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# APPLICATION OF CONDITIONED REVERSE PATH METHOD TO LARGE MULTI-DEGREE-OF-FREEDOM NONLINEAR STRUCTURE

H. M. Y. Norfazrina, P. Muhamad, B. A. Aminudin, A. M. Wahab and M. R. Raihan
Intelligent Dynamics and System (IDS) Research Laboratory, Malaysia-Japan International Institute of Technology (MJIIT), Universiti Teknologi Malaysia Kuala Lumpur, Kuala Lumpur, Malaysia
E-Mail: hmynorfazrina2@live.utm.my

**ABSTRACT**

Nonlinear system identification (NSI) has received a fair amount of attention in recent years. The highly individual nature of nonlinearity makes it important to fully understand the implication of nonlinear behavior. Nevertheless, the application of NSI to a more complex multi-degree-of-freedom (MDOF) system is still scarce and limited. A spectral approach called the Conditioned Reverse Path (CRP) method is chosen to test the capability of said method in identifying underlying linear characteristics. A 240 degree-of-freedom airplane-like structure with a cubic nonlinearity attached between the wings and the engines is modelled using MSC Patran/Nastran software to generate the desired responses. The obtained responses are then imported to Matlab software to proceed with spectral analysis utilizing the CRP method. Unfortunately, the CRP could not depict its capability as one of the advantageous method in the NSI area and further works are necessary to improve the airplane-like modelling thus determine the true underlying linear system. Once the modelling is reliable to be applied in the CRP, a more recent method called the Orthogonalised Reverse Path (ORP) method will be applied as future works.

**Keywords:** conditioned reverse path, nonlinear systems, system identification, multi-degree-of-freedom system.

## INTRODUCTION

Manufactured structures such as high-rise buildings, bridges, automotive vehicles, electronic devices and even furnitures are often subjected to damage and failure. Researchers and engineers have done extensive studies on how to minimize the risk of these failures, bringing Structural Health Monitoring (SHM) to the centre of interest in the engineering world. However, the difficulty increases when nonlinearity is involved in the structural dynamic analysis. The knowledge of nonlinear system identification has expanded these few decades but still limited and vast majority of applications are only concerned about linear structures. It is found that nonlinearity is commonly associated with damage in a system (Worden *et al*. 2008). The identification of the nonlinearity can give certain information and understanding on the behavior and the possible aftermaths of a certain system. It is a good contribution to the SHM area if the nonlinear system identification is reliable in taking preventive action against failures.

The methodology used to identify linear system is inadequate when applied to nonlinear system. The three general phases suggested to nonlinear system identification process are detection, characterization and parameter estimation (Kerschen *et al*., 2006). Several techniques have been developed to identify the presence of nonlinearity and to estimate the involved parameters. Some of the techniques are namely by-passing nonlinearity by linearization, time-domain method, frequency-domain method, time-frequency method and modal methods. Even so, frequency or spectral approach is commonly used in nonlinear system identification for obtaining the frequency response function (FRF). The FRF can characterize the system's dynamic responses and gives direct interpretable information about the response behaviour.

The Duffing oscillator has been widely studied as the classical model of nonlinear system (Duffing, 1918), (Wang *et al*. 2009), (Cao *et al*. 2009). This single degree-of-freedom (DOF) system represents an impediment compared with linear system identification for which the structure of the functional is well defined. It gets more fascinating when nonlinearity existed in a complex multi-degree-of-freedom (MDOF) system. Many has extended the application of nonlinear system identification methodology to MDOF system since most of the real operating structure are actually consist of MDOF structure. For instance, a frequency-domain technique called the harmonic balance nonlinearity identification (HBNID) method was originally proposed by Casas for a single DOF nonlinear system. Since then, few works have been done to further extend its application towards MDOF nonlinear systems. There is also an attempt to apply this method to MDOF fluid-structure systems (Thothadri *et al.* 2003). It is found that the HBNID method can capture well the unknown parameters if the model structure is well defined. The reality is it is quite challenging to have full information of certain model structure when MDOF structure is involved.

