as the speed of rotation was increased and at the same time the increase in the rotational root flexibility makes the beam more sensitive to periodic excitation.

Li *et al.* (2015) conducted dynamic instability of a rotating ship shaft subjected to a periodic axial force induced by a rotating screw propeller where the excitation frequency of the periodic axial force has a relation with spinning speed and the number of blades on the propeller. The periodic axial force here is not of the typical form

VOL. 10, NO. 15, AUGUST 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

where the force is expressed in a series. Through the discrete singular convolution (DSC) with regularized Shannon's delta kernel it was found that the increase of number of blades and damping could improve the dynamic stability of rotating shaft. The increase of constant term in the periodic force has led to dynamic instability regions shifting to lower frequencies, making the shaft more sensitive to periodic force.

Pei *et al.* (2010) modelled a rotating Timoshenko shaft with rigid unsymmetrical disc subjected to a periodic axial force as a parametrically excited system using FEM. Steady state response, resonance and dynamic instability were investigated using the harmonic balanced method. It was found that the fluctuating part of the axial force has caused the dynamic instability of the system and the regions of the dynamic instability were enlarged with increasing amplitude of the fluctuation.

The effects of gyroscopic and rotary inertia towards dynamic instability region has been discussed by researchers (Shahgholi and Khadem, 2012; Abbas and Thomas, 1978; Alugongo, 2011; Mazzei, 2011). Ku and Chen (1992) applied Timoshenko's beam theory while considered the effects of translational and rotatory inertia, gyroscopic moments, bending and shear deformation in studying the parametric instability of a shaft-disk system with localized zones of damage and subjected to axial periodic forces. A Ritz FEM was used to get the parametric equation and generate the instability chart. One of the finding was that if a localized zone of damage is presented, the parametric resonance may occur at lower axial disturbance frequencies.

Considering the sources of parametric excitation to be considered, the types of model to be applied and the beam theory to be utilized if the continuous model is chosen will determine the complexity of the FEM model to be employed in any parametric instability analysis. Thomas *et al.* (1973) categorized elements in the FEM equation into two classes: simple and complex. A simple element has four degrees of freedom, 2-dof at each node. On the other hand, a complex element can have more than four degrees of freedom per element. Figure 5 shows dof that can be considered in a shaft element where at each node, there can be three translational and three rotational degrees of freedom.

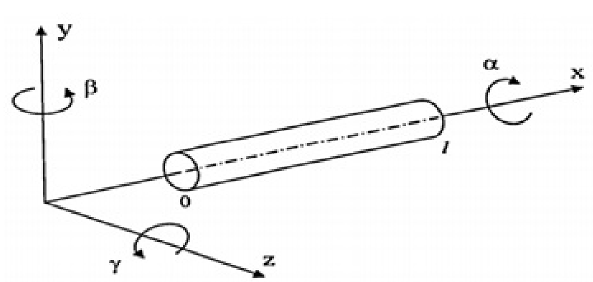

Figure-5. Degree of freedom in one element.

Literature review shows that 8-dof for each element in a FEM model has been used to establish the dynamic instability chart for rotating Timoshenko beam (Abbas, 1986; Ku and Chen, 1992). All these works considered the effect of rotational dof about y and z-axis while the torsional dof about x-axis has often been neglected. The Nelson's beam theory however further generalized the rotating Timoshenko beam element by considering torsional moment (Rao, 2011). In this study each node consists of 5 dof which are two translation dof and three rotational dof about x, y and z directions. However, this FEM analysis that was conducted here was limited to finding the response of vibration due to external force. Therefore, it is reasonable to use the Nelson's beam theory in the future study of parametric instability of rotating shafts interconnected through joint in order to get a model that represents the best of the actual situation.

Sources of parametric excitation

In the shafts interconnected through joint, the sources of parametric excitation are various. Porter (1961) in his dynamic stability study of shafts interconnected via joint stated that the violent oscillation (parametric resonance) was due to critical speed range of the linearized shaft model. The critical speed range aroused because of periodic variation of the velocity ratio of the joint. Then, similar problem was reconsidered through a more accurate non-linear modelling in the work of Porter and Gregory (1963). The investigation showed that under the instability condition, the frequency of violent torsional oscillation was significantly related to two cycles per revolution of the shaft system. The amplitude of the unstable oscillation increased as the frequency of revolution was approximately half of natural frequency of the free vibration. This phenomenon occurred when the system was considered to be in light damping condition. Zahradka (1973) reported that instability could arise in the areas of main, sub-harmonic and sub-ultra-harmonic resonance for non-linear driving system interconnected through Cardan joint. The study found that damping of the system and the angular misalignment of the Cardan joint contributed to the characteristics of the stability diagram.

