



THE AVERAGE SER ANALYSIS OF STBC BASED FSO SYSTEMS

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ABSTRACT

In this paper, a closed-form symbol error rate (SER) for space-time block code (STBC) in free space optical (FSO) systems is derived. Then the asymptotic performance analysis presents the insight values, the diversity order and the SNR gain. We show that the SNR gain worsens as the number of lasers and photo detectors increases.

Keywords: free space optics, space-time block code, symbol error rate.

INTRODUCTION

This Free space optical (FSO) technologies are one of the potential candidates for wideband wireless communications [1-5]. Recently we have seen research works on FSO systems employing diversity schemes offering improved system performances. In [6] a closed-form bit error rate (BER) expression with transmit laser selection has been derived, whereas in [7] orthogonal space-time codes for binary pulse position modulation (BPPM) is proposed for FSO systems. The spatial diversity adopted at the receiver to mitigate the effect of turbulence induced irradiance fluctuation is reported in [8-10]. To that extent, in this paper, an exact closed-form of SER for STBC in FSO systems under the strong turbulence channel is derived. Furthermore, using asymptotic analysis, the diversity order and the signal-to-noise ratio (SNR) gain of the STBC in FSO systems are presented.

SYSTEM MODEL

M -laser and N -photodetector are considered in the proposed systems, see Figure-1. The channel is assumed to be flat and non-frequency selective fading and the perfect channel state information is assumed available at the receiver with the maximum likelihood (ML) detection. The data symbol vector \mathbf{x} is modulated and mapped onto a STBC signal vector $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M]$ with a size of $M \times T$ for a block length T . Note that $\mathcal{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{E}\mathbf{I}_M$ where $\mathcal{E}[\cdot]$ the expectation of the argument, \mathbf{H} is Hermitian operator, E is the total transmitted signal energy and \mathbf{I}_K is $K \times K$ the identity matrix. The received signal vector $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_N]$ is given by

$$\mathbf{Y} = \sqrt{\frac{E}{M}} \mathbf{H} \mathbf{X} + \mathbf{n} \quad (1)$$

where \mathbf{H} is the $N \times M$ channel matrix and \mathbf{n} is the circular complex Gaussian random variables with zero mean and variance $N_0 \mathbf{I}_N$. The effective instantaneous SNR at the receiver is given by

$$\gamma = \frac{E}{R_c M N_0} \|\mathbf{H}\|_F^2 \quad (2)$$

where R_c is the code rate, $\|\mathbf{H}\|_F^2 = \sum_{i=1}^M \sum_{j=1}^N |h_{ij}|^2$ is the squared Frobenius norm of \mathbf{H} , and h_{ij} is the amplitude path gain from the i^{th} laser to the j^{th} photodetector. According to the assumption of strong turbulence regime, h_{ij} is modelled as Rayleigh distribution with $\mathcal{E}[h_{ij}] = 1$. Then, the power path gain $h_{ij}^2 = \Phi^2$ has the negative exponential distribution with $\mathcal{E}[\Phi] = 1$. Let $h = \|\mathbf{H}\|_F^2$, then h is modelled as the chi-squared random variable with $2MN$ degrees of freedom. The probability density function (PDF) of h is given by:

$$f(h) = \frac{1}{2^{MN} \Gamma(MN)} h^{MN-1} e^{-h/2}, h > 0 \quad (3)$$

where $\Gamma(\cdot)$ is a gamma function. When we assume that all channels are identical, the average SNR is

$$\gamma_0 = \mathcal{E}[\gamma] = \frac{E}{R_c M N_0} \mathcal{E}[h] = \frac{2EN}{R_c N_0} \quad (4)$$

ANALYSIS OF SER

Exact Closed Form SER

The exact closed-form SER of STBC in FSO systems for BPPM is derived as follows:

$$\begin{aligned} P_{s,BPPM} &= \int_0^\infty \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{\gamma}{2}}\right) f(h) dh \\ &= \int_0^\infty \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{\gamma_0}{4MN}} h\right) f(h) dh \\ &= \frac{1}{2\sqrt{\pi} D!} \int_0^\infty z^D G_{0,1}^{1,0}\left(z \middle| \begin{smallmatrix} \cdot \\ 0 \end{smallmatrix}\right) G_{1,2}^{2,0}\left(\frac{\gamma_0 z}{2MN} \middle| \begin{smallmatrix} 1 \\ 0, 0.5 \end{smallmatrix}\right) dz \\ &= \frac{1}{2\sqrt{\pi} D!} G_{2,2}^{2,1}\left(\frac{\gamma_0}{2MN} \middle| \begin{smallmatrix} -D, 1 \\ 0, 0.5 \end{smallmatrix}\right) \end{aligned} \quad (5)$$



where erfc is the complementary error function, $h/2 = z$, $D = MN-1$, and $G(\cdot)$ is the Meijer-G function