The frequency-domain nonlinear subspace identification method (FNSI) method is another example of the frequency-domain method in nonlinear system identification area which formulation is based on the feedback formulation. It assumes that an underlying linear vibration system is existed in the system and the location of the nonlinearity is known. One study exploit this

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method to a single DOF and 5-DOF systems with two nonlinearities (Noël and Kerschen 2013). Results show that the FNSI can identify the linear frequency response functions (FRF) of the system and the nonlinear coefficients are well-identified for intermediate excitation levels.

Another frequency-domain technique which has been extended from single DOF system to MDOF system is the reverse path (RP) method. The principal of RP method is mathematically reverse the input to the output from the dynamic system and then proceed with the spectral analysis. Readers can refer to (Bendat, 1998) for a detailed review on RP method. (Rice & Fitzpatrick 1988) have developed the RP approach by generalising it to MDOF system. The nonlinear coefficients are calculated with a physical model of the underlying linear structure. This advancement of RP method is a breakthrough for a better methodology in system identification for a complex MDOF structure. However, the drawback of this RP method is excitations are needed to be applied at every response location.

Since the RP method shows a promising future for nonlinear system identification, more studies have been flourished towards this method. Magnevall has applied the RP method in simulations and experiments on a MDOF continuous nonlinear mechanical systems with multiple nonlinearities (Magnevall *et al.* 2012). The unconditioned inputs are used and the location of the nonlinearities is known beforehand. Only one excitation is needed in one point and it will discretize the continuous nonlinear structure according to the measuring points. Subsequently, the underlying linear system can be estimated using reciprocity and the position vectors of the nonlinearities.

Another enhancement of the RP method termed the conditioned reverse path (CRP) method has shown a relatively accurate values of the nonlinear coefficient and its underlying linear characteristics (Richards and Singh 1998). The CRP method separates the nonlinear elements of the response with a single excitation and obtains the true FRF matrix using conditioned spectral analysis. A lot of research works has applied the CRP method and it is admitted that the CRP method is reliable to detect and to measure nonlinearities in MDOF system (Marchesiello 2003), (Garibaldi 2003), (Kerschen et al. 2003), (Kerschen and Golinval 2005), (Muhamad *et al*. 2012), *(Norfazrina et al*. 2015).

Nevertheless, an issue arised in concern of the CRP method. Recently, it is claimed that error is inevitably accumulated when parameter estimates are extracted during spectral analysis in the CRP method (Muhamad *et al.* 2012). This is due to the recursive process when the spectra are being conditioned. Fortunately, the author has suggested ways to improve the performance without a major change to the CRP algorithm. In this study, a finite element model with large number of DOF will be created using FEA software called MSC Patran/Nastran and the CRP method will be applied to the modelling to test the reliability of the modelling and the robustness of the applied method.

## THE CONDITIONED REVERSE PATH METHOD

### The CRP algorithm

A lot of publications has written in details about the CRP method, but for the convenience of readers a brief explanation on this method will be presented here. In the CRP method, there are two steps need to be fulfilled. First step is to compute the estimates of conditioned spectra which is those of the underlying linear system. In other words, the spectrum is already cleaned from any nonlinearity. The second step is to estimate the nonlinear coefficients using the conditioned spectra. The equation of motion of the nonlinear MDOF system is specified by,

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) + \sum_{j=1}^{n} A_j z_j(t) = x(t) \quad (1)$$

where $m$, $c$ and $k$ are the mass, stiffness and damping matrices, $y(t)$ and $x(t)$ are the output response and applied force vector, respectively. One applies the Fourier transform to Equation (1) to transform to frequency domain.

