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EFFECTS OF PASTERNAK LAYER ON FORCED TRANSVERSE VIBRATION OF A TIMOSHENKO DOUBLE-BEAM SYSTEM WITH COMPRESSIVE AXIAL LOAD

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ABSTRACT

Based on Timoshenko beam theory, the forced transverse vibrations of an elastically connected simply supported Timoshenko double-beam system with Pasternak layer in-between subjected to arbitrarily distributed continuous loads under compressive axial load are investigated. It is assumed that the two beams of the system are continuously joined by a Pasternak layer. The dynamic responses of the system caused by arbitrarily distributed continuous loads are obtained. The effects of Pasternak layer on the forced vibrations of the Timoshenko double-beam system are discussed for one case of particular excitation loading. The properties of the forced transverse vibrations of the system are found to be significantly dependent on the compressive axial load and shear modulus of Pasternak layer. Vibrations caused by the harmonic exciting forces are discussed, and conditions of resonance and dynamic vibration absorption are formulated. The important result that this paper put emphasize on it is that the magnitudes of the steady-state vibration amplitudes become smaller when the shear Pasternak modulus increases and Pasternak layer can reduces the magnitudes of the steady-state vibration amplitudes more than Winkler elastic layer. But base on Timoshenko theory which takes into account the effects of shear deformation and rotary inertia, Pasternak layer doesn't have considerable effect on the magnitudes of the steady-state vibration amplitudes. Thus the Timoshenko beam-type dynamic absorber with Pasternak layer acts with a little more efficiency than Winkler elastic layer. Effects of Pasternak layer on Rayleigh double-beam is more than Timoshenko double-beam. Thus the Timoshenko beam-type dynamic absorber with Pasternak layer can be used to suppress the excessive vibrations of corresponding beam systems instead of those with Winkler elastic layer. Numerical results of the present method are verified by comparing with those available in the literature.

Keywords: forced vibration, Timoshenko double-beam, compressive axial load, Pasternak layer.

1. INTRODUCTION

A great number of mechanical systems are complex structures composed of two or more basic mechanical systems whose dynamic behavior is conditioned by their interaction. The systems connected by an elastic layer constitute one group of such mechanical structures which are commonly encountered in mechanical, construction and aeronautical industry. Mechanical systems formed by elastic connection of their members, due to the nature of the dynamic interaction conditioned by elastic connections are characterized by complex vibration and a higher number of natural frequencies. Since the number of natural frequencies depends on the number of basic elements joint together, such mechanical systems are exposed to an increased likelihood of creating resonance conditions which can cause breakage and damage. Such models are important as they give the initial approximation of the solution and a general insight into a dynamic behavior of the system at slight motion. The problem concerning the vibrations of beams joint by a Winkler elastic layer has attracted the interest of a large group of scientists. The problem of two elastically connected beams joint by the Winkler elastic layer emerged in order to determine the conditions for the behavior of the system acting as a dynamic absorber in technical practice.

A mathematical model was developed by Seelig and Hoppman [1]. They investigated the problem of an impulse load effect on a beam and produced a system of partial differential equations describing its vibration. The obtained theoretical and experimental results confirmed a sound approximation of an analytical solution obtained for slender beams at small transverse motions using the Euler-Bernoulli theory.

Oniszczuk [2, 3] analyzed the problem of free and forced vibration of two elastically connected Euler-Bernoulli beams. He determined analytical solutions for eigen-frequencies, amplitudinous functions and vibration modes. He discussed the effect of stiffness which the elastic interlayer had on the frequencies and amplitudes of the system. He determined the conditions for the occurrence of resonance and the behavior of the system as a dynamic absorber. The analysis of the system composed of two connected beams was carried on by Zhang *et al.* [4, 5]. In their work, free and forced vibrations by two elastically connected Euler-Bernoulli beams affected by axial compression forces are investigated. They presented analytical solutions for natural frequencies of the system in the function of axial compression force impact and their effect on the vibration amplitude. They determined the co-dependency between the system's critical force and the

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Euler critical load in the function of an axial force of the other beam.

It has been shown that the behavior of foundation materials in engineering practice cannot be represented by foundation model which consists of independent linear elastic springs. In order to find a physically close and mathematically simple foundation model, Pasternak proposed a so-called two-parameter foundation model with shear interaction. Wang *et al.* [6, 7], studied natural vibrations of a Timoshenko beam on a Pasternak-type foundation. Frequency equations are derived for beams with different end restraints. A specific example is given to show the effects of rotary inertia, shear deformation, and foundation constants on the natural frequencies of the beam.

Stojanovic *et al.* [8] analyzed free vibration and static stability of two elastically connected beams with Winkler elastic layer in-between with the influence of rotary inertia and transverse shear. The motion of the system is described by a homogeneous set of two partial differential equations, which is solved by using the classical Bernoulli-Fourier method. The boundary value and initial value problems are solved. The natural frequencies and associated amplitude ratios of an elastically connected double-beam complex system and the analytical solution of the critical buckling load are determined. The presented theoretical analysis is illustrated by a numerical example, in which the effect of physical parameters characterizing the vibrating system on the natural frequency, the associated amplitude ratios and the critical buckling load are discussed. Stojanovic et Kozic [9] discussed the case of forced vibration of two elastically connected beams with Winkler elastic layer in-between and the effect of axial compression force on amplitude ratio of system vibration for three types of external forcing (arbitrarily continuous harmonic excitation, uniformly continuous harmonic excitation and concentrated harmonic excitation). They determined general conditions of resonance and dynamic vibration absorption. In paper [10], Stojanovic *et al.* discussed the analytic analysis of static stability of a system consisting of three elastically connected Timoshenko beams on an elastic foundation. They provided expressions for critical force of the system under the influence of elastic Winkler layers. Stojanovic *et al.* [11] using the example of multiple elastically connected Timoshenko and Reddy-Bickford beams, determined the analytical forms of natural frequencies, their change under the effect of axial compression forces and the conditions for static stability for a different number of connected beams.

As an extension of the work of Stojanovic *et al.* [9], Effects of Pasternak layer on forced transverse vibration of a Timoshenko double-beam system with effect of compressive axial load are studied in the present paper.

2. MATHEMATICAL MODEL

The following assumptions is considered: (a) the behavior of the beam material is linear elastic; (b) the cross-section is rigid and constant throughout the length of the beam and has one plane of symmetry; (c) shear deformations of the cross-section of the beam are taken into account while elastic axial deformations are ignored; (d) the equations are derived bearing in mind the geometric axial deformations; (e) the axial forces F acting on the ends of the beam are not changed with time; (f) the two beams have the same effective material constants.

Figure-1 shows Timoshenko double-beam system with Pasternak layer in-between with length of l, that subjected to axial compressions F_1 and F_2 that are positive in compression and arbitrarily distributed transverse continuous loads f_1 and f_2 that are positive when they act downward.

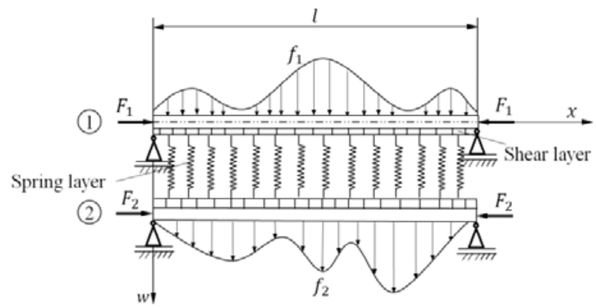

Figure-1. Timoshenko double-beam complex system with Pasternak layer in-between.

An element of deflected differential layered-beam of length dx with Pasternak layer between two cross-sections taken normal to the deflected axis to the beam is shown in Figure-2.

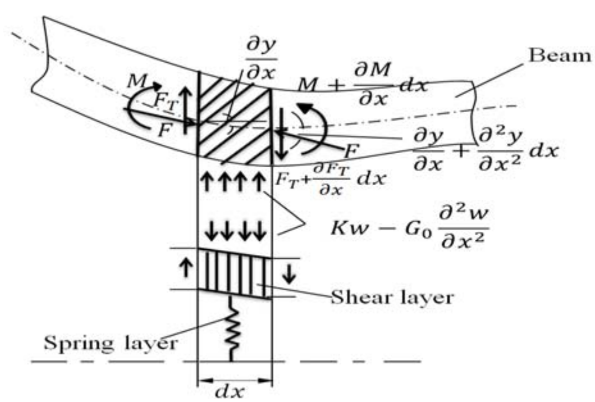

Figure-2. Deflected differential layered-beam element with Pasternak layer in-between.

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The set of coupled differential equations of forced transverse vibration of a double-beam system with effect of compressive axial load with Winkler elastic layer in-between have the following form [9].

$$GA_1k\left(\frac{\partial\psi_1}{\partial x}-\frac{\partial^2 w_1}{\partial x^2}\right)+\rho\,A_1\frac{\partial^2 w_1}{\partial t^2}+F_1\frac{\partial^2 w_1}{\partial x^2}+K\left(w_1-w_2\right)=f_1(x,t),\quad EI_1\frac{\partial^2\psi_1}{\partial x^2}+GA_1k\left(\frac{\partial w_1}{\partial x}-\psi_1\right)-\rho\,I_1\frac{\partial^2\psi_1}{\partial t^2}=0 \tag{1}$$

and

$$GA_2k\left(\frac{\partial\psi_2}{\partial x}-\frac{\partial^2 w_2}{\partial x^2}\right)+\rho\,A_2\frac{\partial^2 w_2}{\partial t^2}+F_2\frac{\partial^2 w_2}{\partial x^2}+K\left(w_2-w_1\right)=f_2(x,t),\quad EI_2\frac{\partial^2\psi_2}{\partial x^2}+GA_2k\left(\frac{\partial w_2}{\partial x}-\psi_2\right)-\rho\,I_2\frac{\partial^2\psi_2}{\partial t^2}=0\cdot \tag{2}$$

