A COMPARISON OF ENSEMBLE KALMAN FILTER AND EXTENDED KALMAN FILTER AS THE ESTIMATION SYSTEM IN SENSORLESS BLDC MOTOR

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ABSTRACT
In this paper, a new filtering algorithm is proposed for system control of the sensorless BLDC motor based on the Ensemble Kalman filter (EnKF). The proposed EnKF algorithm is used to estimate the speed and rotor position of the BLDC motor only using the measurements of terminal voltages and three-phase currents. The speed estimation performance of developed EnKF was compared with the Extended Kalman Filter (EKF) under the same conditions. Results indicate that the proposed EnKF as an observer shows better performance than that of the EKF.

Keywords: speed estimation, sensorless brushless DC motor, ensemble kalman filter, extended kalman filter.

INTRODUCTION
Brushless Direct Current (BLDC) is a motor without brush and electronically control; and recently is one of the motor types that rapidly gaining popularity in a number of industrial applications. Because of its characteristics, such as high starting torque, high efficiency with no excitation losses, low noise (silent) operation and high durability, BLDC motors are used in various applications such as in various types of compressors, in electrical vehicles, hard disc drives and in medical applications (Liu, 2006). To replace the function of commutators and brushes, the BLDC motor requires an inverter and a position sensors such as Hall-effect, resolver, or absolute encoder that detects the rotor position for a proper commutation of current. The rotation of the BLDC motor is based on the feedback of the rotor position which is obtained from a sensor position (i.e., a hall sensors). BLDC motor usually uses three hall sensors for determining the commutation sequence, hence, the installation of these sensors poses several problems for the motor-drive system. As the consequence, various problems occurred on the operation of the BLDC motor such as reduce reliability and system robustness, difficulty on its installation and maintenance, and increasing the size and the expense of the motor (Niapour, 2014).

To overcome the negative effects of the used sensor systems, the researches are doing researches on the possibility of a sensor-less systems. Iizuka e al., proposed a zero-crossing point (ZCP) of the back-EMF and a speed-dependent period of time delay (Iizuka, 1985); however, this method has an error accumulation problem when the motor is operated at a low speed. Another sensorless method is a Flux Calculation Method (Kim, 2004), but as with the previous proposed method, this method has also performed an error accumulation problem for integration at a low speed. This method also demands a lot of computational cost and is sensitive to a parameter variation, then an expensive floating-point processor would be required to handle this complex algorithm. The third method is developed based on the function of an observer (Kim, 2004). Various types of observers are then used to estimate rotor position, especially the Extended Kalman Filter (EKF) (Lenine, 2007). The biggest advantage of using observers is lied on that all of the states in the system can be estimated, including with the states that are hard to obtain by measurements. However, there are also some limitations on using the EKF as an observer, such as the characteristics of the EKF that can only be performed as first-order accuracy, a high computational complexity due to calculation of the Jacobian matrices and its covariance matrix. However, the most important problem related with the used of EKF as an observer is lied on its weak robustness characteristics against parameter detuning.

Figure-1 shows the block diagram of the system operation of a speed control sensorless BLDC motor using a family of Kalman filters, as an observer, for estimating the rotor speed ($\omega$) and the rotor position ($\theta$). To turn the BLDC motor, a DC power supply is necessary to be fed through a three-phase current-controlled voltage-source.
inverter. And to be rotated precisely in a determined sequence of times, a control signal timed by a precisely rotor position is required, i.e. turning the ON/OFF of the active inverter transistor pair.

As can be seen from this figure, the input signal into the controller is the difference between the reference rotor speed \( \omega_\text{ref} \) and the actual output \( \omega \), which in a sensorless system, is provided also from the observer. Thus, the function of the observer, in this case is the family of Kalman Filters, is to provide the estimated of the actual rotor speed \( \omega \) and the rotor position \( \theta \). The PID controller process this speed error signal \( e \) into a torque command \( T_\text{cmd} \), which in the reference current generator, this torque command \( T_\text{cmd} \) is combined with the position signal \( \theta \) to provide the current signal for each phase of the current controller system. In a sensorless system, these positions signal \( \theta \), which are very important in providing the best performance curve of the BLDC motor, is provided also from the observer. Then, the current signal of each phase of the motor is compared with the the feedback BLDC current, then generates the fault currents of each phase of the motor for further fed to the inverter to rotate the motor.