$$B(\omega)Y(\omega) + \sum_{j=1}^{n} A_j Z_j(\omega) = X(\omega) \quad (2)$$

$B(\omega)$ is the linear dynamic stiffness matrix,

$$B(\omega) = -\omega^2 M + i\omega C + K \quad (3)$$

The CRP method will then conditioned the spectra that have had been cleaned consecutively from the nonlinearities. Once the conditioning is complete, the linear frequency response functions (FRFs) can be computed by,

$$H_{C1}(\omega) = G_{YX}(\omega)^{\dagger} G_{XX}(\omega),$$
$$H_{C2}(\omega) = G_{YY}(\omega)^{\dagger} G_{YX}(\omega) \quad (4)$$

where $G_{YY}$ is the power spectral density (PSD) of the response, $G_{XX}$ is the PSD of the force and $G_{YX}$ is the cross PSD between the response and the force. The dagger implies a pseudo-inverse. The nonlinear coefficients are estimated using Equation (5),

$$A_i^T Q_i H^T = G_{ii(-1:i-1)}^{\dagger} G_{iX(-1:i-1)} H^T - G_{iY(-1:i-1)} - \sum_{j=1}^{n} G_{ij(-1:i-1)} A_j^T H^T \quad (5)$$

where $Q_i = G_{ii}^{\dagger} G_{ii}$. The matrix $Q_i$ contains the information on the location of nonlinearity (Muhamad *et al*. 2012). The CRP identifies the coefficients by reversing the computation starting from $A_n$ to $A_1$. Since the nonlinear

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coefficients are frequency dependent, the actual value of the coefficients is computed by taking the spectral mean. In this study, $H_{C2}$ estimator is used to determine the conditioned FRF of the airplane-like structure.

### Application to MDOF system

In previous work, the CRP method was applied to a single DOF system (Duffing oscillator) and a 3-DOF system with a cubic nonlinear stiffness attached between the third mass and the ground (Norfazrina *et al.* 2015). The responses (output) were generated using the Simulink software paired with Matlab. The FRFs for the 3-DOF system using the conventional unconditioned H1 estimator and the conditioned CRP are compared. It is found that the unconditioned H1 estimate shows severe bias because of the nonlinearity while the CRP shows a good agreement with the true underlying linear FRF. Table-1 summarises the nonlinear coefficient spectral means of the system. The estimated nonlinear stiffness coefficients for CRP are found to be close with the true values.

**Table-1.** The estimated nonlinear stiffness coefficients of the 3-DOF system (Norfazrina *et al*. 2015).

| Mode | True $k_3$ (GN/m$^3$) | Estimated $k_3$ CRP (GN/m$^3$) |
|---|---|---|
| 1 | 1.0 | 0.99 |
| 2 | 1.0 | 0.95 |
| 3 | 1.0 | 0.99 |

There were also various different kind of nonlinearities and location manipulated to observe the capability of CRP method in identifying the linear behavior of those nonlinear systems (Richards and Singh 1998), (Richards and Singh, 2000), (Muhamad *et al.* 2012). It is observed that the calculated underlying linear FRF from the CRP method is in a good agreement with that of the true linear one. This proves that the CRP method is capable in identifying the underlying linear structure of nonlinear systems up to 3-DOF system.

The application of CRP method continues with a 7-DOF nonlinear system, only this time it was done experimentally. Figure-1 shows the experimental set-up taken from (Kerschen *et al.*, 2003), (Muhamed *et al.* 2012). Two steel beams are used; a clamped beam with a thin beam attached at the end of the main beam. The thin beam was used to create enough stiffening effect to let the beam behave nonlinearly. The effect of gravity was also examined and understood that better results can be obtained if the excitation is applied horizontally to the steel beam. It is shown that the CRP can lower the distortions produced in the FRFs and can give a close estimation of nonlinear coefficients when compared with that of the linear ones.

