DIRECT COMPARISON OF FSI OPTIMIZED THEODORSEN AND LARRABEE PROPellers

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ABSTRACT

The goal of this study is the analysis of the design process of aircraft propellers which are coupled to a piston engine, aiming to find the best design approach. The first design step is the calculation of the initial geometry. This phase is particularly critical since it will affect the following optimization. Several theories for blade design have been proposed during the years. The most popular are the Larrabee's procedure and the Theodorsen's theory. The Larrabee theory is the most used in recent years, while the Theodorsen was most popular in the WWII era. This work focuses on the differences on the results of the two approaches for a general aviation propeller for light aircrafts. For this aerial vehicle category both the strength and efficiency should be considered, since the production technology cannot be as refined as for larger propellers. As it will be seen, the subsonic nature of these aerial vehicles makes it possible to use both initial design approaches. In a second phase, the evaluation of the effect of aerodynamics and centrifugal loads requires the union of the results that come from CFD (Computational Fluid Dynamics) and the ones come from the CSM (Computational Structural Mechanics), through the execution of several one way FSI (Fluid Structure Interaction) analyses. However the starting point proved to be critical for the final result. The Larrabee procedure proves to be ideal for high speed aircraft propellers manufactured with up-to-date materials and procedures. The "old" Theodorsen theory leads to a stronger blade that can be easily manufactured with wood or simplified technologies. The Theodorsen blade is superior for the centrifugal load bearing capacity. This geometry leads to lighter blades. The efficiency of the Larrabee blade seems to be superior. However, experience proved that the CFD analysis can be tricky and unreliable for efficiency evaluation. The pressures are better distributed along the Larrabee’s blade with better results at high airspeed. Eventually two geometrically optimized blades have been designed, which have a deformed shape (at cruise conditions) similar to the best aerodynamic geometry and comparable technological characteristics. The Larrabee and Theodorsen designs lead to different optimized blades even after the FSI simulation, demonstrating that the optimization procedure is largely influenced by the initial propeller blade design.

Keywords: aircraft propeller; optimization; starting design; CFD; FEA; FSI.

INTRODUCTION

Propellers weight and efficiency are very important design concerns, especially with piston engines. In fact for UL, light vehicles and UAV these parameters are of primary interest for weight reduction and fuel consumption. The design procedure starts from an initial traditional design that is further optimized through FSI (Fluid Structure Interaction) analyses. This technique combines CFD (Computational Fluid Dynamics) result with and the come from the CSM (Computational Structural Mechanics) usually FE (Finite Element) but also BE (Boundary Elements). Unfortunately, in piston engines the torsional vibrations of the crankshaft are a common occurrence. These vibrations can lead to serious structural overload of main parts or auxiliaries, thereby requiring the installation of a specific damper [1].

The described phenomenon concerns almost all aeronautical piston-engines, but in these last years some ultra-light airplanes seems not to suffer of this problems, even if they haven’t got dynamical absorbers. Going into details, it has been noted that this fact is possible if the propeller is made of GFRP (Glass Fibre Reinforced Plastics), while problems come out if the propeller is made of other traditional materials (like CFRP - Carbon Fibre Reinforced Plastic).

Such a fact led to think that a blade made of composite material can be so deformable that the energy generated by shaft vibrations can be absorbed and dissipated by the propeller [1] [2].

In this specific case a practical approach has been followed, in order to verify the correctness of the just described hypothesis, through the analysis of a propeller designed starting from the classic design methods available in the literature.

The original designed have then been optimized to obtain the ideal shape in the deformed cruise condition.
Table-1. Materials properties (Ultimate Tensile Strength).

