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FRACTURE ENERGY OF FOAMED CONCRETE BY MEANS OF THE THREE-POINT BENDING TESTS ON NOTCHED BEAM SPECIMENS

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ABSTRACT

Foamed concrete has recently gained attention as an alternative to normal concrete in structural engineering. Its low range of densities with good strength, high serviceability and lightness that lead to the many applications. In the last few years, many studies have been conducted to determine the mechanical properties and strengths of foamed concrete, especially high-performance characteristics. However, these studies did not examine the fracture energy, which is the important parameter of structural behaviour and damage mechanisms. Fracture energy represents the ability of a structural element to resist crack propagations during the softening phase. This study, therefore, aims to experimentally investigate the fracture energy of foamed concrete. The notched beam specimens of foamed concrete were prepared at densities of 1400kg/m^3 and 1600kg/m^3. The dimension of beam specimen is 700mm × 150mm × 150mm. Two types of notches; U and V shapes were used. The notch has a depth of 30mm and located at the mid-span of beam specimens. Meanwhile, cube samples were also cast to access the compressive strength. All notched beam specimens were tested using the three-point bending test to obtain the load-deflection profiles. The results were used to calculate fracture energy using Hillerborg model. Meanwhile, the fracture energy was also calculated using Bazant, CEB and Oh models. A comparison of these models shows a favourable agreement with the fracture energy of foamed concrete is in the range of 18N/m to 25N/m. surprisingly, the fracture energy is relatively high for compression strength of foamed concrete at a range of 6.4MPa to 14.7MPa.

Keywords: fracture energy, foamed concrete, notched beam specimen, three-point bending test.

INTRODUCTION

It is cogently undeniable that foamed concrete has become the most popular lightweight material in construction and infrastructural industries. This is due to the advantages offered by foamed concrete such as low density, sufficient durability, excellent fire resistance and good thermal conductivity. With its promising density and strength, foamed concrete is a viable solution for reducing loading, an especially important issue that needs to be tackled in the soft soil area. Typical densities of foamed concrete are ranged around 1000kg/m^3 to 1600kg/m^3 (Rammurty *et al*., 2009) while the compressive strength can achieve up to 12MPa. In addition, high-performance foamed concrete has been introduced by many researchers such as Liu and Jiang (2012), Bing *et al.* (2012) and Hilal *et al*. (2015) to surpass conventional concrete. However, the greatest invention was produced by Just and Middenorf (2009) that propose a high-performance foamed concrete with strength up to 105.7MPa using aluminium powder, superplasticizer and microsilica. This type of foamed concrete can be utilised as cast-in-place beams and columns, load bearing walls, sandwich panels, prestressed structures and refractories.

High renewed interests on the application of foamed concrete as structural elements have lead to various investigations on the engineering properties, strengths and structural behaviours of foamed concrete. Many experimental studies were conducted to determine the compression, tensile and flexural strengths. These strengths are associated with the performance and durability of the structure. In terms of serviceability and crack resistance, fracture energy is the most important parameter that governs the cracking and failure of the structure. It also represents the toughness of the material. Until recently, the investigate on fracture energy is still limited to normal concrete, mortar and lightweight aggregate concrete. Early study by Wang and Li (2003) suggested that the fracture energy of foamed concrete is only a fraction of that obtained from normal concrete. Abd Rahman *et al*. (2015) reviewed extensive test on foamed concrete and concluded that the fracture energy is around 18N/m to 25N/m.

Since there are huge demands about the application of foamed concrete as material for structural elements, but lack of knowledge of its fracture energy, this study intends to examine in-depth the fracture energy of foamed concrete using experimental approach. The establishment of specifications and procedures in the investigation of fracture energy of foamed concrete is based on the current standard that practically applied for mortar and normal concrete. This study contributes to a rational prediction of fracture energy of foamed concrete that later can be implemented in the analysis, design and numerical modelling of structural elements.

FRACTURE ENERGY

In general, fracture energy is defined as the total amount of work required to break a specimen completely per unit ligament area (Li *et al*., 1995; Jirasek *et al*., 2004). It can also be referred to as energy required in the incarnation and the formation of cracks as dissipated per unit area. Basically, fracture energy corresponds to the area of the softening phase at a stress-strain curve. Hillerborg (1985) suggested that fracture energy can be simply determined using Equation (1) below:

$$G_f = \frac{U_o + m_g d_o}{B(W - a_o)} \quad (1)$$

where: U_o is area under the softening phase of the stress-strain curve that obtained from bending test, m_g is specimen weight, d_o is load-point deflection at the fracture, *B* is specimen width, *W* is specimen depth, and a_o is notch depth.