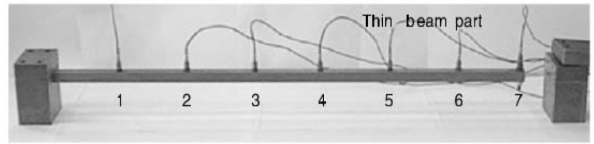


**Figure-1.** Experimental set-up for 7-DOF system taken from (Kerschen *et al*. 2003).

These great achievements of the CRP method has given the motivation to further intensify the application of CRP towards larger nonlinear MDOF system. Therefore, the aims for current study is to develop the algorithm of CRP for larger MDOF nonlinear system and assess the robustness of developed algorithm towards an efficient nonlinear system identification approach.

## APPLICATION TO LARGE MDOF NONLINEAR STRUCTURE

### Airplane-Like structure

Kerschen *et al*. had applied the CRP method to a large MDOF nonlinear system and it was focused on finite element nonlinear model updating techniques. The suggested corrections of the parameters' values were also included (Kerschen and Golinval, 2005). In current study, we will focus on finding the appropriate methodology in modelling the airplane-like structure and test the capability of the CRP method in identifying the underlying linear structure of the nonlinear system with higher number of DOF. There are two steps of methodology taken in this study,

(a) A finite element model of an airplane-like structure is created using an FEA software called the MSC Patran and for solving the analysis, the MSC Nastran (NASA Structural Analysis) solver is used. The required responses (displacement and acceleration) are exploited in the second step.
(b) The responses are manipulated in the CRP method using Matlab software. The conditioned FRF ($H_{C2}$) and nonlinear coefficients ($\beta_1$ for the left engine and $\beta_2$ for the right engine) are then determined.

**Table-2.** Material properties of airplane-like structure.

| Young's modulus, $E$ | 205 GPa |
|---|---|
| Density, $\rho$ | 7850 kg/m$^3$ |
| Poisson ratio, $\upsilon$ | 0.3 |
| Lumped mass (engine) | 0.5 kg |

A numerical case study is conducted to a 240 degree-of-freedom airplane-like structure similar with the work mentioned previously (Kerschen and Golinval 2005). The wing's length is 1.5 m while the airplane body or the

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fuselage is a hollow rectangle with a length of 1.2 m. The material properties of the structure are summarized in Table-1. The structure of each component is modelled using the beam elements and the engines are represented with lumped masses. A multi-point constraints (MPC) is modelled as the rigid connection between the wings and the fuselage. A spring element with cubic nonlinearity is affixed at the junction between both wings and engines.

The nonlinearity affects the translational movement along $y$ direction and the nonlinear functions are written as,

$$z_1 = 3\text{x}10^{10} (y_{17} - y_{39})^3$$
$$z_2 = 3\text{x}10^{10} (y_{23} - y_{40})^3 \quad (6)$$

Nonlinearities $z_1$ and $z_2$ were applied to the left and the right engines, respectively. A white noise with a frequency range of 0 to 150 Hz was applied as the excitation force at the farthest point on the left wing. In conclusion, the modelling of the airplane-like structure was completed with a total of 1 MPC node, 2 lumped masses, 35 beam elements and 40 nodes.

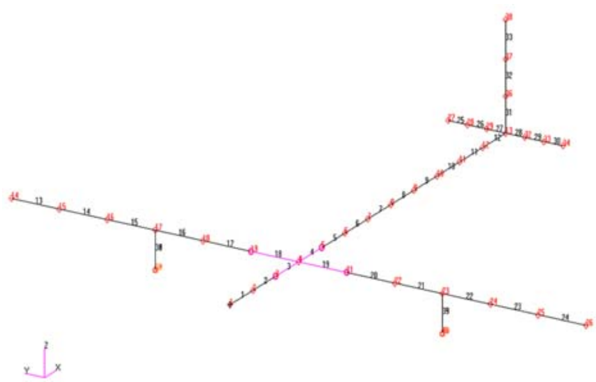

**Figure-2.** The finite element model of the airplane-like structure using MSC Patran/Nastran.

As mentioned previously, the responses are simulated through numerical simulation using MSC Patran/Nastran software. Figure-2 shows the finite element model of the airplane-like structure using the software. Solution type Transient Response (Solution Sequence 112) is applied in order to obtain the “experimental” time histories of the periodic response. The displacement and acceleration are determined at several nodes in the $y$ and $z$ direction; three at the fuselage, five at the wings and three at the tails.