On the other hand, Asokanthan and Wang (1996) studied linearized torsional system that incorporated Hooke's joint. The study identified the dynamic instabilities that corresponded to the sub-harmonic and combination of resonances. Furthermore, similar problem was investigated under linear and non-linear conditions (Asokanthan and Meehan, 2000). Results showed that parametric instabilities that occurred depended on the input shaft speeds and the Hooke's joint angle. The existence of parametric resonance, quasi-periodic and chaotic motions were shown under the non-linear governing equation. Chang (2000) revisited the non-linear single degree of freedom problem in (Porter and Gregory, 1963) to investigate the parametric and external resonances. Parametric resonance occurred as rotating speed was closed to the linear natural frequency of the system. On the other hand, external and secondary parametric resonances aroused as rotating speed was closed to half the linear natural frequency. In this study, it

VOL. 10, NO. 15, AUGUST 2015 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

was identified that the ratio of input and output shaft stiffness determined the dynamic instability characteristics Further study conducted by Bulut and Parlar (2011) identified that misalignment angle, inertia and rigidity ratio contributed to the instability of the linearized two degree of freedom torsional vibration of a shaft interconnected through Hooke's joint. In addition, different types of combination resonance existed and unsafe speed occurred below the fundamental torsional frequency of the system. Besides that, it was reported that bending moment, torque and fluctuating rotating speed were sources of parametric instabilities in a drive system (Mazzei Jr, Argento and Scott, 1999). Bending moment showed the stronger effect to dynamic instability as compared to the torque and fluctuating rotating speed. This phenomenon occurs at driven shaft rather than driving shaft due to joint characteristics.

Looking at a more complicated rotor system that may consist of shafts, disks and bearings besides the joint, literature review shows that there are several other sources of parametric excitation that may have not been considered in the study of the parametric instability of shafts interconnected through joint. Some of these sources of parametric excitation are the geometric asymmetry of the shaft (Boru, 2010; Oncescu, Lakis and Ostiguy, 2001), rotational motion of support (anisotropic bearing) (Dakel, Baguet and Dufour, 2014), gyroscopic moment (Chen and Ku, 1990; Gayen and Roy, 2014), (Chen and Ku, 1990; Gayen and Roy, 2014)(Chen & Ku 1990; Gayen & Roy 2014)(Chen and Ku, 1990; Gayen and Roy, 2014)(Chen & Ku 1990; Gayen & Roy 2014)(Chen & Ku, 1990; Gayen & Roy, 2014)(Chen and Ku, 1990; Gayen and Roy, 2014)(Chen and Ku, 1990; Gayen and Roy, 2014), crack propagation (Chen, Chen and L.W.Chen, 1995), varied stiffness ratio and periodic axial compressive loads (Chen and Peng, 1997; (Raffa and Vatta, 2007).

Axial asymmetry of shaft and disk is one reason why inertia and stiffness of a shaft change periodically. This axial asymmetry of a shaft may occur due to its design itself that requires non-circular shaft or may be due to the occurrences of crack, manufacturing failures, critical speed and misalignment. Axisymmetric rotor system is actually seldom in today's complicated industrial rotor (Lee *et al*. 2007). The periodic axial force varies the stiffness of the system with time, which in turn becomes the source of parametric excitation. In interconnected shafts, the axial force may come from the torque produced by the driver shaft onto the driven shaft. In a recent research work on rotor dynamic conducted by Gayen and Roy (2014), bearing was modeled as spring and viscous damper. Dakel *et al*. (2014) modeled flexible rotor with hydrodynamic journal bearing. The authors reported that hydrodynamic bearings have an influence greater than the rigid bearings on the size of the instability regions of the rotor.

This literature review indicates that the various sources of parametric excitation have been neglected in the dynamic instability studies of shafts interconnected through joint. In future works, it is logical to consider the combined effects of these various sources of parametric excitation as to represent to the closest to the real complicated system of the rotor system in the drive train system.