If we consider Pasternak layer instead of Winkler elastic layer, the above equations can be changed in the following form:

$$GA_1k\left(\frac{\partial\psi_1}{\partial x}-\frac{\partial^2 w_1}{\partial x^2}\right)+\rho A_1\frac{\partial^2 w_1}{\partial t^2}+F_1\frac{\partial^2 w_1}{\partial x^2}+K(w_1-w_2)-G_0\left(\frac{\partial^2 w_1}{\partial x^2}-\frac{\partial^2 w_2}{\partial x^2}\right)=f_1(x,t),\quad EI_1\frac{\partial^2\psi_1}{\partial x^2}+GA_1k\left(\frac{\partial w_1}{\partial x}-\psi_1\right)-\rho\,I_1\frac{\partial^2\psi_1}{\partial t^2}=0 \tag{3}$$

and

$$GA_2k\left(\frac{\partial\psi_2}{\partial x}-\frac{\partial^2 w_2}{\partial x^2}\right)+\rho A_2\frac{\partial^2 w_2}{\partial t^2}+F_2\frac{\partial^2 w_2}{\partial x^2}+K(w_2-w_1)-G_0\left(\frac{\partial^2 w_2}{\partial x^2}-\frac{\partial^2 w_1}{\partial x^2}\right)=f_2(x,t),\quad EI_2\frac{\partial^2\psi_2}{\partial x^2}+GA_2k\left(\frac{\partial w_2}{\partial x}-\psi_2\right)-\rho\,I_2\frac{\partial^2\psi_2}{\partial t^2}=0\cdot \tag{4}$$

By eliminating ψ_1 from equation (3) and ψ_2 from equation (4), the following two fourth-order partial differential equations can be obtain

$$\begin{aligned}&EI_1\left(1-\frac{F_1}{GA_1k}+\frac{G_0}{GA_1k}\right)\frac{\partial^4 w_1}{\partial x^4}-\frac{EI_1G_0}{GA_1k}\frac{\partial^4 w_1}{\partial x^4}+\rho A_1\left(1+\frac{KI_1}{GA_1^2k}\right)\frac{\partial^2 w_1}{\partial t^2}-\frac{K\rho I_1}{GA_1k}\frac{\partial^2 w_2}{\partial t^2}\\&+F_1\left(1-\frac{EI_1K}{F_1GA_1k}-\frac{G_0}{F_1}\right)\frac{\partial^2 w_1}{\partial x^2}+\left(\frac{EI_1K}{GA_1k}+G_0\right)\frac{\partial^2 w_2}{\partial x^2}-\rho I_1\left(1+\frac{E}{Gk}-\frac{F_1}{GA_1k}+\frac{G_0}{GA_1k}\right)\frac{\partial^4 w_1}{\partial x^2\,\partial t^2}\\&+\frac{\rho\,I_1G_0}{GA_1k}\frac{\partial^4 w_2}{\partial x^2\,\partial t^2}+\frac{\rho^2I_1G}{Gk}\frac{\partial^4 w_1}{\partial t^4}+K(w_1-w_2)=f_1(x,t)+\frac{\rho\,I_1}{GA_1k}\frac{\partial^2 f_1(x,t)}{\partial t^2}-\frac{EI_1}{GA_1k}\frac{\partial^2 f_1(x,t)}{\partial x^2}\end{aligned} \tag{5}$$

and

$$EI_2\left(1-\frac{F_2}{GA_2k}+\frac{G_0}{GA_2k}\right)\frac{\partial^4 w_2}{\partial x^4}-\frac{EI_2G_0}{GA_2k}\frac{\partial^4 w_1}{\partial x^4}+\rho A_2\left(1+\frac{KI_2}{GA_2^2k}\right)\frac{\partial^2 w_2}{\partial t^2}-\frac{K\rho I_2}{GA_2k}\frac{\partial^2 w_1}{\partial t^2}$$
$$+F_2\left(1-\frac{EI_2K}{F_2GA_2k}-\frac{G_0}{F_2}\right)\frac{\partial^2 w_2}{\partial x^2}+\left(\frac{EI_2K}{GA_2k}+G_0\right)\frac{\partial^2 w_1}{\partial x^2}-\rho I_2\left(1+\frac{E}{Gk}-\frac{F_2}{GA_2k}+\frac{G_0}{GA_2k}\right)\frac{\partial^4 w_2}{\partial x^2\,\partial t^2} \tag{6}$$
$$+\frac{\rho I_2G_0}{GA_2k}\frac{\partial^4 w_1}{\partial x^2\,\partial t^2}+\frac{\rho^2 I_2G}{Gk}\frac{\partial^4 w_2}{\partial t^4}-K(w_1-w_2)=f_2(x,t)+\frac{\rho I_2}{GA_2k}\frac{\partial^2 f_2(x,t)}{\partial t^2}-\frac{EI_2}{GA_2k}\frac{\partial^2 f_2(x,t)}{\partial x^2}$$

Equations (5) and (6) can be reduced to fourth-order partial differential equations for forced vibration of the Timoshenko double-beam model

$$C_{b1}^2\left[1-\frac{m_1}{C_{s1}^2\,C_{r1}^2}(F_1-G_0)\right]\frac{\partial^4 w_1}{\partial x^4}-m_1\frac{C_{b1}^2\,G_0}{C_{s1}^2\,C_{r1}^2}\frac{\partial^4 w_2}{\partial x^4}+\left(1+\frac{H_1}{C_{r1}^2}\right)\frac{\partial^2 w_1}{\partial t^2}-\frac{H_1}{C_{r1}^2}\frac{\partial^2 w_2}{\partial t^2}$$
$$+\left[m_1(F_1-G_0)-\frac{C_{b1}^2\,H_1}{C_{s1}^2\,C_{r1}^2}\right]\frac{\partial^2 w_1}{\partial x^2}+\left(\frac{C_{b1}^2\,H_1}{C_{s1}^2\,C_{r1}^2}+m_1G_0\right)\frac{\partial^2 w_2}{\partial x^2}$$
$$-\left[C_{r1}^2+\frac{C_{b1}^2}{C_{s1}^2\,C_{r1}^2}-\frac{m_1}{C_{r1}^2}(F_1-G_0)\right]\frac{\partial^4 w_1}{\partial x^2\,\partial t^2}+\frac{m_1G_0}{C_{s1}^2}\frac{\partial^4 w_2}{\partial x^2\,\partial t^2}+\frac{1}{C_{s1}^2}\frac{\partial^4 w_1}{\partial t^4} \tag{7}$$
$$+H_1(w_1-w_2)=m_1\left[f_1(x,t)+\frac{1}{C_{s1}^2}\frac{\partial^2 f_1(x,t)}{\partial t^2}-\frac{C_{b1}^2}{C_{s1}^2\,C_{r1}^2}\frac{\partial^2 f_1(x,t)}{\partial x^2}\right]$$

and

$$C_{b2}^2\left[1-\frac{m_1}{C_{s2}^2\,C_{r2}^2}(F_2-G_0)\right]\frac{\partial^4 w_2}{\partial x^4}-m_2\frac{C_{b2}^2\,G_0}{C_{s2}^2\,C_{r2}^2}\frac{\partial^4 w_1}{\partial x^4}+\left(1+\frac{H_2}{C_{s2}^2}\right)\frac{\partial^2 w_2}{\partial t^2}-\frac{H_2}{C_{s2}^2}\frac{\partial^2 w_1}{\partial t^2}$$
$$+\left[m_2(F_2-G_0)-\frac{C_{b2}^2\,H_1}{C_{s2}^2\,C_{r2}^2}\right]\frac{\partial^2 w_2}{\partial x^2}+\left(\frac{C_{b2}^2\,H_2}{C_{s2}^2\,C_{r2}^2}+m_2G_0\right)\frac{\partial^2 w_1}{\partial x^2}$$
$$-\left[C_{r2}^2+\frac{C_{b2}^2}{C_{s2}^2\,C_{r2}^2}-\frac{m_2}{C_{r2}^2}(F_2-G_0)\right]\frac{\partial^4 w_2}{\partial x^2\,\partial t^2}+\frac{m_2G_0}{C_{s2}^2}\frac{\partial^4 w_1}{\partial x^2\,\partial t^2}+\frac{1}{C_{s2}^2}\frac{\partial^4 w_2}{\partial t^4} \tag{8}$$
$$+H_2(w_2-w_1)=m_2\left[f_2(x,t)+\frac{1}{C_{s2}^2}\frac{\partial^2 f_2(x,t)}{\partial t^2}-\frac{C_{b2}^2}{C_{s2}^2C_{r2}^2}\frac{\partial^2 f_2(x,t)}{\partial x^2}\right].$$

Where,

$$m_i=\frac{1}{\rho A_i}\ ,\quad H_i=\frac{K}{\rho A_i}\ ,\quad C_{bi}=\sqrt{\frac{EI_i}{\rho A_i}}\ ,\quad C_{si}=\sqrt{\frac{GA_ik}{\rho A_i}}\ ,\quad C_{ri}=\sqrt{\frac{I_i}{A_i}}\ ,\quad i=1,2\cdot$$

The boundary conditions for simply supported beams of the same length l are assumed as follows

$$w_i(0,t)=w_i''(0,t)=w_i(l,t)=w_i''(l,t)=0\ ,\quad i=1,2\cdot \tag{9}$$

3. SOLUTION OF EQUATIONS

Equations (7) and (8) representing forced vibrations of a Timoshenko double-beam system with Pasternak layer in-between. The natural frequencies and the corresponding mode shapes of the system should be obtained by solving the undamped free vibration with appropriate boundary conditions. Assuming time harmonic motion and using separation of variables, the solutions to equations (7) and (8) with the governing boundary conditions (9) can be written in the form

$$w_i(x,t)=\sum_{n=1}^{\infty} X_n(x)\,T_{ni}(t)\ ,\quad i=1,2 \tag{10}$$

Where,

$$X_n(x)=\sin(k_n x)\ ,\quad k_n=\frac{n\pi}{l}\ ,\quad n=1,2,3,...$$

By substituting solution (10) into equations. (7) and (8), the following ordinary differential equations for the Timoshenko double-beam model will be obtained

$$\sum_{n=1}^{\infty}\left\{\frac{1}{C_{s1}^2}\frac{d^4T_{n1}}{dt^4}+\left[1+C_{r1}^2\,k_n^2+\frac{C_{b1}^2\,k_n^2}{C_{s1}^2\,C_{r1}^2}+\frac{1}{C_{s1}^2}(H_1-F_1\eta_1+G_0\eta_1)\right]\frac{d^2T_{n1}}{dt^2}\right.$$
$$-\left[\frac{1}{C_{s1}^2}(H_1+G_0\eta_1)\right]\frac{d^2T_{n2}}{dt^2}+\left[C_{b1}^2\,k_n^4+(H_1-F_1\eta_1)\left(1+\frac{C_{b1}^2\,k_n^2}{C_{s1}^2\,C_{r1}^2}\right)+G_0\eta_1\left(1+\frac{C_{b1}^2\,k_n^2}{C_{s1}^2\,C_{r1}^2}\right)\right]T_{n1} \tag{11}$$
$$\left.-\left[H_1\left(1+\frac{C_{b1}^2\,k_n^2}{C_{s1}^2\,C_{r1}^2}\right)+G_0\eta_1\left(1+\frac{C_{b1}^2\,k_n^2}{C_{s1}^2\,C_{r1}^2}\right)\right]T_{n2}\right\}X_n=m_1\left(f_1+\frac{1}{C_{s1}^2}\ddot{f}_1-\frac{C_{b1}^2}{C_{s1}^2\,C_{r1}^2}f_1''\right)$$