Once completed, the responses are then imported into the Matlab software to exploit the CRP method. The signal processing parameters used in this study are as the following; the sampling frequency was 300 Hz, 8192 of Fast Fourier Transform (FFT) points, the time step used throughout the simulation was 6 ms and a Hanning window was used to minimize the effects of leakage.

## RESULTS AND DISCUSSIONS

In simulation study, one needs to give an extra attention on how to add the nonlinearity values in the studied structure. Originally, the beam elements between the wings and engines are modelled as spring elements containing a spatial data of nonlinear values to create a nonlinear behavior. However, the lumped mass, representing the engines, are not moving and behaves like a fixed beam during the simulation. It is observed that there are no FRF generated at the engines. After few tests and some discussions, it is understood that the spring is too soft and has low stiffness which constrains the movement of the engines. Finally, the nonlinear modelling is created by combining one beam element with the spring element and put between the wings and the engines. In order to ensure cubic nonlinearity is generated in the system, the excitation force versus the displacement at the left engine is produced. Figure-3 shows the obtained plot and it is clearly seen that hardening cubic stiffness nonlinearity is induced in the investigated structure.

In the early stage, we conducted the nonlinear system identification on the left side of the structure, assuming that the airplane-like structure is symmetrical. Therefore, we only focus on the left wing with $z_1$ nonlinearity at the left engine. As the nonlinear system identification using the CRP method is conducted, it is realized that it is not possible to consider half of the structure eventhough it is symmetrical. The reason to why we have to include all parts of the structure is that the algorithm of the CRP method is dependent on both force excitation and responses. The excitation force and the location of nonlinearity must be known beforehand to be applied with the CRP algorithm. In this case study, the excitation force is applied only at the left wing tip of the structure and it will completely affect the whole structure’s response and behavior. To truncate the response from the right part of the structure will give incorrect underlying linear behavior.

For comparison purpose, the spring element at the junctions between the wings and engines -which contains the nonlinear functions- are removed and the same methodology is performed, insinuating a “linear” case is generated to the airplane-like modelling. Figure-4 shows the magnitude of the unconditioned and conditioned FRF using

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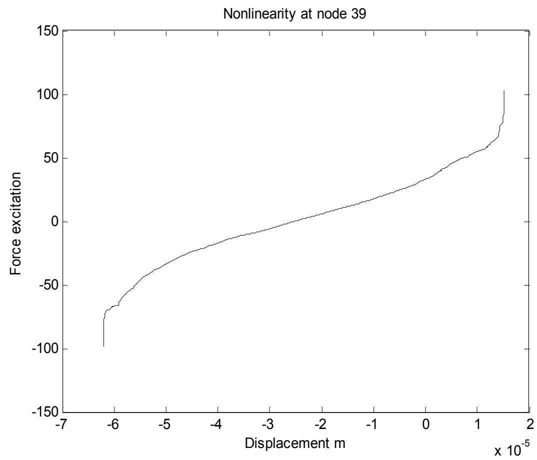