Asymptotic SER

The diversity order and the SNR gain when the PDF $f(h = \gamma/\gamma_0)$ around $h \rightarrow 0$ is approximated by a

single polynomial. Using this asymptotic approach, we approximate the SER and obtain the insight values, the diversity order and the SNR gain of the STBC in FSO systems. By applying Taylor series $e^{-k} = \sum_{i=0}^{\infty} \frac{(-k)^i}{i!}$, the PDF $f(h)$ is rewritten as

$$f(h) = \frac{1}{2^{MN} \Gamma(MN)} h^{MN-1} \sum_{i=0}^{\infty} \frac{(-h/2)^i}{i!} \quad (6)$$

When $h \rightarrow 0$, the first term ($i = 0$) is dominant in (3). Thus

$f(h)$ is approximated by a single polynomial as given by:

$$f(h) \approx \frac{1}{2^{MN} \Gamma(MN)} h^{MN-1} \quad (7)$$

By employing the same process as (2), the asymptotic SER of BPPM $P_{s,BPPM,asym}$ is given by:

$$\begin{aligned} P_{s,BPPM} &\approx \frac{1}{2\sqrt{\pi D!}} \int_0^{\infty} z^D G_{1,2}^{2,0} \left(\frac{\gamma_0 z}{2MN} \middle| \frac{1}{0, 0.5} \right) dz \\ &\approx \frac{1}{2\sqrt{\pi D!}} \left(\frac{\gamma_0}{2MN} \right)^{-MN} \frac{\Gamma(1/2 + MN)}{MN} \end{aligned} \quad (8)$$

When the SER is approximated in the form of $P_s \approx (G_s \gamma_0)^{-G_o}$ at a high SNR, G_o and G_s represent the diversity orders and the SNR gain respectively. Moreover, the slope of the SER curve and its shift relative to a benchmark SER curve of $P_s = \gamma_0^{-G_o}$ represent the diversity order and the SNR gain respectively. From the asymptotic analysis as shown in (5), the diversity order and the SNR gain of the STBC in FSO systems is calculated as $G_o = MN$ and $G_s = \eta^{-1/MN}$, respectively where $\eta = \frac{\Gamma(\frac{1}{2} + MN)(2MN)^D}{\sqrt{\pi D!}}$. As a result, the diversity order can be improved as the number of lasers and photodetectors increases.

NUMERICAL RESULTS

Using the above closed-form equations (2) and (5), Figure-2 represents the average SER as a function of the average SNR for three sets of lasers and photodetectors, $(M, N) \in \{(2, 1), (4, 2), (8, 2)\}$. Figure-2 confirms that the

asymptotic results provide a good approximation of the exact approach results at higher values of SNR. Figure-3 illustrates the SNR gain as a function of the number of lasers M and for a number of photodetectors (e.g. $N = 1, 2$) by using the asymptotic analysis. It can be observed that the SNR gain worsens as the number of lasers and photodetectors increases. This is because the spatial diversity generally reduces the probability of receiving peak signals at the receiver.

CONCLUSIONS

In this paper, we have derived the exact closed-form of SER for the STBC in FSO systems employing the BPPM scheme. Furthermore, using the asymptotic analysis, we approximated the SER and then obtain the diversity order and the SNR gain. The diversity order and the SNR gain is calculated as $G_o = -MN$ and $G_s = \eta^{-1/MN}$ respectively where $\eta = \frac{\Gamma(\frac{1}{2} + MN)(2MN)^D}{\sqrt{\pi D!}}$. We showed that SNR gain worsens as the number of lasers and photodetectors increases.