and

$$\sum_{n=1}^{\infty}\left\{\frac{1}{C_{s2}^2}\frac{d^4T_{n2}}{dt^4}+\left[1+C_{r2}^2\,k_n^2+\frac{C_{b2}^2\,k_n^2}{C_{s2}^2\,C_{r2}^2}+\frac{1}{C_{s2}^2}(H_2-F_2\eta_2+G_0\eta_2)\right]\frac{d^2T_{n2}}{dt^2}\right.$$
$$-\left[\frac{1}{C_{s2}^2}(H_2+G_0\eta_2)\right]\frac{d^2T_{n1}}{dt^2}+\left[C_{b2}^2\,k_n^4+(H_2-F_2\eta_2)\left(1+\frac{C_{b2}^2\,k_n^2}{C_{s2}^2\,C_{r2}^2}\right)+G_0\eta_2\left(1+\frac{C_{b2}^2\,k_n^2}{C_{s2}^2\,C_{r2}^2}\right)\right]T_{n2} \tag{12}$$
$$\left.-\left[H_2\left(1+\frac{C_{b2}^2\,k_n^2}{C_{s2}^2\,C_{r2}^2}\right)+G_0\eta_2\left(1+\frac{C_{b2}^2\,k_n^2}{C_{s2}^2\,C_{r2}^2}\right)\right]T_{n1}\right\}X_n=m_2\left(f_2+\frac{1}{C_{s2}^2}\ddot{f}_2-\frac{C_{b2}^2}{C_{s2}^2\,C_{r2}^2}f_2''\right)$$

Where,

$$\eta_1=\frac{k_n^2}{\rho A_1}\ ,\quad \eta_2=\frac{k_n^2}{\rho A_2}\cdot$$

4. FORCED VIBRATIONS

Solving the undamped free vibration gives four frequencies, two shear frequencies which are associated with a shear vibration and two frequencies associated with a transverse vibration with appropriate boundary conditions of ordinary differential equations for the unknown time functions. Equations (11) and (12) can be simplified as follows

$$\frac{\mathrm{d}^4T_{n1}}{\mathrm{d}t^4}+J_1\frac{\mathrm{d}^2T_{n1}}{\mathrm{d}t^2}-(H_1+G_0\eta_1)\frac{\mathrm{d}^2T_{n2}}{\mathrm{d}t^2}+P_1T_{n1}-Q_1T_{n2}=0 \tag{13}$$

and

$$\frac{\mathrm{d}^4T_{n2}}{\mathrm{d}t^4}+J_2\frac{\mathrm{d}^2T_{n2}}{\mathrm{d}t^2}-(H_2+G_0\eta_2)\frac{\mathrm{d}^2T_{n1}}{\mathrm{d}t^2}+P_2T_{n2}-Q_2T_{n1}=0 \tag{14}$$

Where,

$$J_1=C_{s1}^2\,[R_1+C_{r1}^2\,k_n^2+\frac{1}{C_{s1}^2}(H_1-F_1\eta_1+G_0\eta_1)]\ ,$$
$$J_2=C_{s2}^2\,[R_2+C_{r2}^2\,k_n^2+\frac{1}{C_{s2}^2}(H_2-F_2\eta_2+G_0\eta_2)],$$
$$R_1=1+\frac{C_{b1}^2\,k_n^2}{C_{s1}^2\,C_{r1}^2},\quad R_2=1+\frac{C_{b2}^2\,k_n^2}{C_{s2}^2\,C_{r2}^2},$$
$$P_1=C_{s1}^2\,[C_{b1}^2\,k_n^4+(H_1-F_1\eta_1+G_0\eta_1)R_1],$$
$$P_2=C_{s2}^2\,[C_{b2}^2\,k_n^4+(H_2-F_2\eta_2+G_0\eta_2)R_2],$$
$$Q_1=C_{s1}^2\,[(H_1+G_0\eta_1)]R_1\ ,\quad Q_2=C_{s2}^2\,[(H_2+G_0\eta_2)]R_2\cdot$$

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In order to find natural frequencies of the structural model, the solution of equations (13) and (14) could be expressed as

$$T_{n1} = C_n \, \mathrm{e}^{\mathrm{j}\omega_n t} \ , \ \ T_{n2} = D_n \, \mathrm{e}^{\mathrm{j}\omega_n t} \cdot \tag{15}$$

By substituting of equation (15) into equations (13) and (14), we have

$$(\omega_n^4 - \bar{J}_1 \omega_n^2 + \bar{P}_1)\, C_n - [\bar{Q}_1 - (H_1 + G_0\eta_1)\omega_n^2]\, D_n = 0 \tag{16}$$

and

$$-[\bar{Q}_2 - (H_2 + G_0\eta_2)\omega_n^2]\, C_n + (\omega_n^4 - \bar{J}_2 \omega_n^2 + \bar{P}_2)\, D_n = 0 \cdot \tag{17}$$

When the determinant of the coefficient matrix of the C_n and D_n vanishes, the equations (16) and (17) have non-trivial solutions. Setting the determinant equal to zero yields

$$\begin{aligned} &\omega_n^8 - (J_1 + J_2)\,\omega_n^6 - (\bar{P}_1 + \bar{P}_2 + \bar{J}_1\bar{J}_2 - H_1H_2 - H_1G_0\eta_2 - H_2G_0\eta_1 - G_0^2\,\eta_1\eta_2)\,\omega_n^4 \\ &- (\bar{J}_1\bar{P}_2 + \bar{J}_2\bar{P}_1 - H_1\bar{Q}_2 - G_0\eta_1\bar{Q}_2 - H_2\bar{Q}_1 - G_0\eta_2\bar{Q}_1)\,\omega_n^2 + \bar{P}_1\bar{P}_2 - \bar{Q}_1\bar{Q}_2 = 0 \cdot \end{aligned} \tag{18}$$

From the equation (18), we obtain two natural frequencies which are associated with a transverse vibration $\omega_{nI} = \sqrt{\lambda_1}$, $\omega_{nII} = \sqrt{\lambda_3}$, and two much higher natural shear frequencies which are associated with a shear vibration $\omega_{nI(s)} = \sqrt{\lambda_2}$, $\omega_{nII(s)} = \sqrt{\lambda_4}$. The analytical expressions for the free natural and shear frequencies of the double-beam complex system with the influence of rotary inertia and shear are determined and given in [8]. Shear effect makes two frequencies ω_{nI} and ω_{nII} which are associated with a transverse vibration lower. The two higher shear frequencies $\omega_{nI(s)}$ and $\omega_{nII(s)}$ which are associated with a shear vibration are of much lesser technical interest [8]. Ordinary differential equations under the influence of shear and rotary inertia for the unknown time functions can be written as

$$J_1 \frac{\mathrm{d}^2 T_{n1}}{\mathrm{d}t^2} - (H_1 + G_0\eta_1)\frac{\mathrm{d}^2 T_{n2}}{\mathrm{d}t^2} + P_1 T_{n1} - Q_1 T_{n2} = 0 \tag{19}$$

and

$$J_2 \frac{\mathrm{d}^2 T_{n2}}{\mathrm{d}t^2} - (H_2 + G_0\eta_2)\frac{\mathrm{d}^2 T_{n1}}{\mathrm{d}t^2} + P_2 T_{n2} - Q_2 T_{n1} = 0 \cdot \tag{20}$$

In order to find natural frequencies associated with a transverse vibration of the structural model, the solution of equations (19) and (20) could be expressed as

$$T_{n1} = \bar{C}_n \, \mathrm{e}^{\mathrm{i}\omega_n t} \ , \ \ T_{n2} = \bar{D}_n \, \mathrm{e}^{\mathrm{i}\omega_n t} \cdot \tag{21}$$

By substituting equation (21) into equations (19) and (20), we obtain

$$(-J_1\omega_n^2 + P_1)\,\bar{C}_n - [Q_1 - (H_1 + G_0\eta_1)\omega_n^2]\,\bar{D}_n = 0 \tag{22}$$

and

$$-[Q_2 - (H_2 + G_0\eta_2)\omega_n^2]\,\bar{C}_n + (-J_2\omega_n^2 + P_2)\,\bar{D}_n = 0 \cdot \tag{23}$$

When the determinant of the coefficients in equations (22) and (34) vanishes, non-trivial solutions for the constants $\bar{C}_n$ and $\bar{D}_n$ can be obtained, which yield the following frequency equation:

$$\begin{aligned} &(J_1J_2 - H_1H_2 - H_1G_0\eta_2 - H_2G_0\eta_1 - G_0^2\eta_1\eta_2)\omega_n^4 \\ &+ (H_1Q_2 + G_0\eta_1Q_2 + H_2Q_1 + G_0\eta_2Q_1 - J_1P_2 - J_2P_1)\omega_n^2 + P_1P_2 - Q_1Q_2 = 0 \cdot \end{aligned} \tag{24}$$

Then from the characteristic equation (24), we obtain

$$\begin{aligned} \omega_{nI}^2 = &\frac{J_2P_1 + J_1P_2 - H_2Q_1 - H_1Q_2 - G_0\eta_1Q_2 - G_0\eta_1Q_2}{2(J_1J_2 - H_1H_2 - H_1G_0\eta_2 - H_2G_0\eta_1 - G_0^2\eta_1\eta_2)} \\ &- \frac{\sqrt{D}}{2(J_1J_2 - H_1H_2 - H_1G_0\eta_2 - H_2G_0\eta_1 - G_0^2\eta_1\eta_2)} \end{aligned} \tag{25}$$

and

$$\begin{aligned} \omega_{nII}^2 = &\frac{J_2P_1 + J_1P_2 - H_2Q_1 - H_1Q_2 - G_0\eta_1Q_2 - G_0\eta_1Q_2}{2(J_1J_2 - H_1H_2 - H_1G_0\eta_2 - H_2G_0\eta_1 - G_0^2\eta_1\eta_2)} \\ &+ \frac{\sqrt{D}}{2(J_1J_2 - H_1H_2 - H_1G_0\eta_2 - H_2G_0\eta_1 - G_0^2\eta_1\eta_2)} \end{aligned} \tag{26}$$