**Figure-3.** Force versus displacement at the left engine; a hardening stiffness nonlinearity.

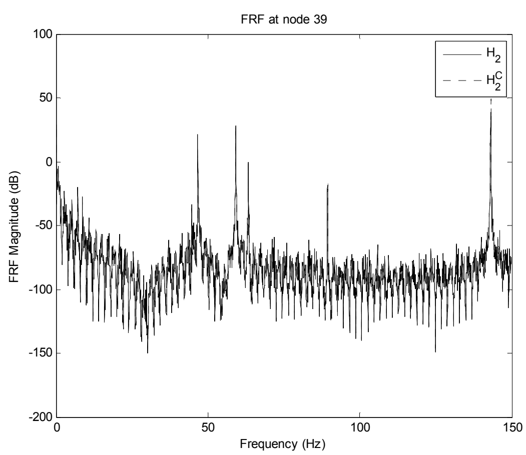


**Figure-4.** Unconditioned ($H_2$) and conditioned ($H_{C2}$) FRF estimator for the left engine.

the conventional $H_2$ estimator and the CRP at the left engine of the airplane-like structure. Although the FRF seems to be rough and polluted, five peaks can be clearly seen from the system. It seemed like the FRF are completely overlap each other for both unconditioned and conditioned FRF, therefore a closer inspection is needed to distinguish the dynamical behavior. Figure-5 shows the first mode of the FRFs at the same left engine. Both FRFs have the same natural frequency (46.7 Hz) and it is also seen, although hardly, that the magnitude from the unconditioned estimators is shifted towards higher magnitude. This is an interesting finding since most of the unconditioned FRF investigated in previous studies, they usually have a shifted natural frequency when nonlinearity existed in the system. It is presumed that the response generated in the system is inaccurate and not completely nonlinear. Therefore, the FRF for unconditioned, conditioned (CRP) and "linear" cases are compared together in Figure 6 to confirm the assumption.

It is shown in Figure 6 that the "linear" FRF are completely overlap with the unconditioned FRF. The main purpose of using CRP method is to extract the underlying linear FRF of a nonlinear system; therefore the FRF from CRP method should be in good agreement with the linear FRF. In addition, the nonlinear coefficients calculated are smaller than the real values of the nonlinearity ($3\times10^{10}$); $-1.12\times10^{-10}+i2.11\times10^{-9}$ and $-3.85\times10^{-9}+i7.19\times10^{-9}$ for $\beta_1$ and $\beta_2$, respectively. The ordinary coherence function is calculated to ensure the model behaves nonlinearly and it also may be regarded as a measure of the model accuracy. However, the ordinary coherence function calculated from the airplane-like structure is below 0.5 and could not identify the frequency of when the nonlinearity starts to affect the modelling.

From the result, it is understood that the CRP could not identify the underlying linear system of the airplane-like structure. Nonetheless, the plausibility of CRP has been recognized by a lot of researchers and it is inequitable to conclude that the CRP could not accurately compute the underlying linear structure and the nonlinear coefficients for a large MDOF system. The question here is whether the added nonlinearity in the modelling is correct and if the MSC Patran/Nastran has generated the precise responses.

MSC Patran is one of the most widely used pre/post-processing software for Finite Element Analysis and the MSC Nastran is the compatible solver to be used with Patran. This software is an industry-tested application, has various choice of solution type and proved to be reliable to evaluate any kind of modelling. Therefore, the results and