Meanwhile, Comite Euro-International du Beton, CEB (1993) proposed a simple empirical formula to determine the fracture energy. The formula as depicted in Equation (2) is based on two main factors known as compressive strength and aggregate size. Similarly, Bazant (2002) introduced the formula as in Equation (3) that include water-cement ratio as an additional parameter.

$$G_F = \left[0.0469\left(D_{max}\right)^2 - 0.5 D_{max} + 26\right] \times \left(\frac{f_c}{10}\right)^{0.7} \quad (2)$$

$$G_F = 2.5\alpha_0 \left(\frac{f_c}{0.058}\right)^{0.40} \left(1 + \frac{D_{max}}{1.94}\right)^{0.43} \left(\frac{w}{c}\right)^{-0.18} \quad (3)$$

where: α_0 is an aggregate shape factor where 1 for the rounded aggregate and 1.12 for the angular aggregate, D_{max} is the maximum aggregate size, f_c is the compressive strength and *w/c* is the water-cement ratio of concrete.

Martin *et al*. (2007) and Arezoumandi *et al*. (2014) performed a comparative study using these three models and found that Bazant and CEB models showed excellent agreement with the Hillerborg test data. A recent study by Abd Rahman *et al*. (2015) also found that the fracture energy of foamed concrete from Hillerborg model falling within 10% of the predicted by Bazant and CEB models. Inspired by tensile strength and modulus of elasticity, a rather simple formula was proposed by Bazant and Oh (1983):

$$G_F = \left(15.53 + 17.72 f_t\right) f_t^2 \left(\frac{d_a}{E_c}\right) \quad (4)$$

On the other hand, Oh *et al*. (1999) simplified the equation as follows:

$$G_F = \frac{54.24 f_t d_a}{E_c} \quad (5)$$

where: f_t is the tensile strength of concrete, d_a is the diameter of aggregate and E_c is the modulus of elasticity. When the values of f_t and E_c of foamed concrete are not measured directly through experiment, the usual relations as suggested by Byun *et al*. (1998), Jones and McCarthy (2005), Ramamurthy and Nambiar (2009) and Jaini et al. (2015) may be used, which read:

$$E_c = 0.42 f_c^{1.18} \quad (6)$$

$$f_t = 1.03\left(f_c\right)^{0.5} \quad (7)$$

For mortar and normal concrete, f_t and E_c can be calculated based on a recommendation by Eurocode 2 (2004). However, it is noticeable that Equations (4) and (5) may not suitable to be used for foamed concrete. Likely due to its empirical formula may mislead the value of tensile strength and the prediction of Young's modulus. In addition, Rahman and Jaini (2013) used this formula to predict the fracture energy of foamed concrete and the value is 40% underestimated the actual fracture energy.

HILLERBORG MODEL

Hillerborg model or also known as the work-of-fracture method is the most commonly and widely used to determine the fracture energy of brittle materials. Fracture energy is computed as the area under the experimental load-deflection response for a notched beam specimen subjected to a point load. Figure-1 shows a graphical representation of this concept. Since introduced by Hillerborg (1985), many other researchers have implemented a similar technique to determine the fracture energy of concrete, as example Malvar and Warren (1988), Abdalla and Karihaloo (2002), and Antico *et al*. (2011). Moreover, the Hillerborg model was also adopted in a standard test method such as RILEM (1990), NT Build 491 (1999) and ASTM E1820-13 (2014). Early in the adoption of this model, one issue of interest is the impact of size effects on fracture energy, especially the influence of dimensions, geometries and notch conditions. Shah *et al*. (1995) investigated the size effects on fracture energy and found that fracture energy exhibit considerable dependence on beam specimen size. Hanson and Ingraffea (2003) found that fracture energy increases with increasing the depth of beam specimen. Sim *et al*. (2014) stated that the notch is a more critical parameter than the depth of beam specimen. It is should be considered here that fracture energy also depends on the size of aggregate (Elices and Rocco, 2008).

