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COMPARISON OF QUALITATIVE ASSESSMENTS OF EMPLOYEES WORK BY RANDOMIZED INDICATORS

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ABSTRACT

The article offers the approach based on creation of randomized quality assessments with the use of stochastic domination methods intended for the quality management in information and communication technologies. It considers the existing approaches to convolution of the quality indicators with the use of Kolmogorov mean (Quasi-arithmetic mean), shows an advantage of the randomization of scales to the solution of various qualimetric tasks. It offers the qualimetric scale of the quality assessment of support agents' work. It considers the modernization of the existing approach based on discrete models of probabilities distribution on integer lattices in relation to the studied objects in information and communication technologies. It shows an advantage of use these methods and gives recommendations to its application for calculation of coefficients of linear verification as the average from randomized coefficients taking into account the set limitations. Comparison of the quality indicators is made on the basis of a domination matrix with the corresponding orgraph creation. It offers the algorithm of casual scales generation on the truncated polytope on the basis of the Dirichlet distribution possessing the linear complexity. It considers the practical examples of the described methods application for comparison of quantitative characteristics of the work quality of support staff. The methodical approaches described in the article are useful for comparison of the objects quality in the conditions of uncertainty.

Keywords: quality, qualimetry, stochastic domination, randomization, information technologies.

INTRODUCTION

Recently in information and communication technologies it is paid close attention to questions concerning the services quality both on the part of public authorities and academic community. In the IT-companies which work at the competitive market, the quality can be theoretically characterized; firstly, by figures estimating the organization itself that renders services, for example, by CMMI, CobIT, etc., secondly, it can be estimated by the consumer's reaction (sales volume, a controlled market share, the results of polls). At the same time in the conditions of the allocated intra holding IT-company the problem of the quality assessment is much more difficult. It is particularly caused by the fact that in most cases the IT-company in the holding conditions is not core business, it has almost guaranteed orders at payment for services that is lower the market level, and it is first of all oriented to the business problems solution sometimes to the detriment of expectations satisfaction of final users of information systems of the average and the lowest unit. The described situation does not allow using both economic figures and the quality assessment methods by all consumers of services to the full. Thus, the task of creation of the unified approach to the complex services quality assessment of the intra holding IT-company as in general and to each its worker considering both objective and subjective indicators is for today insufficiently studied. Within the solution of the specified task for the process of technical support and management of incidents we have carried out the analysis of the key entrances and exits of the processes [2] and we have offered the approach to the quality assessment of the rendered services with the use of numerical characteristics of the process [3]. However the integration of the numerical and non-numerical data characterizing the services quality taking into account the principles of the representational theory of measurements represents the certain complexity owing to the inadmissibility of the arithmetic operators use directly to data where the numbers are used for reflection of an order at the quality level. Thus, the purpose of this work is to develop the methods of the assessment of a complex figure of the IT-company staff work quality on the basis of numerical and non-numerical figures of the quality and its verification on the real data.

For this task solution the use of the method randomized estimates creation of the quality on the basis of discrete models of the probability distribution on integer lattices that is considered in N. N. Rozhkov's [4,5,6] and N. V. Hovanova's works [7, 8, 9] with the subsequent comparison of the objects quality by stochastic domination is reasonable.

MAIN PART

The complex indicator of the quality Q which construction is made by the linear convolution m of the simple indicators X(1)…X(m) with the use of weight coefficients p1…pm is used in the majority of the practical tasks that are solved by qualimetry methods:

$$Q = \sum_{i=1}^{m} p_i X^{(i)} \quad (1)$$

In other words the task of the indicator Q creation can be divided into two stages:

a) A choice of the key performance indicators (KPI) that can be included in the nomenclature of simple indicators of the linear convolution.

b) Determination of weight coefficients p1 … pm reflecting the relative importance of simple indicators X(1) … X(m). Thus all simple indicators have the samefocus of scales and their relative quality increases with the growth of their value.

As it was shown [4], in the absence of the sufficient bases for the unambiguous choice of weight coefficients p1 …pm, this task concerning the services sector can be solved with the use of these coefficients randomization model.