Where,

$$\begin{aligned} D = &(H_1Q_2 + G_0\eta_1Q_2 + H_2Q_1 + G_0\eta_2Q_1 - J_1P_2 - J_2P_1)^2 \\ &+ 4(H_1H_2 + H_1G_0\eta_2 + H_2G_0\eta_1 + G_0^2\eta_1\eta_2 - J_1J_2)(P_1P_2 - Q_1Q_2) \cdot \end{aligned}$$

For each of the natural frequencies, the associated amplitude ratio of vibration modes of the two beams is given by

$$\alpha_{ni}^{-1} = \frac{\bar{C}_n}{\bar{D}_n} = \frac{Q_1 - (H_1 + G_0\eta_1)\omega_{ni}^2}{-J_1\omega_{ni}^2 + P_1} = \frac{P_2 - J_2\omega_{ni}^2}{Q_2 - (H_2 + G_0\eta_2)\omega_{ni}^2} \cdot \tag{27}$$

Following analysis for the undamped free transverse vibration, particular solutions of non-homogeneous differential equations (7) and (8) and

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negligible shear frequencies $\omega_{nI(s)}$ and $\omega_{nII(s)}$ which are described by ordinary differential equations (19) and (20), representing forced vibrations of a Timoshenko double model and can be assumed in the following

$$w_1(x,t)=\sum_{n=1}^{\infty}X_n(x)\sum_{i=I}^{II}S_{ni}(t)\ ,\quad w_2(x,t)=\sum_{n=1}^{\infty}X_n(x)\sum_{i=I}^{II}\alpha_{ni}\,S_{ni}(t)\cdot \tag{28}$$

Substituting of equations (28) and (29) into equations (7) and (8) results in the following form

$$\sum_{n=1}^{\infty}X_n(x)\sum_{i=I}^{II}\{[J_1-\alpha_{ni}(H_1+\eta_1G_0)]\ddot{S}_{ni}+(P_1-\alpha_{ni}\,Q_1)\,S_{ni}\} = m_1\left(f_1+\frac{1}{C_{s1}^2}\ddot{f}_1-\frac{C_{b1}^2}{C_{s1}^2\,C_{r1}^2}f_1''\right) \tag{29}$$

and

$$\sum_{n=1}^{\infty}X_n(x)\sum_{i=I}^{II}\{[J_2-(H_2+\eta_2G_0)\,\alpha_{ni}^{-1}]\ddot{S}_{ni}+(P_2-\alpha_{ni}^{-1}\,Q_2)\,S_{ni}\}\alpha_{ni} = m_2\left(f_2+\frac{1}{C_{s1}^2}\ddot{f}_2-\frac{C_{b2}^2}{C_{s2}^2\,C_{r2}^2}f_2''\right). \tag{30}$$

By multiplying the relations (30) and (31) by X_m, then integrating them with respect to x from 0 to l and using orthogonality condition, we have

$$\sum_{i=I}^{II}\{[J_1-\alpha_{ni}\,(H_1+\eta_1G_0)]\,\ddot{S}_{ni}+(\overline{P}_1-\alpha_{ni}\,Q_1)\,S_{ni}\} = \frac{2m_1}{l}\int_0^l X_n\left(f_1+\frac{1}{C_{s1}^2}\ddot{f}_1-\frac{C_{b1}^2}{C_{s1}^2\,C_{r1}^2}f_1''\right)\mathrm{d}x \tag{31}$$

and

$$\sum_{i=I}^{II}\{[J_2-(H_2+\eta_2G_0)\,\alpha_{ni}^{-1}]\,\ddot{S}_{ni}+(\overline{P}_2-\alpha_{ni}^{-1}\,Q_2)\,S_{ni}\}\,\alpha_{ni} = \frac{2m_2}{l}\int_0^l X_n\left(f_2+\frac{1}{C_{s2}^2}\ddot{f}_2-\frac{C_{b2}^2}{C_{s2}^2\,C_{r2}^2}f_2''\right)\mathrm{d}x\cdot \tag{32}$$

By combining equations (27), (32) and (33), after some algebra, we obtain

$$\ddot{S}_{ni}+\omega_{ni}^2\,S_{ni}=Z_{ni}(t)\ ,\quad i=I,II \tag{33}$$

Where,

$$Z_{nI}(t)=\frac{2m_1[J_2\,\alpha_{nII}-(H_2+\eta_2G_0)]}{l\,(\alpha_{nI}-\alpha_{nII})\,[(H_1+\eta_1G_0)\,(H_2+\eta_2G_0)-J_1J_2]}\int_0^l\left(f_1+\frac{1}{C_{s1}^2}\ddot{f}_1-\frac{C_{b1}^2}{C_{s1}^2\,C_{r1}^2}f_1''\right)\sin\,(k_nx)\,\mathrm{d}x + \frac{2m_2\,[(H_1+\eta_1G_0)\,\alpha_{nII}-J_1]}{l\,(\alpha_{nI}-\alpha_{nII})\,[(H_2+\eta_2G_0)\,(H_1+\eta_1G_0)-J_1J_2]}\int_0^l\left(f_2+\frac{1}{C_{s2}^2}\ddot{f}_2-\frac{C_{b2}^2}{C_{s2}^2\,C_{r2}^2}f_2''\right)\sin\,(k_nx)\,\mathrm{d}x \tag{34}$$

and

$$Z_{nII}(t)=\frac{2m_1[(H_2+\eta_2G_0)-J_2\,\alpha_{nI}]}{l\,(\alpha_{nI}-\alpha_{nII})\,[(H_1+\eta_1G_0)\,(H_2+\eta_2G_0)-J_1J_2]}\int_0^l\left(f_1+\frac{1}{C_{s1}^2}\ddot{f}_1-\frac{C_{b1}^2}{C_{s1}^2\,C_{r1}^2}f_1''\right)\sin\,(k_nx)\,\mathrm{d}x + \frac{2m_2[J_1-(H_1+\eta_1G_0)\,\alpha_{nI}]}{l\,(\alpha_{nI}-\alpha_{nII})\,[(H_2+\eta_2G_0)(H_1+\eta_1G_0)-J_1J_2]}\int_0^l\left(f_2+\frac{1}{C_{s2}^2}\ddot{f}_2-\frac{C_{b2}^2}{C_{s2}^2\,C_{r2}^2}f_2''\right)\sin\,(k_nx)\,\mathrm{d}x\cdot \tag{35}$$

From equation (33) we have

$$S_{ni}(t)=\frac{1}{\omega_{ni}}\int_0^t Z_{ni}(s)\sin[\omega_{ni}(t-s)]\,\mathrm{d}s\ ,\quad i=I,II\cdot \tag{36}$$

By combining equations (28), (29) and (36), the forced vibrations of an elastically connected Timoshenko double-beam system can be described by

$$w_1(x,t)=\sum_{n=1}^{\infty}\sin(k_nx)\sum_{i=I}^{II}\frac{1}{\omega_{ni}}\int_0^t Z_{ni}(s)\sin[\omega_{ni}(t-s)]\,\mathrm{d}s \tag{37}$$

and

$$w_2(x,t)=\sum_{n=1}^{\infty}\sin(k_nx)\sum_{i=I}^{II}\frac{\alpha_{ni}}{\omega_{ni}}\int_0^t Z_{ni}(s)\sin[\omega_{ni}(t-s)]\,\mathrm{d}s\cdot \tag{38}$$

Now these general solutions (37) and (38) are used to find the vibrations of the two coupled Timoshenko beams for certain exciting loadings.

In the following, we conduct an analysis of forced vibrations for case of uniformly distributed continuous harmonic load. For simplicity of further analysis, it is assumed that only one of the two beams is subjected to the exciting load (see Figure-3). Without loss of generality, we suppose

$$f_1(x,t)=q\sin(\Omega t)\ ,\quad f_2(x,t)=0\cdot \tag{39}$$

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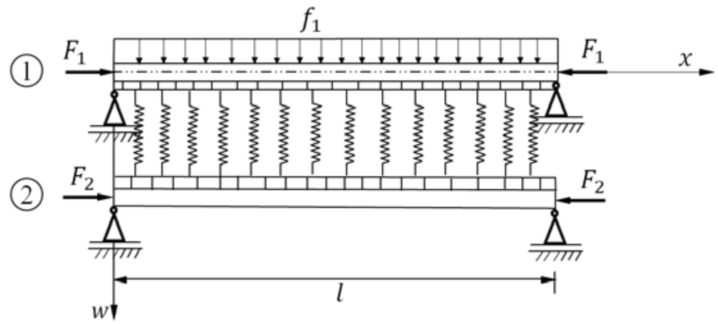

Figure-3. Timoshenko double-beam complex system with Pasternak layer in-between subjected to harmonic uniform distributed continuous load.