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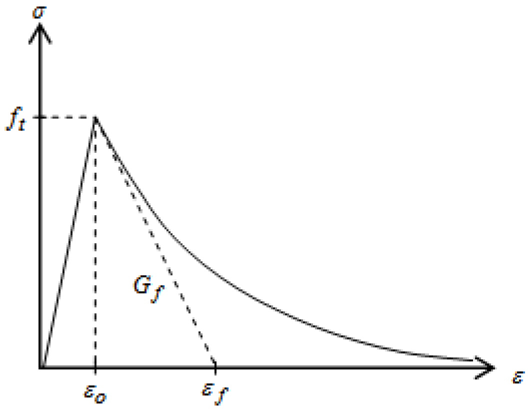


Figure-1. Fracture energy under the stress-strain curve.

According to Prasad and Sagar (2008), size effects may be defined by considering geometrically similar structures or specimens of different sizes, where the nominal strength at the failure stage is taken as:

$$\sigma_N = c_N \frac{P_u}{td} \tag{8}$$

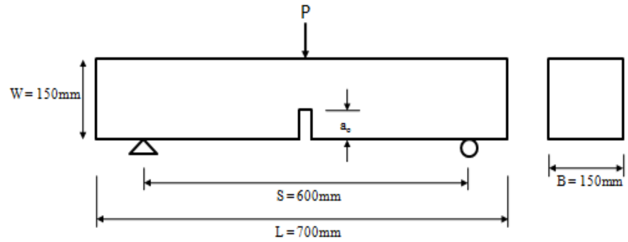
where: c_N is arbitrary constant, P_u is maximum load, t is thickness and d is a characteristic dimension of the beam specimen. Hu and Wittman (2000) and Cifuentes *et al.* (2013) proposed a method of measuring the size independent fracture energy of concrete. Meanwhile, Lee and Lopez (2014) proposed notched beam specimen with counterweight as experimental correction to the current practice in determining the fracture energy.

EXPERIMENTAL DETAILS

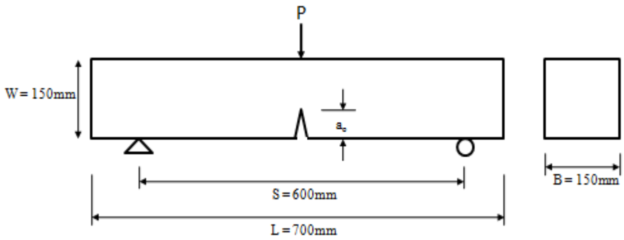
An experimental study to determine the fracture energy of foamed concrete was mainly conducted using the Hillerborg model. Unlike normal concrete and mortar, there are no details of specifications and procedures that can be referred to conduct the experimental study. Therefore, the current standard test method was adopted.

Specimen design

The design of notched beam specimens is based on two standard test methods of RILEM (1990) and ASTM E1820-13 (2014). The effective span to depth ratio of beam specimen was setup as S/W= 4. Hence, the dimension of beam specimen is 700mm × 150mm × 150mm. The depth ratio of the notch was taken as a_0/W = 0.2 as suggested by Oh *et al*. (1997), hence, the depth of the notch is a_0= 30mm. Two types of U and V-notch were used and positioned at the center of beam specimen. The specifications of beam specimen can be referred in Figure-2.



a) U-notched beam specimen



b) V-notched beam specimen

Figure-2. Schematic design of the beam specimens, where depth and width of beam, B and W= 150mm, span of beam, S= 700mm and depth of notch, a_0= 30mm.

In this experimental study, a total of 16 notched beam specimens were prepared for the three-point bending test. It means that each type of concrete has two U-notched beam specimens and another two of V-notched beam specimens. Table-1 shows the quantity of beam specimens.

Table-1. Specimen preparation for the three-point bending test

Type of Concrete	Density (kg/m^3)	Number of specimen	
		U-Notch	V-Notch
Foamed (FC1400)	1400	2	2
Foamed (FC1600)	1400	2	2
Mortar (MR2000)	2000	2	2
Normal (NC2400)	2000	2	2
Total of specimen =		8	8

Mix design and materials

The mix design based on the Department of Environment (DOE) was utilized to evaluate the proportions of foamed concrete, mortar and normal concrete. The proposed densities of foamed concrete are 1400 kg/m^3 and 1600 kg/m^3. Meanwhile, mortar and normal concrete have densities of 2000 kg/m^3 and 2400 kg/m^3, respectively. In order to achieve the targeted compressive strength, especially for foamed concrete, the

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cement-sand ratio, water-cement ratio and foam-cement ratio were fixed according the requirement. The concentration of foam agent is 140kg/liter. Fine and coarse aggregates have maximum diameters of 3mm and 20mm respectively. Table-2 shows the mix design of foamed concrete, mortar and normal concrete. This study was used the Type 1 of Ordinary Portland Cement (OPC) for all types of concrete.