Let p1,…, pm – weight coefficients which are used in the complex quality model creation, then besides usually imposed conditions:

$$p_i \geq 0 \text{ (for all i=1,… m)}, \ \sum_{i=1}^{m} p_i = 1 \quad (2)$$

the weights coefficients also satisfy the following limitation: all pi accept the values only from the set of numbers:

$$\{0, n^{-1}, 2n^{-1}, \ldots, (n-1)n^{-1}, 1\}, \quad (3)$$

where n – the set integer which determines the accuracy with which weight will be further estimated. The number of all possible vectors in that case further indicated as Nn(0) is described by the following combinatory formula (4):

$$N_n(0) = \binom{n+m-1}{m-1} \quad (4)$$

In case if all Nn (0) of sets (p1,…, pm) are admissible in the linear convolution, otherwise, in the absence of the restrictions imposed by the indicators hierarchy, we choice the model based on the uniform distribution when each set of scales can be accepted with probability of Nn (0)-1. For the first time this model was studied in N. V. Hovanov's works [7, 8, 9]. At such approach weight coefficients of pi are random values, further indicated Pi and as it is shown in the work [4], asymptotically meet on distribution function to beta-distribution, with density of distribution described by the function (5):

$$f_{\beta}(x;1;m-1) = (m-1)(1-x)^{m-2}, \quad (5)$$

The work [4] considers the mathematical apparatus supporting these methods in more detail.

The unstrict inequality pi≥0 in the formula (2) means that some of the weights coefficients can be equal to zero, i.e. do not make a contribution to the final quality of the object. As the solved task is connected with the key indicators of productivity, it is reasonable to use the strict inequality that will transform the formula (2) and (3) to the form:

$$p_i \geq \frac{1}{n} \text{ (for all i=1,… m)}, \ \sum_{i=1}^{m} p_i = 1 \quad (6)$$

The set of various scales which meets such conditions contains the number $N_n(1)$, where:

$$N_n(1) = \binom{n-1}{m-1} \quad (7)$$

This model of the casual scales distribution is also based on the random distribution when each set of scales can be accepted with the probability $N_n(1)^{-1}$. Thus the mathematical expectation of the random variable Pi for this distribution law is proper equal to:

$$E(P_i) = \frac{1}{m} \quad (8)$$

Let the values of the randomized complex indicator Q for the objects quality (A) and (B) can be calculated using formula (1). The decision of one object to another can be made by comparison QA and QB. However, as a result of randomization the performance of the inequality QA > QB is a casual event with the

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probability P(QA > QB). If the specified probability surpasses some threshold value γ, it is possible to speak about significant stochastic domination of the object A over the object B.

We will consider the practical task of the analysis of support staff work quality of the IT-company on a fragment of the developed system of the quality complex analysis. As the quality indicators we will consider the indicators presented in Table-1.

Table-1. The quality indicators of the support staff work quality (KPI).

The quality indicator	Designation	The way of measurement/assessment
The average mark	$X^{(1)}$	The average assessment which is put down by the user after the service rendering and the address closing. The assessment is in the range [1:5], the average grade will be transformed to 10-point scale by multiplication by
The share of addresses solved in time	$X^{(2)}$	Relation of the addresses number solved by the employee in target date to the general number of addresses [0:1]. It is transferred to 10 -point scale by multiplication by 10.
The share of addresses without complaints	$X^{(3)}$	The number of appeals without complaints to the general number of addresses [0:1]. It is transferred to 10-piont scale by multiplication by 10.
Intensity of work	$X^{(4)}$	Expert assessment [1:10] is exposed by the unit manager on the basis of the order qualimetrical scale
Responsibility	$X^{(5)}$	Expert assessment [1:10] is exposed by the unit manager on the basis of the order qualimetrical scale
The labor discipline	$X^{(6)}$	Expert assessment [1:10] is exposed by the unit manager on the basis of the order qualimetrical scale

Thus, the quality model can be described by the following linear convolution:

$Q=P_1X^{(1)}+P_2X^{(2)}+P_3X^{(3)}+P_4X^{(4)}+P_5X^{(5)}+P_6X^{(6)}+ P_7X^{(7)}+P_8X^{(8)}$.