Substituting equation (39) into equations (34) and (35), we obtain

$$Z_{nI}(t)=\frac{4qm_1[J_2\,\alpha_{nII}-(H_2+\eta_2G_0)]\sin(\Omega t)}{n\pi\,(\alpha_{nI}-\alpha_{nII})[(H_1+\eta_1G_0)(H_2+\eta_2G_0)-J_1J_2]}\left(1-\frac{\Omega^2}{C_{s1}^2}\right) \tag{40}$$

and

$$Z_{nII}(t)=\frac{4qm_1[(H_2+\eta_2G_0)-J_2\,\alpha_{nI}]\sin(\Omega t)}{n\pi(\alpha_{nI}-\alpha_{nII})[(H_1+\eta_1G_0)(H_2+\eta_2G_0)-J_1J_2]}\left(1-\frac{\Omega^2}{C_{s1}^2}\right). \tag{41}$$

Substituting equations (40) and (41) into equations (37) and (38) gives

$$w_1(x,t)=\sum_{n=1}^{\infty}\sin(k_nx)\left[A_{n1}\sin(\Omega t)+\sum_{i=I}^{II}B_{ni}\sin(\omega_nt)\right],\quad n=1,3,5,... \tag{42}$$

and

$$w_2(x,t)=\sum_{n=1}^{\infty}\sin(k_nx)\left[A_{n2}\sin(\Omega t)+\sum_{i=I}^{II}\alpha_{ni}\,B_{ni}\sin(\omega_nt)\right],\quad n=1,3,5,... \tag{43}$$

Where,

$$A_{n1}=M_T\left\{\frac{[J_2\,\alpha_{nII}-(H_2+\eta_2G_0)]}{(\omega_{nI}^2-\Omega^2)(\alpha_{nI}-\alpha_{nII})[(H_1+\eta_1G_0)(H_2+\eta_2G_0)-J_1J_2]}+\frac{[(H_2+\eta_2G_0)-J_2\alpha_{nI}]}{(\omega_{nII}^2-\Omega^2)(\alpha_{nI}-\alpha_{nII})[(H_1+\eta_1G_0)(H_2+\eta_2G_0)-J_1J_2]}\right\},$$

$$A_{n2}=M_T\left\{\frac{\alpha_{nI}[J_2\,\alpha_{nII}-(H_2+\eta_2G_0)]}{(\omega_{nI}^2-\Omega^2)(\alpha_{nI}-\alpha_{nII})[(H_1+\eta_1G_0)(H_2+\eta_2G_0)-J_1J_2]}+\frac{\alpha_{nII}[(H_2+\eta_2G_0)-J_2\alpha_{nI}]}{(\omega_{nII}^2-\Omega^2)(\alpha_{nI}-\alpha_{nII})[(H_1+\eta_1G_0)(H_2+\eta_2G_0)-J_1J_2]}\right\}$$

and

$$B_{nI}=\frac{\Omega[J_2\,\alpha_{nII}-(H_2+\eta_2G_0)]}{\omega_{nI}(\omega_{nI}^2-\Omega^2)(\alpha_{nII}-\alpha_{nI})[(H_1+\eta_1G_0)(H_2+\eta_2G_0)-J_1J_2]}M_T,$$

$$B_{nII}=\frac{\Omega[(H_2+\eta_2G_0)-J_2\,\alpha_{nI}]}{\omega_{nII}(\omega_{nII}^2-\Omega^2)(\alpha_{nII}-\alpha_{nI})[(H_1+\eta_1G_0)(H_2+\eta_2G_0)-J_1J_2]}M_T$$

Also,

$$M_T=\frac{4qm_1[J_2\,\alpha_{nII}-(H_2+\eta_2G_0)]}{n\pi\,(\alpha_{nI}-\alpha_{nII})[(H_1+\eta_1G_0)(H_2+\eta_2G_0)-J_1J_2]}\left(1-\frac{\Omega^2}{C_{s1}^2}\right).$$

Ignoring the free response, the forced vibrations of the Timoshenko double-beam system can be obtained by

$$w_1(x,t) = \sin(\Omega t) \sum_{n=1}^{\infty} A_{n1} \sin(k_n x) \ , \quad n = 1, 3, 5, ... \quad (44)$$

and

$$w_2(x,t) = \sin(\Omega t) \sum_{n=1}^{\infty} A_{n2} \sin(k_n x) \ , \quad n = 1, 3, 5, ... \quad (45)$$

The following fundamental conditions of resonance and dynamic vibration absorption have practical significance:

1- Resonance

$$\Omega = \omega_{ni} \ , \quad n = 1, 3, 5, ... \quad (46)$$

2- Dynamic vibration absorption

$$\Omega^2 = \frac{\omega_{nI}^2 \left[(H_2 + \eta_2 G_0) - J_2 \alpha_{nII} \right] - \omega_{nII}^2 \left[J_2 \alpha_{nI} - (H_2 + \eta_2 G_0) \right]}{J_2 (\alpha_{nI} - \alpha_{nII})}, \quad (47)$$

$$A_{n2} = M_{Tn} \frac{J_2 (\alpha_{nI} - \alpha_{nII})^2}{\omega_{nII}^2 - \omega_{nI}^2}, \quad A_{n1} = 0 \cdot$$

5. NUMERICAL RESULTS AND DISCUSSIONS

In the following, the effects of Pasternak layer on the steady-state vibration amplitudes A_{n1} and A_{n2} are discussed. For simplicity, it is assumed that both beams are geometrically and physically identical [9]. The values of the parameters characterizing properties of the system are shown in Table-1.

$$H_1 = H_2 \ , \quad \eta_1 = \eta_2 \ , \quad F_2 = \zeta F_1 \ , \quad 0 \le \zeta \le 1 \quad (48)$$

Table-1. Values of the parameters characterizing properties of the system.

l	E	A	K
10 m	1×10^{10} Nm^{-2}	5×10^{-2} m^2	2×10^5 Nm^{-2}
k	ρ	I	G
5/6	2×10^3 kgm^{-3}	4×10^{-4} m^4	0.417×10^{10} Nm^{-2}

If the axial compressions vanish, for the Timoshenko double-beam we have

$$\omega_{nI}^2 = \frac{J_2 P_1 + J_1 P_1 - H_2 Q_1 + G_0 \eta_1 Q_2 + G_0 \eta_2 Q_1}{2 (J_1 J_2 - H_1 H_2 - H_1 G_0 \eta_2 - H_2 G_0 \eta_1 - G_0^2 \eta_1 \eta_2)} - \frac{\sqrt{D'}}{2 (J_1 J_2 - H_1 H_2 - H_1 G_0 \eta_2 - H_2 G_0 \eta_1 - G_0^2 \eta_1 \eta_2)} \quad (49)$$

and

$$\omega_{nII}^2 = \frac{J_2 P_1 + J_1 P_1 - H_2 Q_1 + G_0 \eta_1 Q_2 + G_0 \eta_2 Q_1}{2 (J_1 J_2 - H_1 H_2 - H_1 G_0 \eta_2 - H_2 G_0 \eta_1 - G_0^2 \eta_1 \eta_2)} + \frac{\sqrt{D'}}{2 (J_1 J_2 - H_1 H_2 - H_1 G_0 \eta_2 - H_2 G_0 \eta_1 - G_0^2 \eta_1 \eta_2)} \quad (50)$$

Also,

$$\alpha_{nI}^{-1} = \frac{\overline{C}_n}{\overline{D}_n} = \frac{Q_1 - (H_1 + G_0 \eta_1)\, \omega_{nI}^2}{-J_1 \omega_{nI}^2 + P_1} = \frac{P_2 - J_2 \omega_{nI}^2}{Q_2 - (H_2 + G_0 \eta_2)\, \omega_{nI}^2} \quad (51)$$

and

$$\alpha_{nII}^{-1} = \frac{\overline{C}_n}{\overline{D}_n} = \frac{Q_1 - (H_1 + G_0 \eta_1)\, \omega_{nII}^2}{-J_1 \omega_{nII}^2 + P_1} = \frac{P_2 - J_2 \omega_{nII}^2}{Q_2 - (H_2 + G_0 \eta_2)\, \omega_{nII}^2} \quad (52)$$

Where,

$$D' = (H_1 Q_2 + G_0 \eta_1 Q_2 + H_2 Q_1 + G_0 \eta_2 Q_1 - J_1 P_2 - J_2 P_1)^2 + 4 (H_1 H_2 + H_1 G_0 \eta_2 + H_2 G_0 \eta_1 + G_0^2 \eta_1 \eta_2 - J_1 J_2)(P_1 P_2 - Q_1 Q_2)$$

To determine the effect of compressive axial load on the steady-state vibration amplitudes A_{n1} and A_{n2} of the system, the results under compressive axial load and those without axial load are compared. Introducing the relation

$$\varphi_1 = \frac{A_{n1}}{A_{n1}^0} \ , \quad \varphi_2 = \frac{A_{n2}}{A_{n2}^0} \quad (53)$$

Using non-dimensional ratio

$$s = \frac{F_1}{P} \quad (54)$$

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Where, $P = P_T = \dfrac{(EI\pi^2/l^2)}{1+(EI\pi^2/GAkl^2)}$ is known as the Euler load for Timoshenko beam, which is the smallest load at which the single beam ceases to be in stable equilibrium under axial compression [12]. For the case of uniformly distributed harmonic load, the steady-state vibration amplitudes for the Timoshenko beams $(A_{n1}, A_{n2}, A_{n1}^0, A_{n2}^0)$ can be determined. We accounted the results for three case of shear modulus of Pasternak layer $G_0 = 0,\ 1000,\ 100000$. If we introduce $G_0 = 0$ in the whole equations of this paper we can obtain the vibration equations of the Timoshenko double-beam system on Winkler elastic layer that the results are verified by comparing with those available in Ref [9]. With the vibration mode number $n = 3$ and the exciting frequency $\Omega = 0.6\omega_{nII}$, the effects Pasternak layer on the steady-state vibration amplitudes A_{n1} and A_{n2} of the Timoshenko beam represented by the ratios φ_1 and φ_2, shown in Figures-4 to 15, respectively. As can be seen, the ratio φ_1 decreases with the increase of the axial compression, which implies that the magnitude of the steady-state vibration amplitude A_{n1} become smaller when the axial compression increases and the ratio φ_2 increases with the increase of the axial compression, which implies that the magnitude of the steady-state vibration amplitude A_{n2} become larger when the axial compression increases . It can be observed that the effect of compressive axial load on the magnitude of A_{n1} is almost independent of the axial compression ratio ζ of the two beams whereas it is significantly dependent on the magnitude of A_{n2} [9]. As we said before in this paper the effects of Pasternak layer on forced transverse vibration of a Timoshenko double-beam system with effect of compressive axial load is analyzed.

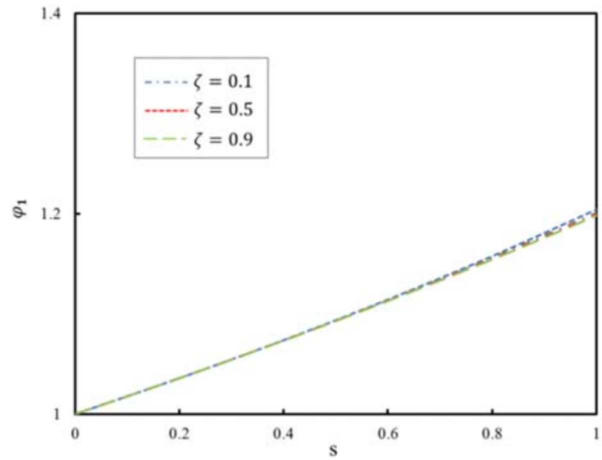

Figure-4. Relationship between ratio φ_1 and dimensionless parameter s for different axial compression ratio ζ , $G_0 = 0$.