Table-2. Mixed design of foamed concrete, mortar and normal concrete.

Mixture	FC1400	FC1600	MR2000	NC2400
Cement-Sand ratio (C/S)	0.5	0.5	0.5	0.5
Foamed-Cement ratio (F/C)	0.7	0.7	-	-
Water-Cement ratio (W/C)	0.55	0.55	0.5	0.5
Cement (kg)	26	32	40.5	450
Fine aggregate (kg)	50	64	81	930
Coarse aggregate (kg)	-	-	-	968
Water (liter)	15	17.6	20.25	225
Foam (liter)	26	22.4	-	-

Compression test

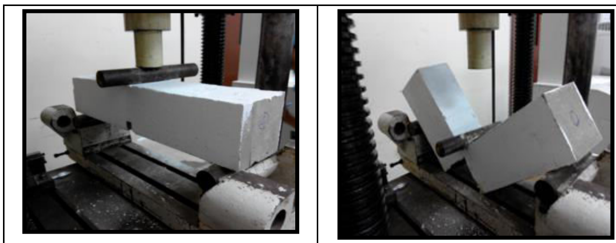
Cube samples with size of 150mm × 150mm × 150mm were casting for the compressive test. Each type of concrete was allocated with six cube samples. The cube samples were placed under compressive loading after 7, 14 and 28 days long curing process to determine the compressive strength. All cube samples were cured using air curing techniques at the ambient temperature. Figure-3 depicts the compressive test on the cube samples.

a) Test setup | b) Failure

Figure-3. Compression test for cube samples.

Three-Point bending test

The three-point bending test was conducted for all beam specimens after 28 days curing period. The universal testing machine was setup to the minimum speed at 0.05mm/min to provide sufficient time during which the propagation of the crack can be detected and to prevent sudden catastrophic failure. The rate of loading should be maintained until failure of beam specimens. Both supports were hinged with rollers. The beam specimens must be handled carefully during the placement on the testing machine to avoid pre-crack growth. The load, deflection and crack formation of beam specimens were recorded to plot the load-deflection profile and crack-load history. Figures 3 and 4 illustrate the test setup for both U-notched and V-notched beam specimens. In this experimental study, the load and deflection were measured directly from the universal testing machine. The stroke of universal testing machine was calibrated with the linear variable displacement transducer (LVDT) to ensure the validity of data.



a) Test setup | b) Failure

Figure-4. Three-point bending test for U-notched beam specimen.

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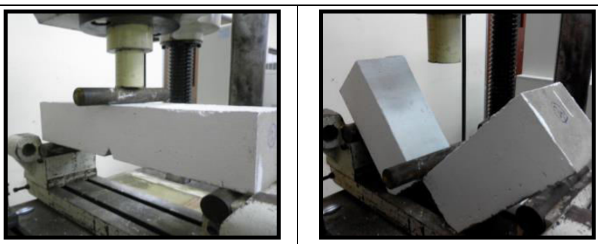

a) Test setup b) Failure

Figure-5. Three point bending test for V-notched beam specimen.

EXPERIMENTAL RESULTS

Compressive strengths and load-deflection profiles were obtained from compression test and three-point bending test respectively. These experimental results are paramount important for prediction of fracture energy. A comparison of fracture energy, calculated using a Hillerborg model with Bazant, CEB and Oh models were performed to appraise the results from the three-point bending test.

Compressive strength

Compressive strength of foamed concrete, mortar and normal concrete at 7, 14 and 28 days is presented in Figure-6. The strength of foamed concrete at densities of 1400kg/m^3 and 1600kg/m^3 has shown a standard compressive strength at 6.4MPa and 14.7MPa respectively. The compressive strengths of foamed concrete are considerable accepted as suggested by Aldrige (2000) and the British Cement Association (1994) where the strength should in the range of 6.0MPa to 8.0MPa for density 1400kg/m^3 and 7.5MPa to 10.0MPa for density 1600kg/m^3. Meanwhile, mortar and normal concrete achieved the respective target characteristic strength of 25MPa. The compressive strength of mortar at a density of 2000kg/m^3 is 23.5MPa while normal concrete is 26.3MPa for density of 2400kg/m^3.