In this paper, to provide a better estimation of the actual rotor speed \( \omega \) and the rotor position \( \theta \), an Ensemble Kalman Filter (EnKF) as an observer is proposed. The characteristics of the EnKF are then compared with that of the EKF in an experimental procedure, with all of the programs are executed in MATLAB environment. As can be seen in our experimental results, it is proved that our proposed EnKF estimation system outperformed the EKF system, especially on its characteristics performance at various speed references.

This paper is organized as follows. Section II presents description of the Brushless DC motor system, including with its mathematical representations. Section III discusses the development of the Ensemble Kalman Filter based controller in detail, including with its comparison with that of the Extended Kalman Filter. Section IV presents the experimental results and discussion, follows by the conclusions that is presented in Section V.

**MATHEMATICAL MODEL OF THE BLDC**

As stated early, information regarding the rotor speed \( \omega \) and the rotor position \( \theta \) signals are necessary in order to control the BLDC motor. In this paper, a three-phase BLDC motor with star connection is considered (see Figure 2) and used as the reference. The mathematical model of the BLDC motor is derived, in order to provide the mathematical relationship between the rotor speed \( \omega \), the rotor position \( \theta \) and the BLDC motor input current, by which the Kalman Filter could provided the estimated-information for the controller.

The general voltage equation of BLDC motor can be written as follows (Sheel, 2012):

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
= 
\begin{bmatrix}
R_a & 0 & 0 \\
0 & R_b & 0 \\
0 & 0 & R_c
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
+ 
\begin{bmatrix}
L_a & 0 & 0 \\
0 & L_b & 0 \\
0 & 0 & L_c
\end{bmatrix}
\begin{bmatrix}
e_a \\
e_b \\
e_c
\end{bmatrix}
\]

\[(1)
\]

which the induced backs EMFs are trapezoidal and their peak values are equal to \( \lambda_\text{m}\omega \)

The electromagnetic torque can be calculated by

\[
T_e = \frac{f_a i_a + f_b i_b + f_c i_c}{\omega} = \lambda_m(f_a i_a + f_b i_b + f_c i_c)
\]

\[(2)
\]

where \( f_a, f_b, f_c \) have shapes like \( e_a, e_b, e_c \), respectively, and their maximum values are one. The equation of motion for a simple system with an inertia \( J \), a friction coefficient \( B \), and a load torque \( T_L \) can be written as:

\[
J \frac{d\omega}{dt} + B \omega = T_e - T_L
\]

\[(3)
\]

![Figure-2. Circuit of a BLDC motor drive.](image)

The rotor speed \( \omega \) and the rotor position \( \theta \), can be written as:

\[
\frac{d\theta}{dt} = \frac{P}{2} \omega
\]

\[(4)
\]

where \( P \) is number of pole on rotor.

While the state space form of such system can be defined as:

\[
\dot{x} = Ax + Bu
\]

\[(5)
\]

where

\[
x = [i_a \ i_b \ i_c \ \omega \ \theta]^T
\]

\[(6)
\]

\[
u = [v_a \ v_b \ v_c \ T_m]^T
\]

\[(7)
\]
The speed controller of the BLDC motor

The most important parameter for controlling the BLDC motor is the speed control, and the most effective controller for the BLDC motor is the PID (Proportional, Integral, and Derivative) controller. The equation of a PID controller can be written as

\[ \tau_{ref} = K_p e + \frac{K_i}{T_i} \int e \, dt + K_d \frac{de}{dt} \]  \hspace{1cm} (10)

with \( e \) the error input signal, \( \tau_{ref} \) the manipulated output signal, \( K_p \) the proportional gain, \( T_i \) the integral time sequence, and \( T_d \) the derivative time sequence. These parameters, \( K_p, T_i, \) and \( T_d \) are carefully chosen to meet the best prescribed performance criteria.

In order to use the PID controller, the parameters related with its operation must be firstly tuned. This tuning process is utilized to synchronize the controller with the controlled variable, thus allowing the process of the plant to be optimized according to the desired operating condition. Standard methods for tuning the controllers and the criteria for judging the loop tuning process have been investigated for many years. Some of them are Mathematical criteria, Cohen-coon Method, Trial and error method, Continuous cycling method, Relay feedback method, Kappa-Tau tuning method, and Chien-Hrones-Reswick (CHR) PID tuning method. As the CHR method performed better compare with that of Cohen-coon or Ziegler-Nichols methods (Gireesh, 2014), (Xue, 2007), especially for a tracking control and for a disturbance rejection problems, in this paper, we have used CHR technique to find the optimal values of \( K_p, T_i \) and \( T_d \) of the PID controller for the BLDC motor speed control system.