<table>
<thead>
<tr>
<th></th>
<th>Carbon fibre</th>
<th>Glass fibre</th>
<th>Aramid</th>
</tr>
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<tbody>
<tr>
<td>Young modulus</td>
<td>160 GPa</td>
<td>40 GPa</td>
<td>75 GPa</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.3</td>
<td>0.25</td>
<td>0.34</td>
</tr>
<tr>
<td>UTS (MPa)</td>
<td>1000</td>
<td>1000</td>
<td>1300</td>
</tr>
<tr>
<td>Density (g/cm³)</td>
<td>1.6</td>
<td>1.9</td>
<td>1.4</td>
</tr>
</tbody>
</table>

DESIGN OF THE BLADES
Two different procedures to design the preliminary blades were implemented, in order to be able to study possible differences of their structural and aerodynamic performances:

- Larrabee procedure [5], based on the theory of blade element. In this specific case scripts made by Hepperle have been used [6]; they get very thin blades distinguished by very large chord of the profiles.
- Crigler procedure [7], based on Theodersen's theory [8][9]. This way of calculation generates much tapered blades, which moreover present the section of maximum chord closer to the hub in comparison with the other kinds of blade; it allows the achievement of a configuration much more rigid than the ones mentioned in other theories.

Basic data for both procedures are based on the characteristics of the engine, of the aircraft and of the air. These data were set to:

- Propeller speed: 2800 rpm.
- \( V_\infty \) : 100 m/s.
- Engine power: 200 HP.
- Air density: 1.225 kg/m³.
- Kinematic viscosity of the air: 1.46*10⁻⁵ m²/s.
- Speed of sound: 340.29 m/s.
- Environmental: ISA-50K, ISA+20K, RH 40%

Other two features have been also arbitrarily set:

- Two-blade propeller.
- Clark Y airfoil.

Both procedures require inputting the diameter of the propeller. From several catalogs of different manufacturers the "optimum" propeller for this type of engine varies from 50" (1270 mm) up to 75" (1905 mm). From the general propeller theory, the fastest aircraft require the smaller propeller and the longer Take Off run (Figure-1).

In our case the cruise data suggest that the aircraft flies at low altitudes at Mach 0.3@ISA-50K (International Standard Atmosphere -50K offset). This temperature is the worst condition for compressibility effects, since the Mach number is proportional to the square root of the temperature.

Even if it is indeed possible to have supersonic tip speed (see for example, the Tupolev TU 95), for efficiency consideration it is better to avoid this condition.

The tip velocity chosen is 0.81 Mach at ISA-50K cruise altitude. This seems to be an acceptable compromise between take-off static thrust and cruise drag. This choice results in a diameter of 1,700 mm and an angular speed of 2,800 rpm.

GENERAL CONSIDERATIONS
In 1919 Betz theory showed that the minimum loss induced by a propeller occurs when its wake has a constant axial velocity and each section (of the wake) rotates around the axis of the propeller, behaving like an ideal uniform disk composed by infinite blades. However blades are finished and Goldstein demonstrated that the swirling flows, which detach from the trailing edge of the blade, cannot be considered as rigid bodies.

A real propeller converts part of the power into traction, while the remaining part is added to the internal energy of the wake. The wake has then an axial and a rotational component. The vortex theory of the propeller considers only the axial velocity component of the wake. In this the ideal efficiency is:

\[
\eta_i = \frac{1}{1 + \frac{W_a}{V_\infty}}
\]  

However, it can be adapted to a true (axial + rotational) wake by substituting the true velocity \( w_j \) to the axial velocity \( w_a \). By assuming a rigid rotation it is then possible to write...
\[ \eta_{\text{lr}} = \frac{1 - \frac{\omega_s}{2\omega}}{1 + \frac{w_j}{2V_{\infty}}} \]  

(2)

\[ \frac{\omega_s}{\omega} \approx \frac{2w_j (V_{\infty} + \frac{w_j}{2})}{R^2 \omega^2} \]  

(3)

and, with high values of \( V_{\infty} \)

\[ \eta_{\text{lr}} \approx \frac{1}{1 + \frac{w_j}{V_{\infty}} (1 + 2\gamma_f^2)} \]  

(4)

Usually \( \gamma_f \) is around 0.3; therefore the rotation component is small.

The Larrabee Approach

Larrabee proposed an approximate method in which the propeller radius \( R \) was estimated arbitrarily, the rotational wake energy is assumed to be negligible and \( w_j \) is constant for the blade element. So, the total propeller efficiency can be evaluated with (5) and (6).

\[ \eta = \eta_i \int_0^\frac{1}{T} \frac{\tan \phi}{\tan(\phi + \delta)} \frac{dT}{d\xi} d\xi \]  

(5)

with

\[ \delta = \frac{C_d}{C_i}, \xi = \frac{r}{R}, \tan \phi = \frac{V_{\infty}}{\omega r} \eta_i \]  

(6)

The wake rotation component can be seen as

In our case the Hepperle's datasheet were used for the initial design. These datasheets are an extremely optimized method of the Larrabee procedure. The result can be seen in Figure-3.