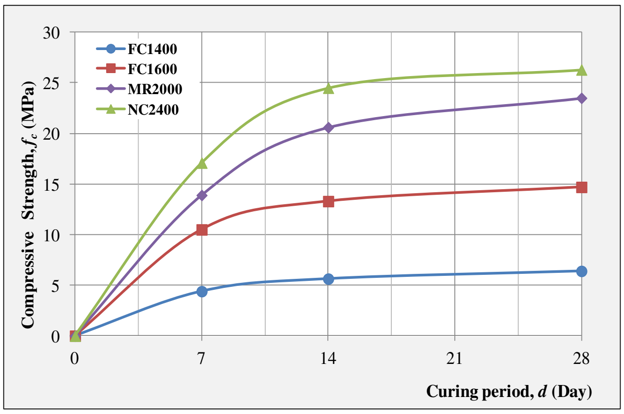

Figure-6. Compressive strengths of the cube specimens for foamed concrete (FC1400 and FC1600), mortar (MR2000) and normal concrete (NC2400).

Load-Deflection profile

In Hillerborg model, the load-deflection profile is the important parameters that need to be determined. The load-deflection profiles for U-notched and V-notched beam specimens are displayed in Figures 7 and 8. It was found that the ultimate load of U-notched and V-notched beam specimens are different, where U-notched beam specimens produce significantly high value compared to V-notched beam specimens. This indicates that high stress occurs at the narrow point of V-notch and leads to the faster formation of cracks. However, the total area under the load-deflection profiles produced by V-notched beam specimens are slightly larger. Unlike normal concrete and mortar that experience catastrophic failure, foamed concrete shows complete softening phase. The residual strength due to the complex microstructure allow it to sustain loading until the beam specimens broken into two halves.

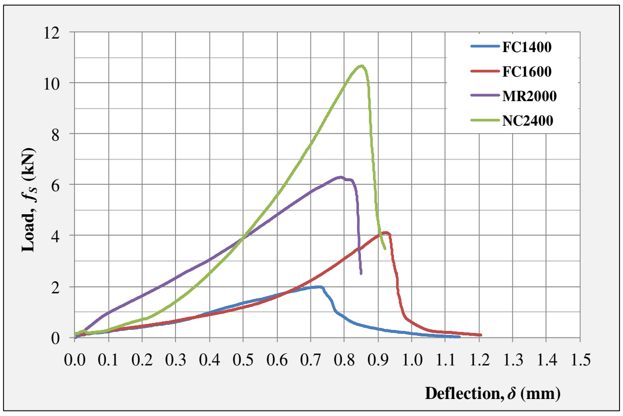

Figure-7. Load-deflection profiles of the U-notched beam specimens for foamed concrete (FC1400 and FC1600), mortar (MR2000) and normal concrete (NC2400).

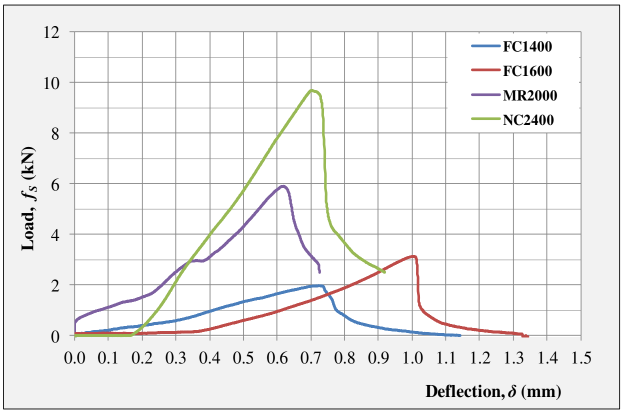

Figure-8. Load-deflection profiles of the V-notched beam specimens for foamed concrete (FC1400 and FC1600), mortar (MR2000) and normal concrete (NC2400).

Fracture energy

The determination of fracture energy using Hillerborg model requires parameters such as total area under load-deflection profile, weight of beam specimen, dimension of beam and the depth of notch. Meanwhile, compressive strength becomes an important parameter for Bazant and CEB models. Oh model requires tensile strength and Young's modulus that can be determined by experimental study or empirical formula. The data of compressive strength, ultimate strength and maximum deflection for all types of concrete are shown in Table-3. The values of fracture energy were calculated using Equations (1) to (3) and Equation (5). Table-4 shows the fracture energy for all types of concrete using U-notched and V-notched beam specimens.