CONCLUSIONS

A brief history of modelling work of shaft interconnected through joint under parametric excitation is presented. The behaviour of dynamic instability in shafts interconnected through joint has been studied corresponds to from the very simple model of massless shaft, single degree of freedom to continuous model. The sources of the instability have been identified as well. It is found that the model developed for the shafts interconnected through joint thus far did not considered all necessary degrees of freedom such as considered in the Nelson's model. Furthermore, the model for the shafts interconnected with joints thus far lacked in considering several sources of parametric excitation such as the anisotropic bearings and asymmetric shafts. It is thus logical to study the instability of a drive train with shafts interconnected with joint that considers the effect of anisotropic bearing, gyroscopic effect and asymmetric shaft using the model of Nelson's. This constitutes the current work of the authors.

REFERENCES

[1] Abbas B. 1986. 'Dynamic stability of a rotating Timoshenko beam with a flexible root', Journal of Sound and Vibration, Vol. 108, No. 1, pp. 25–32.

[2] Abbas B. and Thomas J. 1978. 'Dynamic stability of Timoshenko beams resting on an elastic foundation', Journal of Sound and Vibration, Vol. 60, pp. 33–44.

[3] Adewusi S. a. and Al-Bedoor B. O. 2001. 'Wavelet Analysis of Vibration Signals of an Overhang Rotor with a Propagating Transverse Crack', Journal of Sound and Vibration, Vol. 246, No. (5), pp. 777–793.

[4] Alugongo A. 2011. 'A nonlinear torsional vibration model of a propeller shaft with a crack-induced parametric excitation and a Hookes joint type of kinematic constraint', IEEE AFRICON 2011, Livingstone, Zambia.

[5] Asokanthan S. F. and Meehan P. a. 2000. 'Non-Linear Vibration of a Torsional System Driven By a Hooke'S Joint', Journal of Sound and Vibration, Vol. 233, No. 2, pp. 297–310.

[6] Asokanthan S. F. and Wang X.-H. 1996. 'Characterization of Torsional Instabilities in a Hooke'S Joint Driven System Via Maximal Lyapunov Exponents', Journal of Sound and Vibration, Vol. 194, No. 1, pp. 83–91.

[7] Bazehhour B. G., Mousavi S. M. and Farshidianfar A. 2014. 'Free vibration of high-speed rotating

Timoshenko shaft with various boundary conditions: effect of centrifugally induced axial force', Archive of Applied Mechanics, Vol. 84, No. 12, pp. 1691–1700.

[8] Beliaev N. 1924. 'Stability of prismatic rods subjected to variable longitudinal forces', Collection of papers: Engineering Construction and Structural Mechanic, Leningrad, Put'.

[9] Bolotin V. 1964. The dynamic stability of elastic systems, Holden–Day, San Frascisco, USA.

[10] Boru F. 2010. Numerical and experimental response and stability investigations of anisotropic rotor-bearing systems, PhD Thesis, University of Kassel, Germany.

[11] Brown J. E., Hutt J. M. and Salama A. E. 1968. 'Finite element solution to dynamic stability of bars.', AIAA Journal, Vol.6, No. 7, pp. 1423–1425.

[12] Bulut G. 2014. 'Dynamic stability analysis of torsional vibrations of a shaft system connected by a Hooke's joint through a continuous system model', Journal of Sound and Vibration, Vol. 333, No. 16, pp. 3691–3701.

[13] Bulut G. and Parlar Z. 2011. 'Dynamic stability of a shaft system connected through a Hooke's joint', Mechanism and Machine Theory, Vol. 46, No. 11, pp. 1689–1695.

[14] Chang S. I. 2000. 'Torsional Instabilities and Non-Linear Oscillation of a System Incorporating a Hooke'S Joint', Journal of Sound and Vibration, Vol. 229, No. 4, pp. 993–1002.

[15] Chen L. and Ku D. 1990. 'Dynamic stability analysis of a rotating shaft by the finite element method', Journal of Sound and Vibration, Vol. 143, pp. 143–151.

[16] Chen L. and Peng W. 1997. 'Stability analyses of a Timoshenko shaft with dissimilar lateral moments of inertia', Journal of Sound and Vibration, Vol. 207, pp. pp. 33–46.