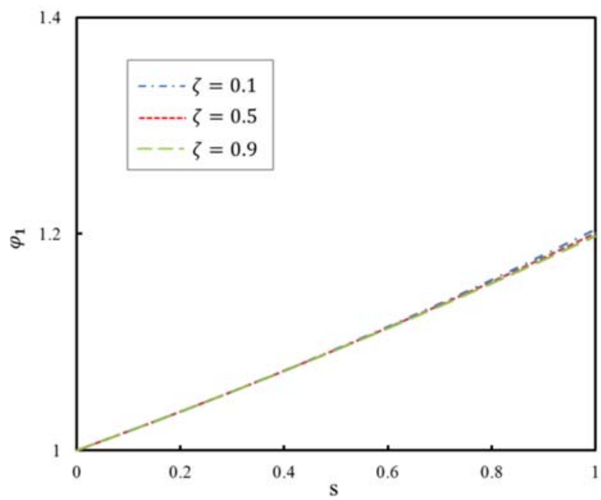

Figure-5. Relationship between ratio φ_1 and dimensionless parameter s for different axial compression ratio ζ , $G_0 = 1000$.

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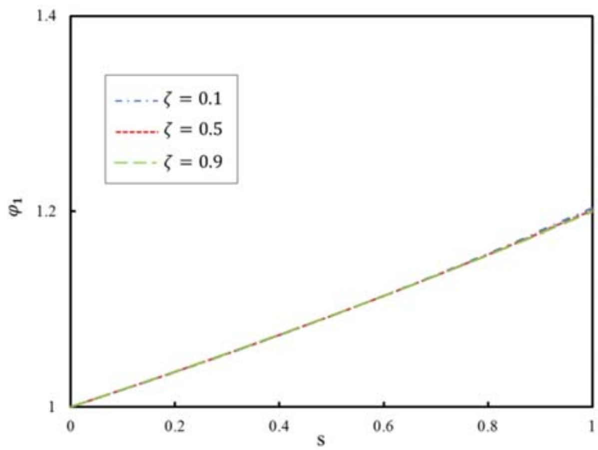

Figure-6. Relationship between ratio φ_1 and parameter s for different axial compression ratio ζ , $G_0 = 100000$.

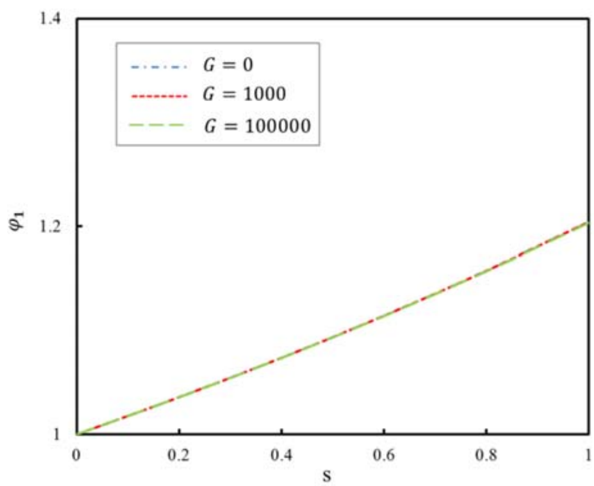

Figure-7. Relationship between ratio φ_1 and parameter s for different shear modulus of Pasternak layer, $\zeta = 0.1$.

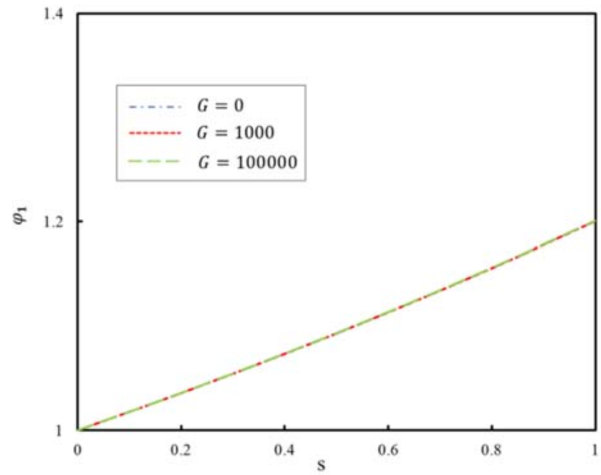

Figure-8. Relationship between ratio φ_1 and parameter s for different shear modulus of Pasternak layer, $\zeta = 0.5$.

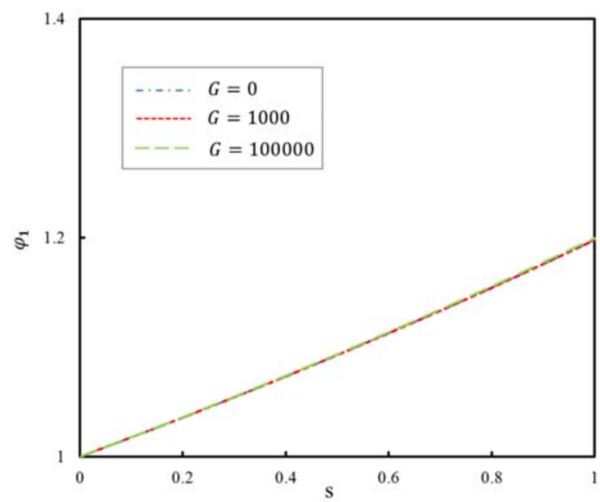

Figure-9. Relationship between ratio φ_1 and parameter s for different shear modulus of Pasternak layer, $\zeta = 0.9$.

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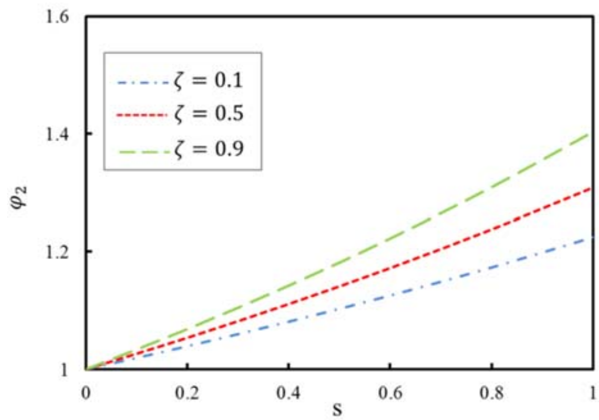

Figure-10. Relationship between ratio φ_2 and dimensionless parameter s for different axial compression ratio ζ , $G_0 = 0$.

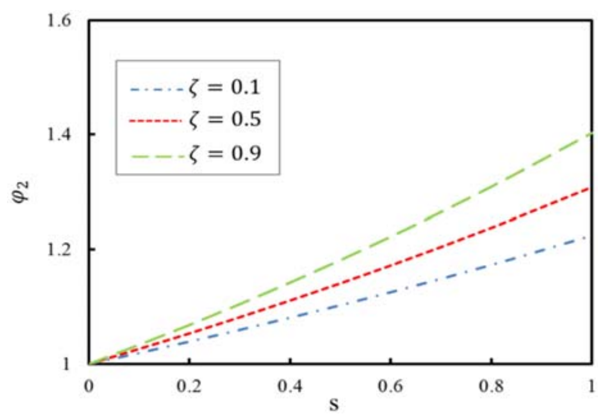

Figure-11. Relationship between ratio φ_2 and parameter s for different axial compression ratio ζ , $G_0 = 1000$.

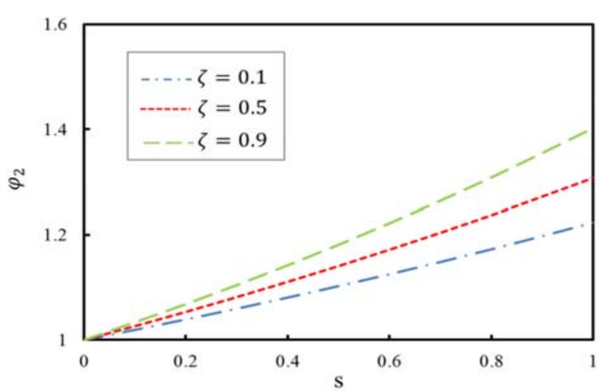

Figure-12. Relationship between ratio φ_2 and parameter s for different axial compression ratio ζ , $G_0 = 100000$.

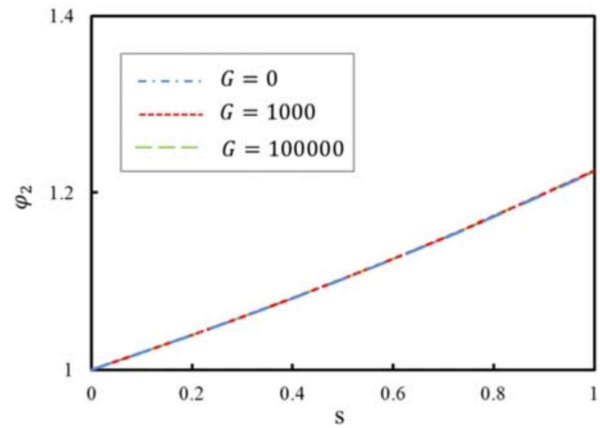

Figure-13. Relationship between ratio φ_2 and parameter s for different shear modulus of Pasternak layer, $\zeta = 0.1$.

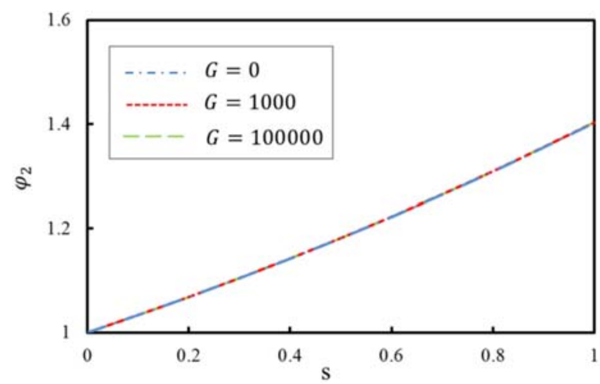

Figure-14. Relationship between ratio φ_2 and parameter s for different shear modulus of Pasternak layer, $\zeta = 0.5$.