We will consider two versions of convolution: without limitations and with them. We will form limitations in the form of the following inequality of the weights coefficients p3>p1, p1>p2, p6>p4, p5>p4, p2>p4. Considering the fact that within the randomization method the weights coefficients become random variables, all five inequalities should be considered as the events which have to be carried out with probability one. Having accepted n-1=0.05 accuracy degree we receive a set from 11628 coefficients for the case without limitations. After the limitations application we receive among admissible only 65 possible values.

We will consider indicators of seven employees (the line "2" - "8" in Table 2) and minimal acceptable level of the quality (the line "1" in Table-2).

Table-2. Values of the quality indicators.

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$\bar{Q}_{cp}$
1	6,00	9,50	9,90	7,00	7,00	8,00	7,90
2	7,00	9,84	10,00	10,00	8,00	10,00	9,14
3	6,00	10,00	10,00	6,00	8,00	10,00	8,33
4	6,00	10,00	9,50	7,00	8,00	9,00	8,25
5	6,89	9,59	10,00	6,00	7,00	8,00	7,91
6	7,00	10,00	10,00	8,00	6,00	9,00	8,33
7	6,00	9,10	10,00	8,00	5,00	8,00	7,68
8	7,50	9,81	10,00	9,00	9,00	7,00	8,72

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Calculation of the weights coefficients and also the following analysis are made with the use of the GNU R [10-12] language. As a result we receive properly 11628 and 65 calculated values of the randomized criterion for the versions described above. The descriptive statistics for the results of these calculations is given in Table-3.

Table-3. The descriptive statistics of the randomized criteria.

	Min	1 Qt	$\tilde{\tilde{X}}$	$\bar{X}$	3 Qt	Max	σ
Without the limitations							
$X(V_1)$	6,57	7,58	7,89	7,9	8,21	9,3	0,44
$X(V_2)$	7,64	8,89	9,18	9,14	9,43	9,74	0,38
$X(V_3)$	6,70	7,90	8,40	8,33	8,80	9,50	0,57
$X(V_4)$	6,68	7,95	8,28	8,25	8,58	9,48	0,45
$X(V_5)$	6,57	7,58	7,89	7,91	8,23	9,37	0,46
$X(V_6)$	6,70	8,00	8,35	8,33	8,65	9,50	0,47
$X(V_7)$	5,81	7,32	7,71	7,68	8,07	9,31	0,54
$X(V_8)$	7,52	8,48	8,74	8,72	8,98	9,62	0,35
Within the limitations							
$X(V_1)$	7,78	8,03	8,15	8,16	8,27	8,65	0,18
$X(V_2)$	8,73	8,98	9,08	9,08	9,18	9,33	0,15
$X(V_3)$	8,40	8,60	8,70	8,71	8,80	9,00	0,17
$X(V_4)$	8,18	8,35	8,43	8,43	8,53	8,70	0,12
$X(V_5)$	7,89	8,19	8,32	8,31	8,44	8,79	0,19
$X(V_6)$	7,75	8,35	8,50	8,48	8,65	8,95	0,25
$X(V_7)$	7,01	7,67	7,91	7,85	8,06	8,51	0,30
$X(V_8)$	8,26	8,67	8,81	8,78	8,90	9,16	0,18

Graphically distributions of the randomized criteria values are presented in Figure-1a and 1в with the help of the "violin plot" as this graphic form of representation gives more information about nature of the distribution than "box-and-whisker plot" because besides the data on a median and quartiles it also reflects indicators of nuclear density of the distribution [13]. As Figure-1 shows, the limitations introduction allows distinguishing more authentically levels of the work quality for various employees. Comparing the general level of the quality for each of possible values, it is possible to calculate the probability of stochastic domination $P(X(V_i)>X(V_j))$. The results of calculations are given in Table-3 and Table-4.