The Theodorsen Approach

The Theodorsen approach uses the velocity triangles of Figure-4:

\[ \text{Figure-2. Model schematics.} \]

\[ \text{Figure-3. CAD Model of the Larrabee blade.} \]

\[ \text{Figure-4. Theodorsen single blade model.} \]
Figure 5. CAD Model of the Theodorsen blade.

The authors have the technology for a composite hollow blade. Therefore, a constant thickness was considered for simplified manufacturing with a range from 1 mm to 4 mm. This range was set considering the characteristics of the propellers already available. A set of suitable materials is summarized in Table 1. During the optimization process the material was considered isotropic. This is a simplification often used in the first steps of a project, but, obviously, once obtained a nearly optimized blade, the orthotropy of the material must be included in the simulation.

FSI (Fluid Structure Interaction) analysis

The effect of loads acting on the propeller, in operating conditions, causes a modification of the shape and natural frequencies of the system. In order to realize the pre-stressed modal analyses, the execution of fluid-structure interaction study was necessary to evaluate the effect of different kinds of load.

The first step concerned the analysis of the aerodynamic load through the use of the CFD solver, therefore a computation domain has been defined (Figure 6).

As shown in Figure 6, the domain is divided in two sub-domains: the inner one is rotating at the propeller speed, while the external one is static. The rotational speed imposed to the inner domain was 2,800 rpm.

The boundary conditions imposed were:
- Velocity Inlet on surface 1 of Figure 6: 100 m/s that is aircraft TAS (True Air Speed)
- Velocity Inlet on surface 2 of Figure 6: 0 m/s.
- Null relative Pressure Outlet on surfaces 3 and 4 of Figure 6.

The CFD (Computational Fluid Dynamic) analysis on these two models (the Larrabee and the Theodorsen one) outputted the pressures acting on the blades surfaces. These results have been exported to the pre-processor of the FE (Finite Element) solver for FSI (Fluid Structural Integrated) analysis.

The two models have been balanced in order to obtain the same thrust (460 N) for a single blade.

The second step involved the setting of the static structural analysis, where the aerodynamic load has been coupled whit the inertia load due to the rotation to perform in conclusion the pre-stressed modal analyses.

Considering the constraints, the two blades have been coupled to the hub model through two Fixed Joint (contact problem was not considered), while the hub has been constrained using a Fixed Support (Figure 7).

Optimization

From the results of the several FSI analyses it has been possible to see that GFRP is the material of choice, being extremely economical and sufficiently performed for the application [1],[2]. However, also aramid fibres blades can be manufactured with higher costs.

The need to optimize the shape concerns the fact that the original CAD models can be further optimized, in order to obtain maximum efficiency (our case) or maximum thrust. Unfortunately, in operative conditions the blade configuration changes due to the loads effect, so the deformed shape is used in the CFD in the optimization process.

The interactive correction with the displacement of nodes coordinates obtains this result. In fact, the
orientation of the various profiles is altered to obtain a blade with the optimum that in operative conditions (2,800 rpm and 100 m/s) has a deformed shape that approximates the maximum efficiency condition (Figure-8).

Figure-8. Deformed propeller in operative conditions.

A practical design rule says that a propeller must have a Safety Factor (SF) larger than 15 in every point of the system. This SF corresponds to a statistical failure probability of 1x10^-9 in 3,000h. In this way, it is possible to find the optimum wall thickness for the blade. A different approach is to increase the rotational speed in the FEA structural analysis. In our case the propeller is loaded with a centrifugal force equal to the twice of the maximum rotational speed in real conditions. Therefore, the speed of 5600 rpm is applied to the structure. In this way, the Von Mises equivalent higher stress calculated is 987.8 MPa. This value is lower than the UTS (Ultimate Tensile Strength) of the material (Figure-10). However, the SF=15 rule is not fulfilled (Figure-9).