[17] Chen, L.-W., Chen, H.-K. and L.w.Chen (1995) 'Stability analyses of a cracked shaft subjected to the end load', Journal of Sound and Vibration, Vol. 188, No. 4, pp. 497–513.

[18] Chen W.-R. and Chen C.-S. 2014. 'Effects of Locally Distributed Kelvin–Voigt Damping on Parametric Instability of Timoshenko Beams', International Journal of Structural Stability and Dynamics, Vol. 14, No. 6, pp. 1450-1474.

[19] Dakel M., Baguet S. and Dufour R. 2014. 'Nonlinear dynamics of a support-excited flexible rotor with hydrodynamic journal bearings', Journal of Sound and Vibration, Vol. 333, No. 10, pp. 2774–2799.

[20] Ecker H. and Pumhossel T. 2012. 'Vibration suppression and energy transfer by parametric excitation in drive systems', Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, Vol. 226, No. 8, pp. 2000–2014.

[21] Faraday M. 1831. 'On a peculiar class of acoustical figures; and on certain forms assumed by groups of particles upon vibrating elastic surfaces', Philosophical Transactions of the Royal Society of London, Vol.121, pp. 299–318.

[22] Fossen T. I. and Nijmeijer H. 2011. Parametric resonance in dynamical systems, Springer Verlag.

[23] Gayen D. and Roy T. 2014. 'Finite element based vibration analysis of functionally graded spinning shaft system', Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, Vol. 228, No. 18, pp. 3306-3321.

[24] Han S., Benaroya H. and Wei T. 1999. 'Dynamics of transversely vibrating beams using four engineering theories', Journal of Sound and Vibration, Vol. 225, pp. 935–988.

[25] Hosseini S. a a and Zamanian M. 2013. 'Multiple scales solution for free vibrations of a rotating shaft with stretching nonlinearity', Scientia Iranica. Vol. 20, No. 1, pp. 131–140.

[26] Hosseini S. a. a. and Khadem S. E. 2009. 'Combination resonances in a rotating shaft', Mechanism and Machine Theory. Vol. 44, No. 8, pp. 1535–1547.

[27] Hosseini S. a. a. and Khadem S. E. 2009. 'Free vibrations analysis of a rotating shaft with nonlinearities in curvature and inertia', Mechanism and Machine Theory. Vol. 44, No. 1, pp. 272–288.

[28] Khadem S. E., Shahgholi M. and Hosseini S. a a 2010. 'Primary resonances of a nonlinear in-extensional rotating shaft', Mechanism and Machine Theory, Vol. 45, No. 8, pp. 1067–1081.

[29] Ku D. and Chen L. 1992. 'Dynamic stability of a shaft-disk system with flaws', Computers & Structures, Vol. 43.

[30] Lacarbonara W. and Antman S. S. 2008. 'What Is a Parametric Excitation in Structural Dynamics ?', ENOC-2008. ENOC, Saint Petersburg, Russia.

[31] Lee C.-W., Han D.-J., Suh J.-H. and Hong S.-W. 2007. 'Modal analysis of periodically time-varying linear rotor systems', Journal of Sound and Vibration, Vol. 303, No. 3, pp. 553–574.

[32] Lee H. 1995. 'Dynamic response of a rotating Timoshenko shaft subject to axial forces and moving loads', Journal of Sound and Vibration, Vol. 181, pp. 169–177.

[33] Lee H. 1997. 'Dynamic Stability of Spinning Timoshenko Shafts With a Time-Dependent Spin Rate', Journal of Sound and Vibration, Vol. 199, pp. 401–415.

[34] Li W., Song Z., Gao X. and Chen Z. 2015. 'Dynamic Instability Analysis of a Rotating Ship Shaft under a Periodic Axial Force by Discrete Singular Convolution', 2015.

[35] Mandal D. K. and Mahapatra M. C. 2007. Dynamic Stability of Rotating Shaft under Parametric Excitation subjected to Specified Boundary Conditions, Degree Thesis, National Institute of Technology Rourkela, India.

[36] Mazzei a. J. 2011. 'Passage through resonance in a universal joint driveline system', Journal of Vibration and Control, Vol. 17, pp. 667–677.

[37] Mazzei Jr A. J., Argento A. and Scott R. A. 1999. 'Dynamic Stability of a Rotating Shaft Driven through a Universal Joint', Journal of Sound and Vibration, Vol. 222, No. 1, pp. 19–47.