Using Hillerborg model, the prediction of fracture energy for U-notched and V-notched beam specimens are almost identical, especially for foamed concrete and mortar. Fracture energy using V-notched beam specimens gave slightly higher value around 15% to 25% for foamed concrete and mortar, while approximately 32% for normal concrete. The correspondence of fracture energy proof that the type of notch plays insignificant role. In view that the strength of foamed concrete is mostly governed by the amount of sand, sand-cement ratio and particle size distribution of sand as stated by Ravindra *et al*. (2005), hence, the fracture energy of foamed concrete is lower than mortar and normal concrete. The present of voids on foamed concrete also reduces the strength, consequently affected the crack resistance. Although the fracture energy of foamed concrete is only around 18N/m to 25N/m, it is indicates relatively high level despite lower compressive strength.

Table-3. Parameter of strengths and maximum deflection.

Type of concrete	Density (kg/m^3)	f_c (MPa)		f_s (kN)		δ_{max} (mm)	
		U	V	U	V	U	V
Foamed	1400	6.40	6.40	10.60	9.66	0.85	0.72
Foamed	1600	14.70	14.70	6.30	5.90	0.79	0.62
Mortar	2000	23.50	23.50	4.14	3.13	0.92	1.00
Normal	2400	26.30	26.30	1.98	1.97	0.72	0.73

Table-4. Fracture energy of foamed concrete, mortar and normal concrete using U-notched and V-notched beam specimens.

Type of concrete	Density (kg/m^3)	Fracture energy (N/m)				
		Hillerborg		Bazant	CEB	Oh
		U	V			
Foamed	1400	18.20	18.13	18.28	19.02	26.41
Foamed	1600	20.44	25.35	25.50	34.05	15.00
Mortar	2000	35.96	31.12	31.30	47.28	12.90
Normal	2400	63.13	83.36	32.86	51.14	90.39

Referring to Hillerborg model, the fracture energy of mortar is in the range of 31N/m to 35N/m. This fracture energy is close to that suggested by Wittmann (2002). On the other hand, normal concrete has fracture energy around 75N/m to 100N/m (Muralidhara, 2011). Results from experimental study revealed a favourable agreement. Despite the mortar has almost similar compressive strength with normal concrete, but the fracture energy is very noticeably low. According to Darwin *et al*. (2001), the size of aggregate is likely influences the fracture energy where the small size of aggregate tends to produce low and minimum fracture energy. Moreover, Ishiguro (2001) found that the fracture energy of mortar is slightly increased by increasing the compressive strength.

A comparison of fracture energy predicted using a Hillerborg model with Bazant, CEB and Oh models are shown in Figure-9. The fracture energy significantly increase by density. At the lower densities from 1400kg/m^3 to 2000kg/m^3, Bazant model has provided results at almost exactly as Hillerborg model. However, both Bazant and CEB models underestimated the fracture energy of normal concrete. Although Oh model not perfectly determine the fracture energy of foamed concrete and mortar, but the fracture energy of normal concrete is within the expected range. As mention earlier, Oh model may only suitable for normal concrete that has established

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method in predicting the tensile strength and modulus of elasticity.

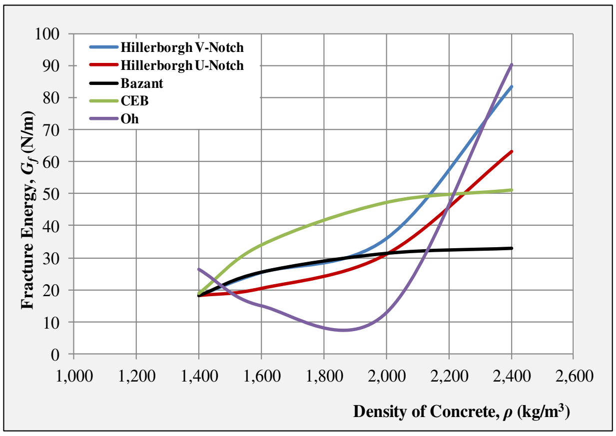

Figure-9. A comparison of fracture energy of foamed concrete, mortar and normal concrete using various models.

CONCLUSIONS

An experimental study was conducted on the fracture energy of foamed concrete using the three-point bending test. This test mainly measured the amount of energy absorbed when the beam specimen broke in half. Compression strengths and load-displacement profiles were obtained and analyzed. The Hillerborg model was used to determine fracture energy, where the area of load-deflection profile and maximum deflection become the predominant parameters. The results were compared with that calculated using Bazant, CEB and Oh models. It is found that the fracture energy of mortar and normal concrete are at acceptable range of 31N/m to 84N/m. This indicates that the specifications and procedures for the three-point bending test were properly designed and conducted. In addition, the fracture energy of foamed concrete ranged from 18N/m to 25N/m. Although, the fracture energy of foamed concrete is a fraction of normal concrete, it is relatively high for compressive strength around 6.4MPa to 14MPa. It is found that the fracture energy significantly increase by density and compressive strength.