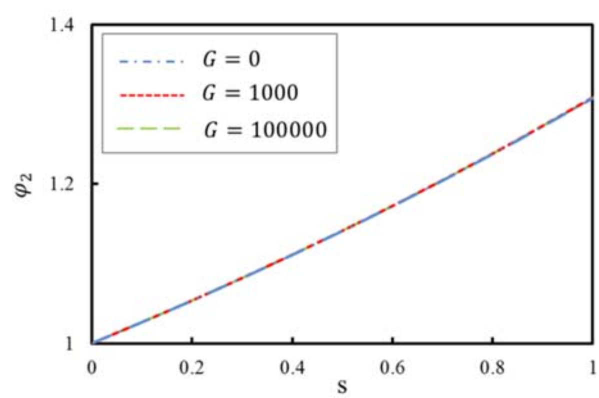

Figure-15. Relationship between ratio φ_2 and parameter s for different shear modulus of Pasternak layer, $\zeta = 0.9$.

Effects of Pasternak layer cannot be observed on Figures 4 to 15. Numerical values of the ratios φ_1 and φ_2 for different shear modulus of Pasternak layer with different axial compression ratio ζ of the two beams is shown in Table-2. Numerical results of the ratios φ_1 and φ_2 show that the difference with ratios φ_1 and φ_2 decrease with increasing shear modulus of Pasternak layer for different axial compression ratio ζ. But the differences among the results is too little and when we compare them with Rayleigh double beam, can be found that the effects of Pasternak layer on Rayleigh double beam is more than Timoshenko double beam.

Table-2. Effects of shear modulus of Pasternak layer on the steady-state vibration amplitudes ratios φ_1 and φ_2 for different compressive axial ratio ζ.

	s					
	0	**0.2**	**0.4**	**0.6**	**0.8**	**1**
$\varphi_1(\zeta=0.1)$						
$(G_0=0)$	1	1.0358	1.0737	1.1142	1.1575	1.2041
$(G_0=1000)$	1	1.0358	1.0737	1.1142	1.1575	1.2041
$(G_0=100000)$	1	1.0357	1.0737	1.1141	1.1573	1.2040
$\varphi_1(\zeta=0.5)$						
$(G_0=0)$	1	1.0358	1.0735	1.1134	1.1557	1.2006
$(G_0=1000)$	1	1.0358	1.0735	1.1134	1.1557	1.2006
$(G_0=100000)$	1	1.0358	1.0735	1.1134	1.1557	1.2005
$\varphi_1(\zeta=0.9)$						
$(G_0=0)$	1	1.0358	1.0734	1.1129	1.1543	1.198
$(G_0=1000)$	1	1.0358	1.0734	1.1129	1.1543	1.198
$(G_0=100000)$	1	1.0357	1.0735	1.1129	1.1544	1.198
$\varphi_2(\zeta=0.1)$						
$(G_0=0)$	1	1.0393	1.0811	1.1256	1.1733	1.2247
$(G_0=1000)$	1	1.0393	1.0811	1.1256	1.1733	1.2246
$(G_0=100000)$	1	1.0393	1.0810	1.1255	1.1732	1.2245
$\varphi_2(\zeta=0.5)$						
$(G_0=0)$	1	1.0537	1.1111	1.1723	1.2379	1.3083
$(G_0=1000)$	1	1.0537	1.1111	1.1723	1.2379	1.3083
$(G_0=100000)$	1	1.0537	1.1110	1.1722	1.2378	1.3082
$\varphi_2(\zeta=0.9)$						
$(G_0=0)$	1	1.0685	1.1425	1.2225	1.3091	1.403
$(G_0=1000)$	1	1.0685	1.1425	1.2225	1.3091	1.403
$(G_0=100000)$	1	1.0685	1.1424	1.2224	1.3090	1.4028

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On the other hand, with the axial compression ratio $\zeta = 0.5$ and the exciting frequency $\Omega = 0.6\omega_{nII}$, the effects of Pasternak layer on the steady-state vibration amplitudes A_{n1} and A_{n2} are shown in Figures-16 to 27 for different mode shape number n, respectively. It can be seen that, with the same axial compression, the ratios φ_1 and φ_2 diminish with the increasing vibration mode number n, which implies that the magnitudes of the steady-state vibration amplitudes A_{n1} and A_{n2} get smaller when the vibration mode number n becomes larger [9].

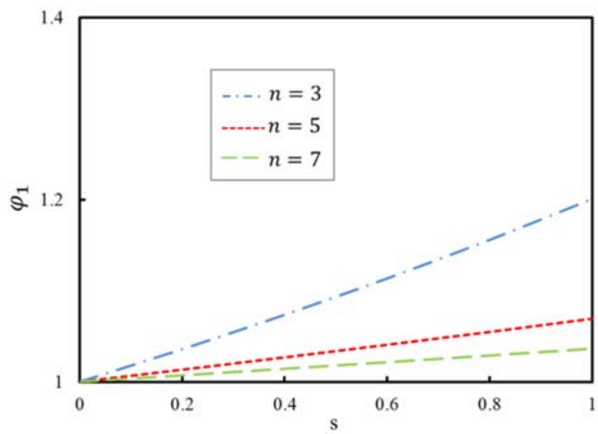

Figure-16. Relationship between ratio φ_1 and dimensionless parameter s for different mode number n for $G_0 = 0$.

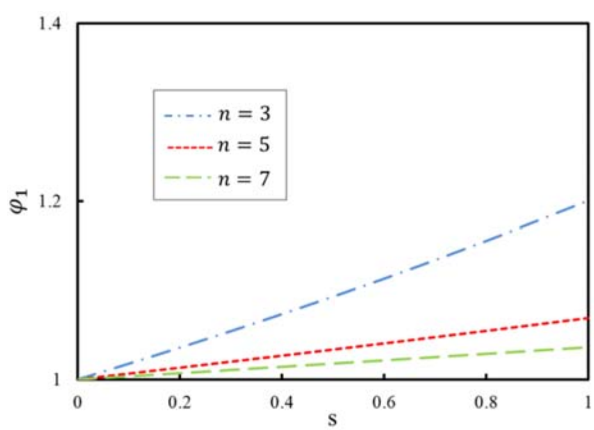

Figure-17. Relationship between ratio φ_1 and dimensionless parameter s for different mode number n for $G_0 = 1000$.

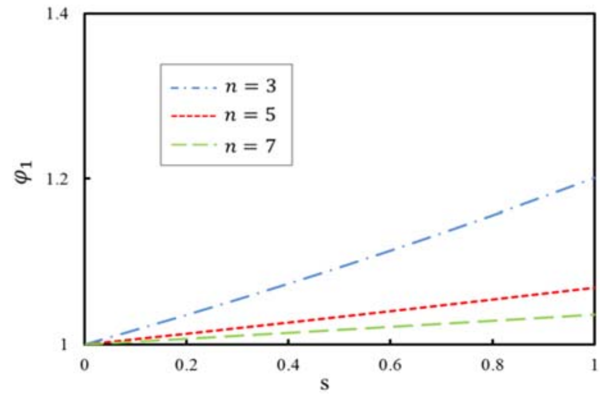

Figure-18. Relationship between ratio φ_1 and dimensionless parameter s for different mode number n for $G_0 = 100000$.

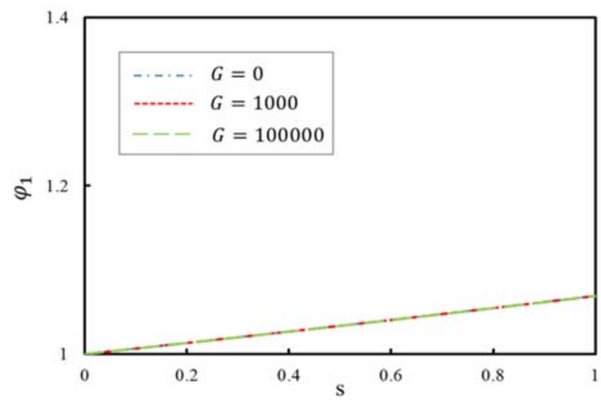

Figure-19. Relationship between ratio φ_1 and parameter s for different shear modulus of Pasternak layer, $n = 3$.

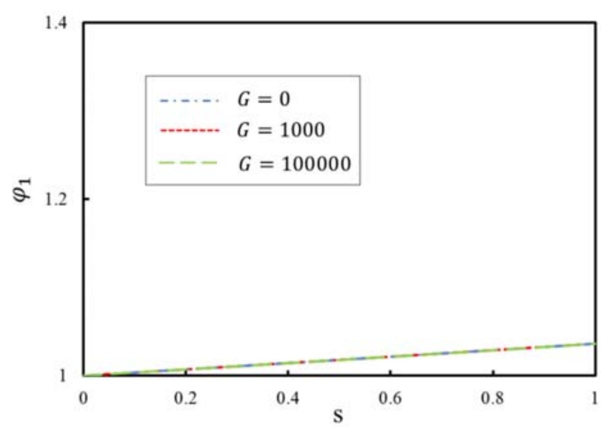

Figure-20. Relationship between ratio φ_1 and parameter s for different shear modulus of Pasternak layer, $n = 5$.

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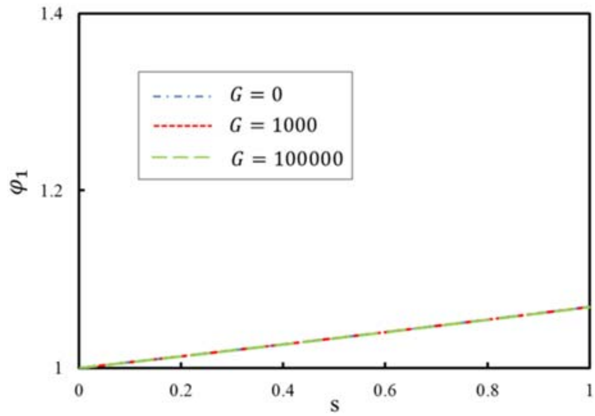

Figure-21. Relationship between ratio φ_1 and parameter s for different shear modulus of Pasternak layer, $n = 7$.

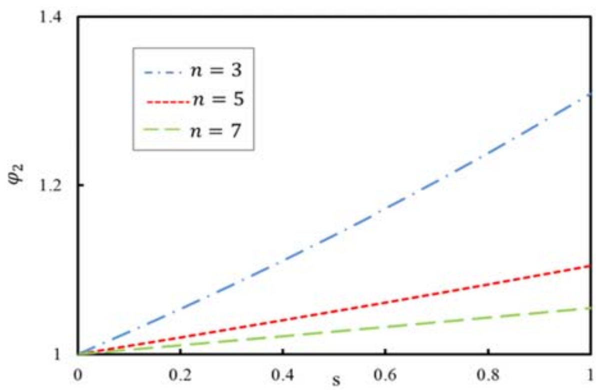

Figure-22. Relationship between ratio φ_2 and dimensionless parameter s for different mode number n for $G_0 = 0$.