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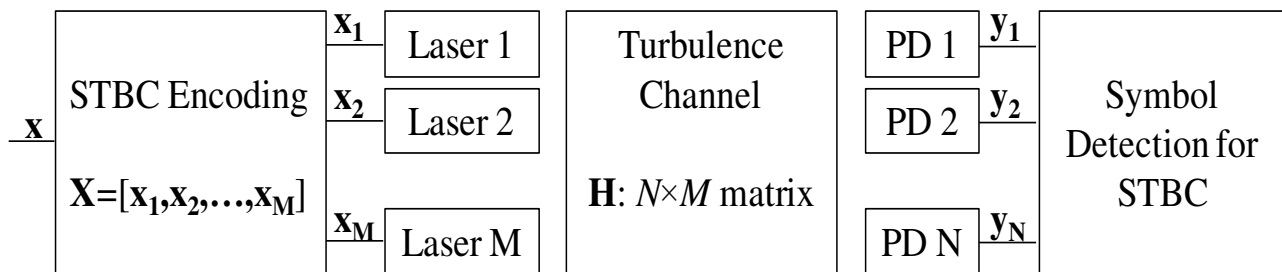


Figure-1. Block diagram for STBC based FSO systems.

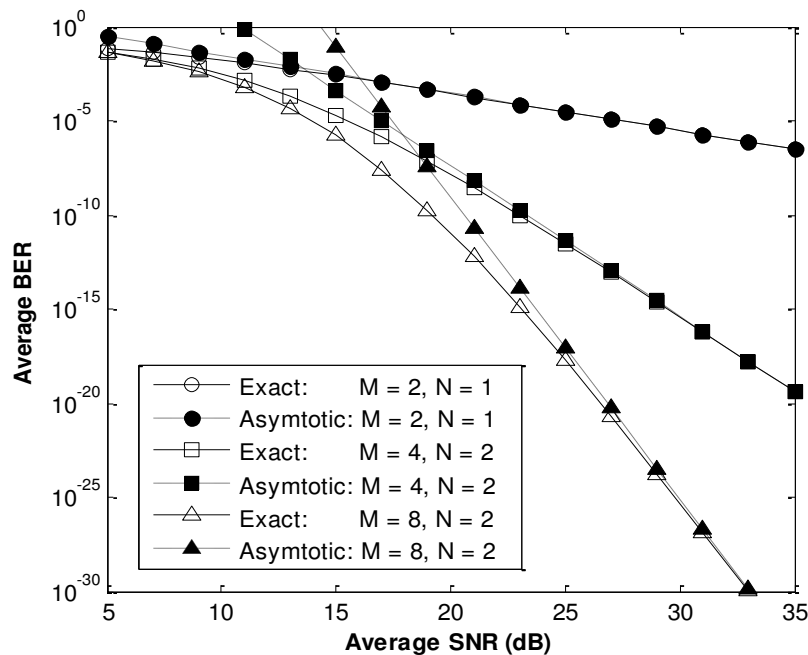


Figure-2. The average BER against the average SNR for exact and asymptotic SERs.

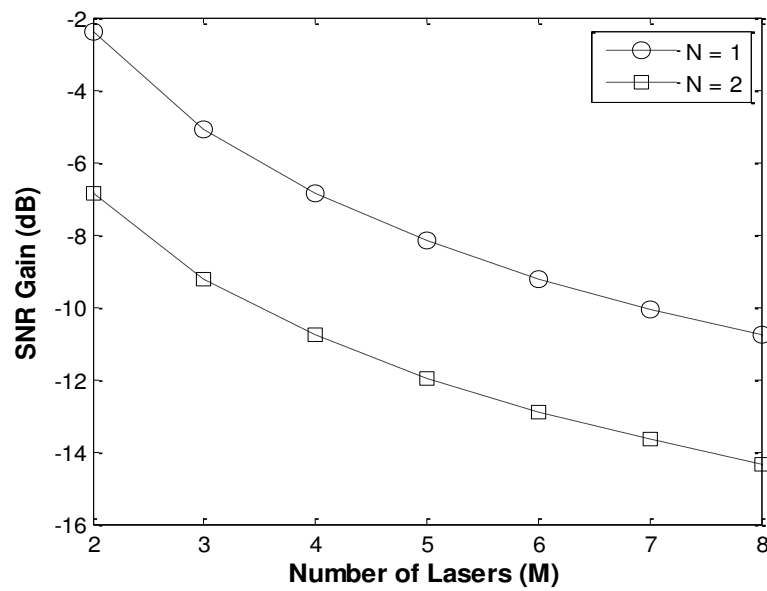


Figure-3. SNR gain as a function of the number of lasers M and for a number of photodetectors N .