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REFERENCES

Abdalla, H.M. and Karihaloo, B.L. 2003. Determination of size-independent specific fracture energy of concrete from three-point bend and wedge splitting tests. Magazine of Concrete Research. 55(2), pp. 133-141.

Abd Rahman, N., Jaini, Z.M., Abd Rahim, N.A. and Abd Razak, S.A. 2015. An experimental study on the fracture energy of foamed concrete using v-notched beams. Proceeding of the International Civil and Infrastructure Engineering Conference, Springer, Singapore. pp. 97-108.

Aldrige, D. 2000. Foamed concrete: properties, applications and potential. Proceeding of One Day Awareness on Foamed Concrete, Dundee, United Kingdom.

Antico, F., De la Varga, I. and Pour-Ghaz, M. 2011. Experimental and numerical analysis of the failure of notched concrete beams. Report CE597-NLFM, Purdue, United States.

Arezoumandi, M., Ezzell, M. and Volz, S.J. 2014. A comparative study of the mechanical properties, fracture behavior, creep, and shrinkage of chemically based self-consolidating concrete. Frontiers Structural and Civil Engineering. 8(1), pp. 35-45.

ASTM E1820-13. 2014. Standard test method for measurement of fracture toughness. ASTM International: West Conshohocken, United States.

Bazant, Z.P. and Oh, B.H. 1983. Crack band theory for fracture of concrete. Materials and Structures. 16, pp. 155-177.

Bazant, Z.P. 2002. Concrete fracture models testing and practice. Engineering Fracture Mechanics. 69, pp. 165-205.

Bing, C., Zhen, W. and Ning, L. 2012. Experimental research on properties of high-strength foamed concrete. Journal of Materials in Civil Engineering. 24(1), pp. 113-118.

British Cement Association. 1994. Foamed concrete: composition and properties. Report 46.042, Slough, United Kingdom.

Byun, K.J., Song, H.W. and Park, S.S. 1998. Development of structural lightweight foamed concrete using polymer foam agent. Proceeding of the 9th International Congress on Polymers in Concrete, Bologna, Italy.

Cifuentes, H., Alcalde, M. and Medina, F. 2013. Measuring the size-independent fracture energy of concrete. International Journal for Experimental Mechanics. 49, pp. 54-59.

Comite Euro-International du Beton. 1993. CEB-FIB Model Code 1990. Thomas Telford: Lausanne, Switzerland.

Darwin, D., Barham, S., Kozul, R. and Luan, S. 2001. Fracture energy of high-strength concrete. ACI Material Journal. 98(5), pp. 410-418.

Elices, M. and Rocco, C.G. 2008. Effect of aggregate size on the fracture and mechanical properties of simple concrete. Engineering Fracture Mechanics. 75(13), pp. 3839-3851.

Eurocode 2. 2004. Design of concrete structures-Part 1-1: General rules and rules for building. British Standards: London, United Kingdom.

Gospodinov, G. and Kerelezova, I. 1999. Numerical modelling and analysis of plain concrete notched beams by means of fracture mechanics. Annuaire de L'universite D'architecture, de Genie Civil et de Geodesie. 40(2), pp. 1-13.

Hanson, J.H. and Ingraffea, A.R. 2003. Using numerical simulations to compare the fracture toughness values for concrete from the size-effect, two-parameter and fictitious crack model. Engineering Fracture Mechanics, 70, pp. 1015-1027.

Hilal, A.A., Thom, N.H. and Dawson, A.R. 2015. The use of additives to enhance properties of pre-formed foamed concrete. International Journal of Engineering and Technology. 7(4), pp.286-293.

Hillerborg, A. 1985. The theoretical basis of a method to determine the fracture energy of concrete. Materials and Structure. 18(4), pp.291-296.

Hu, X. and Wittmann, F. 2000. Size effect on toughness induced by crack close to free surface. Engineering Fracture Mechanics. 65, pp. 209-221.

Ishiguro, S. 2007. Experimental and analysis of fracture properties of grouting mortars. Proceeding of Fracture Mechanics of Concrete and Concrete Structures. Taylor and Francis, Catania. pp. 293-298.