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Table-4. A matrix of stochastic domination probabilities $P(X(V_i)>X(V_j))$ for the randomized complex indicator of the quality level without limitations.

i \ j	1	2	3	4	5	6	7	8
1	0.00	0.00	0.06	0.01	0.43	0.05	0.79	0.01
2	1.00	0.00	1.00	1.00	1.00	1.00	1.00	0.86
3	0.93	0.00	0.00	0.67	0.94	0.48	0.90	0.23
4	0.99	0.00	0.29	0.00	0.90	0.33	0.93	0.13
5	0.57	0.00	0.06	0.10	0.00	0.07	0.75	0.02
6	0.95	0.00	0.48	0.65	0.93	0.00	1.00	0.19
7	0.21	0.00	0.10	0.07	0.25	0.00	0.00	0.01
8	0.99	0.14	0.77	0.87	0.98	0.81	0.99	0.00

Table-5. A matrix of stochastic domination $P(X(V_i)>X(V_j))$ for the randomized complex indicator of the quality level taking into account limitations.

i \ j	1	2	3	4	5	6	7	8
1	0.00	0.00	0.00	0.00	0.00	0.02	1	0.00
2	1.00	0.00	1.00	1.00	1.00	1.00	1	0.94
3	1.00	0.00	0.00	1.00	1.00	0.89	1	0.32
4	1.00	0.00	0.00	0.00	0.77	0.31	1	0.08
5	1.00	0.00	0.00	0.23	0.00	0.08	1	0.00
6	0.98	0.00	0.06	0.63	0.92	0.00	1	0.14
7	0.00	0.00	0.00	0.00	0.00	0.00	0	0.00
8	1.00	0.06	0.68	0.92	1.00	0.86	1	0.00

On the basis of the carried-out calculations for $P(X(V_i)>X(V_j))>0.6$ the orgraph presented in Figure-1b and Figure-1g is constructed. The constructed columns allow easily ranging the levels of quality of employees on the basis of the analysis of the inbound and outgoing edges for each block. Thus, as Table-4 shows, the probabilities of stochastic domination accept the following values:

P(X(“2”)>X(“8”))=0.86; P(X(“8”)>X(“3”))=0,77; P(X(“3”)>X(“6”)) =0.48; P(X(“6”)>X(“4”)) =0.65; P(X(“4”)>X(“5”))=0.90; P(X(“5”)>X(“1”))=0.57; P(X(“1”)>X(“7”))=0.79, that can be described in the form of the following complex quality indicators hierarchy: Q(“2”)>Q(“8”)>Q(“3”)~Q(“6”)>Q(“4”)> Q(“5”)≥Q(“1”)>Q(“7”), where Q(“1”), as it was noted above, is the minimal admissible level of quality.

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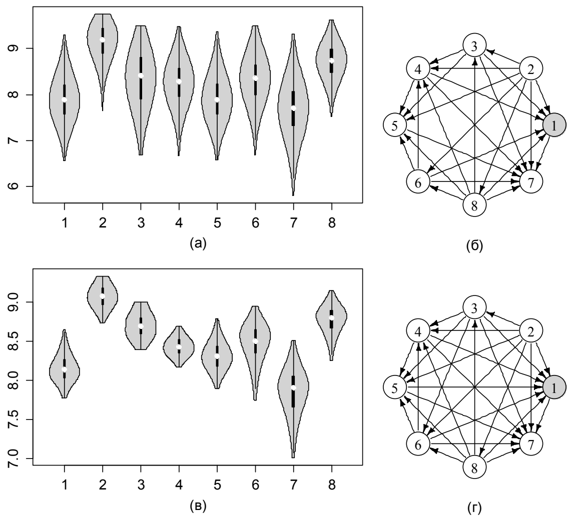

Figure-1. The density of distributions of the randomized complex quality indicator and the column of stochastic domination.

Where: (a) The "violin plot" for the linear convolution of indicators without limitations. (б) The orgraph of stochastic domination for this convolution. (в) The "violin plot" for the linear convolution taking into account limitations. (г) The orgraph of stochastic domination taking into account the limitations. The edges in columns leave the dominating objects, the allocated top "1" corresponds to the minimal acceptable level of quality.

From Table-5, similar to Table 4, we have the following probabilities of stochastic domination: P(X("2")>X("8"))=0.94; P(X("8")>X("3"))=0.68; P(X("3")>X("6"))=0.89; P(X("6")>X("4"))=0.63; P(X("4")>X("5"))=0.77; P(X("5")>X("1"))=1; P(X("1")>X("7"))=1 that is described by the following relation of the quality indicators: Q("2")>Q("8")>Q("3")>Q("6")>Q("4")>Q("5")>Q("1")> Q("7").