Figure-9. The safety factor.

COMPARISON OF THE TWO BLADES

The Theodorsen blade is 25% lighter of the Larrabee one. However the pressure distribution is more even for the Larrabee design (Figures 11 and 12).

Figure-10. Static analysis at 5,600 rpm.

Figure-11. Larrabee pressure.

The Theodorsen blade tends to concentrate the thrust (pressure) on blade tip.

Figure-12. Theodorsen pressure.

This fact is confirmed by the velocity distribution (Figures 13 and 14).

Figure-13. Larrabee absolute velocity.

Practically the Theodorsen design should require a rounded thinner tip with slight reduction in the net thrust. In fact, Theodorsen tip velocity is higher with noise and the real possibility to reach supersonic values during flight. This velocity pattern of Theodorsen is the true reason for a slightly lower efficiency.
However the Larrabee pressure pattern brings the aerodynamic pressure centre closer to the hub. This means that the fuselage or the nacelle behind or in front of the propeller will reduce the thrust more than the Theodorsen design. Therefore, the Larrabee propellers need more slender shapes to keep the high performance of the isolated propeller.

Moreover, the Theodorsen design leads to an FSI optimized blade that is 25% lighter than the Larrabee one. Just to have an idea of what happens with the two design philosophies, the Von Mises stresses are shown in Figures 15 and 16. Von Mises equivalent stresses are however meaningless for a Safety Factor evaluation. Fatigue criteria for composites should be used instead. In our case maximum stress and interlaminar shear were considered. Figures 15 and 16 shows that the Theodorsen blade has a better shape for stress except for the tip that should be rounded for satisfactory results. This is due to the fact that the main stress is given by the centrifugal load and the approximately trapezoidal shape of the Theodorsen blade has larger section near the hub.

The same happens to natural frequencies, where the much heavier Larrabee blade behaves better (figures 17 and 18).

CONCLUSIONS
The present study demonstrates that the initial design of the blade leads to very different results in terms of performance of the FSI optimized propeller. The Theodorsen and the Larrabee theories were used for the initial design. The two propellers were then optimized with an undeformed shape that leads to optimum performance under cruise aerodynamic loads. For this purpose GFRP hollow propeller were considered as suitable manufacturing method. Almost the same results can be obtained with Aramid fibres with better results in terms of weight. CFRP blades are much stiffer and lighter but their performance is more critical for impact loads. The Larrabee based propeller performs slightly better in terms of performance, noise and efficiency. However, it weighs 25% more (whit the same thickness) and the centre of pressure is nearer to the hub. So for bulky fuselages or nacelles the performances are nearly the same. Therefore, the Larrabee procedure is better as a starting point for high speed general aviation aircrafts. For lightweight or ultralight aircraft the Theodorsen approach is still convenient due to lighter construction and the pressure centre closer to the blade tip.

Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>$V_{\infty}$</td>
<td>TAS aircraft</td>
<td>m/s</td>
</tr>
<tr>
<td>$w_j$</td>
<td>Helical flux velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$\eta_i, \eta_R$</td>
<td>Propeller efficiency</td>
<td>-</td>
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<tr>
<td>$\omega_i$</td>
<td>Wake angular velocity</td>
<td>rad/s</td>
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### REFERENCES


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<th>$\omega$</th>
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<tr>
<td>$\gamma_f$</td>
<td>Working ratio</td>
<td>$-$</td>
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<tr>
<td>$R$</td>
<td>Propeller radius</td>
<td>$\text{m}$</td>
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<tr>
<td>$T$</td>
<td>Thrust</td>
<td>$\text{N}$</td>
</tr>
<tr>
<td>$c_d$</td>
<td>Drag Coefficient</td>
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</tr>
<tr>
<td>$c_l$</td>
<td>Lift Coefficient</td>
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high pressure centrifugal compressor for a small common rail diesel engine. Asian Research Publishing Network (ARPN). Journal of Engineering and Applied Sciences. ISSN 1819-6608, 10(4): 1623-1630, EBSCO Publishing, 10 Estes Street, P.O. Box 682, Ipswich, MA 01938, USA.