[38] McLachlan N. 1951. 'Theory and application of Mathieu functions', The Mathematical Gazette, Vol. 31, No. 297, p. 300.

[39] Melde F. 1860. 'Über die Erregung stehender Wellen eines fadenförmigen Körpers', Annalen der Physik. Wiley Online Library, Vol. 187, No. 12, pp. 513–537.

[40] Minorsky N. 1951 'Parametric Excitation', Journal of Applied Physics, Vol. 22, No. 1, p. 49.

[41] Miranda W. and Faria M. 2014. 'Finite Element Method Applied to the Eigenvalue Analysis of Flexible Rotors Supported by Journal Bearings', Engineering, Vol. 6, No. 3, pp. 127–137.

[42] Nayfeh A. H. and Mook D. T. 2008. Nonlinear oscillations. John Wiley & Sons.

[43] Oncescu F., Lakis A. A. and Ostiguy G. 2001. 'Investigation of the stability and steady state response of asymmetric rotors, using finite element formulation', Journal of Sound and Vibration. Vol. 245, No. 2, pp. 303–328.

[44] Ouyang H. and Wang M. 2007. 'A dynamic model for a rotating beam subjected to axially moving forces', Journal of Sound and Vibration, Vol. 308, No. 3-5, pp. 674–682.

[45] Pei Y.-C., Lu H. and Chatwin C. 2010. 'Dynamics of a rotating shaft—disc under a periodic axial force', Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics. Vol. 224, No. 2, pp. 211–219.

[46] Porter B. 1961. 'A theoretical analysis of the torsional oscillation of a system incorporating a Hooke's joint', Journal of Mechanical Engineering Science, Vol. 3, No.4, pp. 324-329.

[47] Porter B. and Gregory R. 1963. 'Non-linear torsional oscillation of a system incorporating a Hooke's joint', Journal of Mechanical Engineering, Vol. 5, No.2, pp. 191-209.

[48] Raffa F. and Vatta F. 2007. 'Dynamic instability of axially loaded shafts in the Mathieu map', Meccanica. Springer Netherlands, Vol. 42, No. 6, pp. 547–553.

[49] Rao J. S. 2011. 'Finite Element Methods for Rotor Dynamics', in History of Rotating Machinery Dynamics SE - 16. Springer Netherlands (History of Mechanism and Machine Science), pp. 269–297.

[50] Sabuncu M. and Evran K. 2005. 'Dynamic stability of a rotating asymmetric cross-section Timoshenko beam subjected to an axial periodic force', Finite Elements in Analysis and Design, Vol. 41, No. 11-12, pp. 1011–1026.

[51] Sabuncu M. and Evran K. 2006. 'The dynamic stability of a rotating asymmetric cross-section Timoshenko beam subjected to lateral parametric excitation', Finite Elements in Analysis and Design, Vol. 42, No. 5, pp. 454–469.

[52] Shahgholi M. and Khadem S. E. 2012. 'Stability analysis of a nonlinear rotating asymmetrical shaft near the resonances', Nonlinear Dynamics, Vol. 70, No. 2, pp. 1311–1325.

[53] Shastry B. P. P. and Venkateswara Rao G. 1987. 'Stability boundaries of a cantilever column subjected to an intermediate periodic concentrated axial load', Journal of Sound and Vibration, Vol. 116, No. 1, pp. 195–198.

[54] Shastry B. and Rao G. 1984. 'Dynamic Stability of Bars considering shear deformation and rotarary inertia', Computers & Structures, Vol. 19, No. 516, pp. 823–827.

ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

[55] Shastry B. and Rao G. 1986. 'Dynamics Stability of short Cantilever Columns subjected to distributed axial loads', Computers & Structures, pp. 1063–1064.

[56] Thomas D., Wilson J. and Wilson R. 1973. 'Timoshenko beam finite elements', Journal of Sound and Vibration, Vol. 31, pp. 315–330.

[57] Wang Z. and Wang D. 2000. 'Dynamic characteristics of a rolling mill drive system with backlash in rolling slippage', Journal of Materials Processing Technology, Vol. 97, No. 1-3, pp. 69–73.

[58] Zahradka J. 1973. 'Torsional vibrations of a non-linear driving system with cardan shafts', Journal of Sound and Vibration, Vol. 26, No. 4, pp. 533–550.