As we see, the order in this case did not change, however there is an expressed domination of the complex quality indicator at the employee "3" over the indicator of the employee "6", the similar picture is observed for employees "5" and "1". It is to be noted that as experiment shows the order of indicators ranging when using limitations can also change that is especially typical for cases of a small gap between the indicators.

Using the mathematical expectation in the formula (1) from the right and left part we receive:

$$E[Q]=\sum_{i=1}^{m} E[p_i]X^{(i)} \qquad (6)$$

The formula (6) allows finding the mathematical expectation of the quality level by mathematical expectations of the randomized scales taken on all sets corresponding to the set limitations. In case if limitations are not used, the task is reduced to the trivial and the average value of coefficients according to the formula (8) will be equal 1/m, in this case 1/6 or 0.17. In case of use of the limitations described above, we receive the following values of scales with the demanded accuracy up to 0.01: p1=0.19, p2=0.11, p3=0.31, p4=0.05, p5=0.17, p6=0.17. Thus, within the express assessment for expeditious adoption of the administrative decisions also, it is possible to use the offered average weights coefficients. However, if this administrative decision affects ranging of employees within the motivation system considering the different dispersion for the randomized scales it is reasonable to use the method of the stochastic domination described above.
It is to be noted that the use of a large number of indicators as it is presented in the formula (7) raises considerably requirements to computing capacities, or leads to calculations accuracy decrease. As can be shown from the given example, the probability of casual selection of admissible on limitations coefficient of all set meeting the condition (6) is only 0.0056. Therefore in this case it is advisable to use the algorithmic approach based on selection of the casual set of coefficients taking into account the set limitations from above and from below. At the same time the power complexity of the algorithm described above complicates its application at a large number of process quality indicators in practical activities. On the basis of it the algorithm of casual points on a polytope taking into account the set limitations considered in P. Rubin's (Rubin) [10] work was adapted for carrying out calculations.

Let the above described coefficients p1... pm are random variables, then their population forms m-vertex simplex in m-dimensional space:

$$S^m=\{(p_1\dots p_m):\sum_{i=1}^{m} p_i=1; p_i .. 0, i=1,\dots m\},$$

because of it the task is reduced to generation of the casual points which are evenly distributed on a surface of a standard simplex. It is known that the Dirichlet distribution meets this condition, the density probability of which for k > 2 and αi>0 is described by the formula:

$$f\left(x_1,\cdots,x_{K-1};\alpha_1,\cdots,\alpha_K\right)=\frac{1}{B(\alpha)}\prod_{i=1}^{K} x_i^{\alpha_i-1},$$

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where B (α) - the multidimensional beta function. As many specialized mathematical software allow generating the casual vectors with the Dirichlet distribution, the task is trivial.

The other way of generation of an initial matrix of the random numbers is based on the known task connected with the Dirichlet distribution to cut the threads: if a1... am−1 - the random independent evenly distributed variables on a piece [0,1] - the same sizes sorted in ascending order, and then a m-dimensional point of the form is the value of the casual m-dimensional vector which is evenly distributed in a simplex.

In case of introduction for coefficients p the form limitations; we receive the wrong simplex (polytope):

$$\dot{S}^m = \{(p_1 \ldots p_m) : \sum_{i=1}^{m} p_i = 1; p_i \ldots 0, i = 1, \ldots m; p_a \ldots p_b, \ldots, p_c \ldots p_d\},$$

where $\dot{S}^m \subseteq S^m$. Reflecting the points $(P \mid P \in S \square \dot{S})$ concerning the planes corresponding to the inequalities, we receive the points which are evenly distributed in the set polytope.

CONCLUSIONS

Received thus Z casual sets of coefficients it is possible to use for calculations according the methods described above. Realization of the specified approach allows considerably lowering requirements to computing capacities when carrying out calculations in comparison with the approach based on generation of all set of coefficients with the set accuracy and their filtrations on the basis of the set limitations.