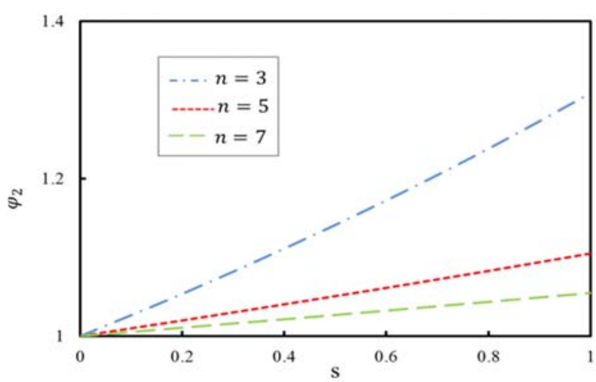

Figure-23. Relationship between ratio φ_2 and dimensionless parameter s for different mode number n for $G_0 = 1000$.

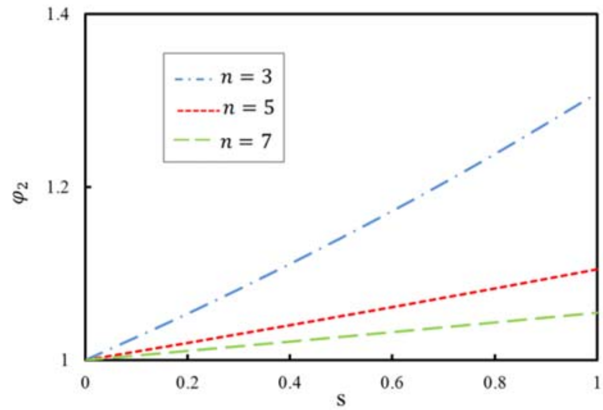

Figure-24. Relationship between ratio φ_2 and dimensionless parameter s for different mode number n for $G_0 = 100000$.

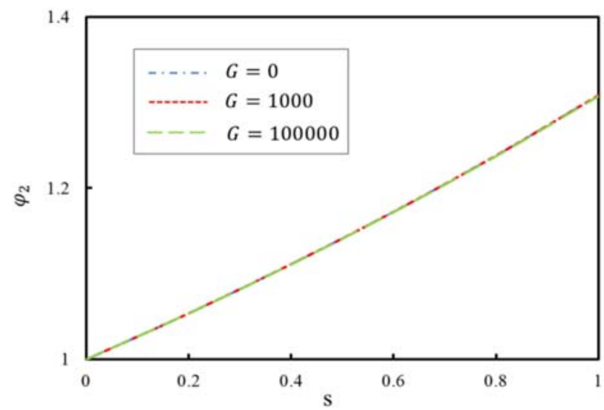

Figure-25. Relationship between ratio φ_2 and parameter s for different shear modulus of Pasternak layer, $n = 3$.

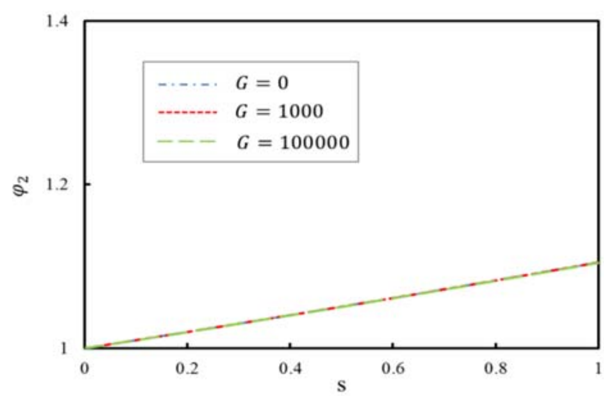

Figure-26. Relationship between ratio φ_2 and parameter s for different shear modulus of Pasternak layer, $n = 5$.

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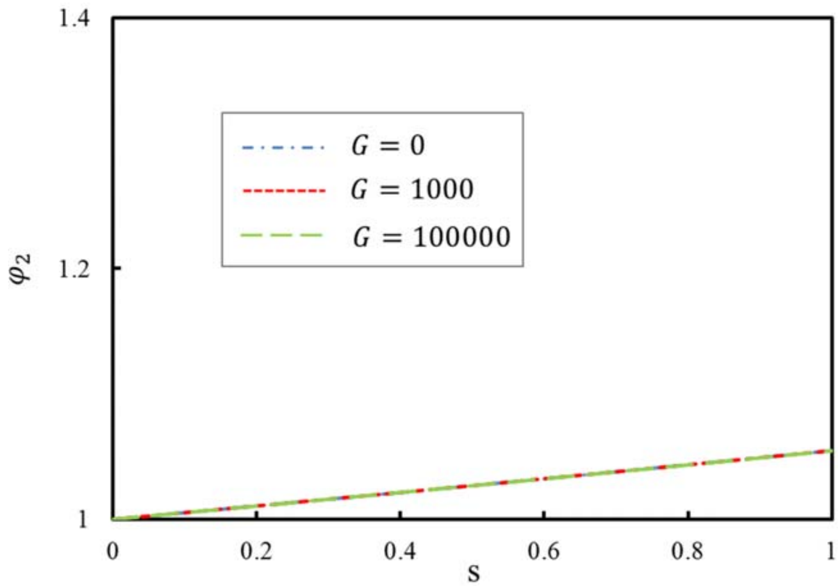

Figure-27. Relationship between ratio φ_2 and parameter s for different shear modulus of Pasternak layer, $n = 7$.

Table-3 shows the effects of compressive axial load and shear modulus of Pasternak layer on difference between the steady-state vibration amplitudes ratios φ_1 and φ_2 of the Rayleigh beam for different mode number n. It can be observed that the differences between ratios φ_1 and φ_2 of the Timoshenko beam increase with increasing the dimensionless parameter s. Also we can see that the ratios φ_1 and φ_2 decrease with increasing of the shear modulus of Pasternak layer for different mode shape numbers.

Table-3. Effects of shear modulus of Pasternak layer on the steady-state vibration amplitudes ratios φ_1 and φ_2 for different mode shape number n.

	s					
	0	**0.2**	**0.4**	**0.6**	**0.8**	**1**
$\varphi_1(n=3)$						
$(G_0=0)$	1	1.0358	1.0735	1.1134	1.1557	1.2006
$(G_0=1000)$	1	1.0358	1.0735	1.1134	1.1557	1.2006
$(G_0=100000)$	1	1.0358	1.0735	1.1134	1.1557	1.2005
$\varphi_1(n=5)$						
$(G_0=0)$	1	1.0133	1.0268	1.0406	1.0547	1.0692
$(G_0=1000)$	1	1.0133	1.0268	1.0406	1.0547	1.0691
$(G_0=100000)$	1	1.0133	1.0268	1.0406	1.0547	1.0691
$\varphi_1(n=7)$						
$(G_0=0)$	1	1.0071	1.0143	1.0215	1.0289	1.0363
$(G_0=1000)$	1	1.0071	1.0143	1.0215	1.0289	1.0363
$(G_0=100000)$	1	1.0071	1.0143	1.0215	1.0289	1.0363
$\varphi_2(n=3)$						
$(G_0=0)$	1	1.0537	1.1111	1.1723	1.2379	1.3038
$(G_0=1000)$	1	1.0537	1.1111	1.1723	1.2379	1.3038
$(G_0=100000)$	1	1.0537	1.1110	1.1722	1.2378	1.3037
$\varphi_2(n=5)$						
$(G_0=0)$	1	1.0199	1.0404	1.0613	1.0828	1.1049
$(G_0=1000)$	1	1.0199	1.0404	1.0613	1.0828	1.1049
$(G_0=100000)$	1	1.0199	1.0403	1.0613	1.0828	1.1048
$\varphi_2(n=7)$						
$(G_0=0)$	1	1.0107	1.0214	1.0324	1.0435	1.0548
$(G_0=1000)$	1	1.0107	1.0214	1.0324	1.0435	1.0548
$(G_0=100000)$	1	1.0106	1.0214	1.0324	1.0435	1.0547

6. CONCLUSIONS

Based on the Timoshenko beam theory, the effects of Pasternak layer on forced transverse vibration of an elastically connected simply supported Timoshenko double-beam, under compressive axial loading for one case of particular excitation loading are studied. The dynamic response of the system caused by arbitrarily distributed continuous loads is obtained. The magnitudes of the steady-state vibration amplitudes of the beam are dependent on the axial compression and shear modulus of Pasternak layer. The properties of the forced transverse vibrations of the system are found to be significantly dependent on the compressive axial load and shear modulus of Pasternak layer. The important result that this paper put emphasize on it is that the magnitudes of the steady-state vibration amplitudes become smaller when the shear Pasternak modulus increases and Pasternak layer can reduces the magnitudes of the steady-state vibration amplitudes more than Winkler elastic layer. But base on Timoshenko theory which takes into account the effects of shear deformation and rotary inertia, Pasternak layer doesn't have considerable effect on the magnitudes of the steady-state vibration amplitudes. Thus the Timoshenko beam-type dynamic absorber with Pasternak layer acts with a little more efficiency than Winkler elastic layer. Effects of Pasternak layer on Rayleigh double-beam is more than Timoshenko double-beam. Thus the Timoshenko beam-type dynamic absorber with Pasternak layer can be used to suppress the excessive vibrations of corresponding beam systems instead of those with Winkler elastic layer. Analytical forms found can be used in the optimal design of a new type of a dynamic vibration absorber. The beam-type dynamic damper is an accepted concept for a continuous dynamic vibration absorber (CDVA).

Nomenclature

$w = w(x,t)$	Transverse displacements of the beams
$\partial w/\partial x$	Global rotation of the cross-section
$T_{ni}(t)$	Unknown time function
$X_n(x)$	Known mode shape function for simply supported single beam
$\psi = \psi(x,t)$	Bending rotation
$S_{ni}(t)\,(i = I, II)$	Unknown time function corresponding to the natural frequency ω_{ni} .
X_m	Eigen-function
$q(x)$	Amplitude of the load
Ω	Exciting frequency of the load
A_{n1}^0, A_{n2}^0	Steady-state vibration amplitudes of the two beams without axial compression
ω_n	Natural frequencies of the system
G	Shear modulus
A	Area of the beam cross-section
k	Shear factor
M	Bending moments
E	Young modulus
I	Second moment of the area of the cross-section
ρ	Mass density
K	Spring constant
G_0	Shear modulus of Pasternak layer

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