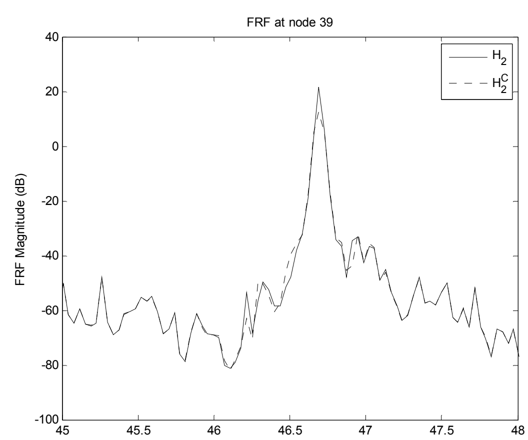


**Figure-5.** A closer look on the details of resonance peak for unconditioned ($H_2$) and conditioned FRF ($H_{C2}$).

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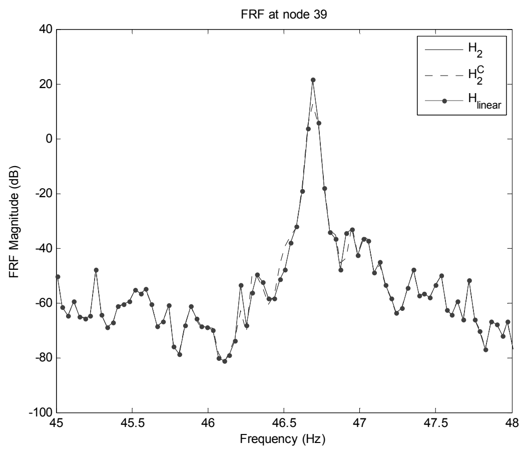


**Figure-6.** The FRFs for unconditioned, conditioned and linear cases for a frequency range between 45 and 48 Hz.

performance is based on the comprehension of the user.

After some thorough investigation and an in-depth discussion, it is believed that the method taken to add the nonlinearity in this particular study is vague and obscuring. Since the nonlinearity is added as a spring element, one should make sure that the stiffness of the spring is much higher than the stiffness of the beam junction between the wing and engine. The spring should be stiff enough to hold the engine and can tolerate with the vibration from the external force (white noise). It is worth pointing out that from Figure-6, the reason why the unconditioned ($H_2$) and the "linear" FRFs completely overlap each other is probably because the nonlinear function is not embedded in the system at all. Both cases give the same exact value, leaving the results from CRP isolated in its own with no consistent data to be compared.

Although it is claimed that nonlinearity is not accurately added in the system, Figure-3 shows that the hardening stiffness nonlinearity has already been induced in the structure. The airplane-like structure is an initially nonlinear system for it is excited with a nonlinear white-noise sequence force. Therefore, the hardening stiffness nonlinearity is originated from the applied force, not from the nonlinearity embedded in the junction beam of the structure. Furthermore, the choice of solution type is also an important factor to be considered in this study. Since the airplane-like structure is already an initially nonlinear structure, it is advisable to avoid using the Nonlinear Transient solution type (Solution Sequence 129) otherwise an unnecessary nonlinearity will be added into the structure. There are also a few nonlinear solutions to be chosen from the MSC Patran/Nastran; SOL 400, 600, 700 and 900. Further studies need to be conducted to determine the difference between these solutions.

**Further works**

A thorough investigation need to be done in order to fully understand the fundamental and the nature of works for both MSC Patran/Nastran and the CRP method. The stiffness of the spring element which contains the nonlinearity will be increased so that the underlying linear system can be identified accurately. Once the modelling of the airplane-like structure is impeccable and the underlying linear system is proven to be consistent with the CRP results, another approach will be applied to continue with the previous works (Norfazrina *et al*. 2015).

Another alternative method used in nonlinear system identification called the orthogonalised reverse path (ORP) method will be applied in order to develop the algorithm for larger MDOF nonlinear system and examine the capability of the ORP algorithm in identifying the underlying linear system of structure in interest. The ORP eliminates the effect of nonlinearity in the time domain and continue with the spectral analysis to extract the linear information of the structure. It is believed that the ORP method can perform as well as the CRP and become distinguished as one of the reliable approach in nonlinear system identification for a large nonlinear MDOF system.

## CONCLUSIONS

The current results are unfortunately did not portray the true capability of the CRP method in identifying the underlying linear FRF and nonlinear coefficients of the airplane-like structure. The conditioned FRF deviates from the linear FRF and the nonlinear coefficients obtained are much smaller than the true values. These drawbacks are due to the unexplored applications and utilization of the MSC Patran/Nastran software. It is also assumed that the impotence of developing the CRP algorithm itself may have contributed in the variation of results. Extra cautions will be taken in modelling the large MDOF nonlinear structure for future application with the ORP method.

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