Paying attention to the above it is possible to make a conclusion that for the qualimetrical problem solution of summary quality indicator finding it is reasonable to use the following methods:

- to define the key indicators (KPI) of the measured process;
- to transform indicators to the uniform range of measurement or an assessment, for example [0:10];
- to define priorities of rather comparative importance of indicators, having presented them in the form of inequalities and limitations for the unknown weights coefficients;
- to allocate from all set of sets of scales with the set accuracy that meet the specified conditions;
- to define a population mean for each of scales and further to use it for an express assessment of a level of quality, for example, by means of control cards of Shukhart;
- to range objects by the stochastic domination among themselves and in relation to "reference" object.

The offered methods of complex estimation and the software product developed at its realization can be, besides the analysis of the employees work quality and the quality of IT-processes, solutions of a problem of determination of the unknown weights coefficients for the set limitations are used for creation the employees motivation system. Its use at the solution of the economic tasks connected with the analysis of alternatives in projects which are described not only economic indicators, for example, ranging within carrying out competitions (tenders) is also possible. The considered methods can be also potentially used as the instrument of decision-making at the conclusion of outsourcing contracts. The received results can be useful first of all for the practicians who are engaged in the quality management of services in information and communication technologies.

REFERENCES

[1] Peter Brooks. 2006. Metrics for IT Service Management/Van Haren Publishing. p. 202.

[2] Tushavin V.A. 2010. Menedzhment kachestva sluzhby podderzhki pol’zovatelei v oblasti informatsionnykh technologiy//Informatsionno-upravlyayushchie sistemy. 4(42): 69-71.

[3] Semenova, E.G., Smirnova M.S. and Tushavin V.A. 2014. Decision making support system in multi-objective issues of quality management in the field of information technology//Research Journal of Applied Sciences. 9(12): 1078-1081. DOI: 10.3923/rjasci.2014.1078.1081.

[4] Rozhkov N. N. Kvalimetricheskie metody I modeli v zadachakh upravleniya kachestvom v sfere obrazovaniya / N. N. Rozhkov. - SPb.: FGBOUVPO of "SPGUTD", 2011. p. 218.

[5] Rozhkov N.N. 2011. Kvalimetrichesky podkhod k zadache kompleksnogo otsenivaniya kachestva gosudarstvennykh uslug//Upravlencheskoe konsul’tiovanie. (3): 26-32.

[6] Rozhkov N.N. 1991. Randomizirovannyi kriteriy sravneniya kachestva slozhnykh ob’’ektov //Ekonomika I matematicheskie metody. 27(3): 597-600.

ARPN Journal of Engineering and Applied Sciences

[7] Hovanov N.V. 1982. Matematicheskie osnovy teorii schkal izmereniya kachestva/N.V. Hovanov. L. Iz-vo LGU. pp. 188.

[8] Hovanov N.V. 1979. Stohasticheskie protsessy i polya s ravnoveroyatnymi diskretnymi monotonnymi realizatsiyami//Upravlenie, nadezhnost' i navigatsiya. Saransk. 5: 136-139.

[9] Hovanov N.V. 1988. ASPID - sistema kvalimetricheskih metodov otsenivaniya v usloviyah defitsita informatsii kachestva slozhnykh tehnicheskikh ob'ektov//Metodologiya i praktika otsenivaniya kachestva produktsii. L.: LDNTP. pp. 56-61.

[10] Core R. and Team R 2015. A language and environment for statistical computing/R Foundation for Statistical Computing, Vienna, Austria. http://www.R-project.org

[11] Daniel Adler. 2005. vioplot: Violin plot. R package version 0.2./2005. - URL:http://wsopuppenkiste.wiso.uni-goettingen.de/~dadler.

[12] Csardi, G and Nepusz T. 2006. The igraph software package for complex network research/ InterJournal, Complex Systems. pp. 1695. URL: http://igraph.sf.net.

[13] Hintze, J.L. and Nelson R.D., 1998. Violin Plots: A Box Plot-Density Trace Synergism / Jerry L. Hintze, Ray D. Nelson // The American Statistician. 52: 181-184.

[14] Rubin, Paul A. 1984. Generating random points in a polytope. Communications in Statistics - Simulation and Computation. 13(3): 375-396.