Jaini, Z.M., Mokhatar, S.N., Feng, Y.T. and Seman, M.A. 2015. 2D multi-scale simulation and homogenization of foamed concrete containing rubber bars. Proceeding of the International Civil and Infrastructure Engineering Conference, Springer, Singapore. pp. 625-638.

Jirasek, M., Rolshoven, S. and Grassl, P. 2004. Size effect on fracture energy induced by non-localit. International Journal for Numerical and Analytical Methods in Geomechanics. 28, pp. 653-670.

Jones, M.R. and McCarthy, A. 2005. Preliminary views on the potential of foamed concrete as a structural material. Magazine of Concrete Research. 57(1), pp. 21-31.

Just, A. and Middenorf, B. 2009. Microstructure of high-strength foamed concrete. Material Characterization. 60(7), pp. 741-748.

Lee, J. and Lopez, M. M. 2014. An experimental study on fracture energy of plain concrete. International Journal of Concrete Structures and Materials. 8(2), pp. 129-139.

Li, Y.N., Hong, A.P. and Bazant, Z.P. 1995. Initiation of parallel cracks from surface of elastic half-plane, International Journal of Fracture. 69, pp. 357-369.

Liu, X. and Jiang, Y. 2012. Preparation of foamed concrete with high-strength and low-shrinkage. Applied Mechanics and Materials. 174-177, pp. 1299-1305.

Oh, B.H., Jang, S.Y. and Byun H.K. 1999. Prediction of fracture energy of concrete. KCI Concrete Journal. 11(3), pp. 211-221.

Malvar, J. and Warren, G. 1988. Fracture energy of three-point bending tests on single-edge notched beams. Experimental Mechanics. 28(3), pp.266-272.

Martin, J., Stanton, J., Mitra, N. and Lowes, L.N. 2007. Experimental testing to determine concrete fracture energy using simple laboratory test setup. ACI Materials Journal. 104(6), pp.575-584.

Muralidhara, S., Prasad, B.K.R., Karihaloo, B.L. and Singh, R.K. 2011 Size independent fracture energy in plain concrete beams using tri-linear model. Journal of Construction and Buildings Material, 25, pp.3051-3058.

NT Build 491. 1999. Concrete and mortar, hardened: fracture energy mode 1-three-point bending tests on notched beams. Nordtest: Espoo, Finland.

Prasad, B.K.R. and Sagar, R.V. 2008. Relationship between AE energy and fracture energy of plain concrete beams: experimental study. Journal of Materials in Civil Engineering. 20, pp. 212-220.

Rahman, M.H.A. and Jaini, Z.M. 2013. The combined finite-discrete element analysis of precast lightweight foamed-concrete sandwich panel (PLFP) under axial load. Proceeding of the International Conference on Advances in Structural, Civil and Environmental Engineering, Kuala Lumpur, Malaysia.

Rammurthhy, K., Kunhanndan Nambiar, E.K. and Indu Siva Ranjani, G. 2009. A classification of studies on properties of foam concrete. Journal of Cement and Concrete, 31(6), pp. 388-396.

Ramamurthy, K. and Nambiar, E.K.K. 2009. A classification of studies on properties of foam concrete. Cement and Concrete Composites. 31(6), pp. 388-396.

Ravindra, K.D., Moray, D.N. and McCarthy, A. 2005. Use of foamed concrete in construction. Thomas Telford: London, United Kingdom.

RILEM. 1990. Fracture mechanics of concrete-test methods, determination of fracture parameters of plain concrete using three-point bend tests. Materials and Structures, 23, pp. 457-460.

Shah, S.P, Swartz, S.E. and Ouyang, C. 1995. Fracture mechanics of concrete: applications of fracture mechanics to concrete, rock and other quasi-brittle materials. Wiley: New York, United States.

Sim, J.I., Yang, K.H., Lee, E.T. and Yi, S.T. 2014. Effects of aggregate and specimen size on lightweight concrete fracture energy. Journal of Materials in Civil Engineering. 26, pp. 845-854.

Wang, S. and Li, V. C. 2003. Lightweight engineered cementitious composites. Proceeding of 4th International RILEM Workshop on High Performance Fiber-Reinforced Cement Composite, Michigan, United States.

Wittman, F.H. 2002. Crack formation and fracture energy of normal and high strength concrete. Material and Structures. 27(4), pp. 413-423.