\[ A = \begin{bmatrix} -\frac{L_s}{R_s} & 0 & 0 & \frac{\lambda_m}{L_s} F(\theta_e) \\ 0 & -\frac{L_s}{R_s} & 0 & \frac{\lambda_m}{L_s} F \left( \theta_e + \frac{4\pi}{3} \right) \\ 0 & 0 & -\frac{R_s}{L_s} & \frac{\lambda_m}{L_s} F \left( \theta_e + \frac{2\pi}{3} \right) \\ 0 & 0 & 0 & -\frac{R_s}{L_s} \end{bmatrix} \] \hspace{1cm} (8)

\[ B = \begin{bmatrix} \frac{1}{L_s} & 0 & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 & 0 \\ 0 & 0 & \frac{1}{L_s} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \] \hspace{1cm} (9)

**Figure-3**. CHR step response tuning method.

Figure-3 shows a simple approach on calculating the time constant \( T \), the delay time \( L \), the controller gain \( k \) and determining the constant value of \( \alpha = kL/T \) of the CHR method, derived from the step time response of the system under investigation. Table-1 shows the CHR method formula for determining those parameters constant for the P, PI, and PID controllers, more specifically using a 0% overshoot and 20% overshoot phenomena. Please note that the quickest periodic response of the system under step response input is labeled with 0% overshoot, while the quickest oscillatory process is labeled with 20% overshoot. Using those parameters value such as in the Table-1, the proportional controller gain, the integral time, and the derivative time of the P, PI, and the PID controllers using the CHR tuning rules for the determined reference speed of 1500 rpm are calculated and depicted in Table-2.

Figure-4 shows speed response parameters of the closed loop system with the P, PI, and the PID controllers for the determined reference speed of 1500 rpm with no load condition and Table-3 shows the time domain analysis of the speed control of the BLDC motor performance calculated from these experiments. Results show that the PID controller performed the best overshoot performance, especially when using the 0% overshoot constants.

**ENSEMBLE KALMAN FILTER AS AN OBSERVER**

The fundamental idea of an estimation system is to use a mathematical model derived from the observed plant or a system to calculate the estimated output parameters value from a measured input parameters. As long as there is a difference between the estimated outputs value and the measured inputs, this error is fed back to the
estimation system for reducing this difference by correcting the estimated output values. The best estimator from the family of the Kalman Filters so far is the Extended Kalman filter (EKF), which has been usually used to estimate the instantaneous system state variables and stator resistance of the BLDC motor by using the actual voltages and currents derived through the mathematical model of the BLDC motor. To improve the performance characteristics of the EKF estimator, we proposed in this paper the Ensemble Kalman Filter (EnKF) as an estimation system that will be described later.

Extended Kalman Filter

The EKF estimation system is estimator developed based on a Taylor expansion series which can handle almost every nonlinear system. Generally, estimation system by EKF can be divided into two stages: the prediction step and the correction step. In the first stage, the predicted value of the state variables and the predicted state covariance matrix, which is expressed by \( P_{k|k-1} \), can be obtained, and in the second stage, the correction step, a correction term is added to the predicted \( \hat{x}_{k|k-1} \).

The estimation procedures for the EKF can be listed as follow:

\[
\begin{align*}
\text{Selection of initial values for } P, Q, R 	ext{ and } X(0). \\
\text{State vector prediction} \\
\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_{k-1}) \quad (11) \\
\text{with } \hat{x}_{k|k-1} \text{ is calculated through (5)} \\
\text{Prediction of error covariance matrix} \\
P_{k|k-1} = F_{k-1}P_{k-1|k-1}F_{k-1}^T + Q_{k-1} \quad (12) \\
\text{Calculation of correction factor of EKF} \\
S_{k} = H_{k}P_{k}H_{k}^T + R_{k} \quad (13) \\
K_{k} = S_{k}H_{k}^T S_{k}^{-1} \quad (14)
\end{align*}
\]

Table-1. CHR tuning formula for set point regulation.

<table>
<thead>
<tr>
<th>Controller Type</th>
<th>With 0% overshoot</th>
<th>With 20% overshoot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( K_p )</td>
<td>( T_i )</td>
</tr>
<tr>
<td>P</td>
<td>0.3/( \alpha )</td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>0.35/( \alpha )</td>
<td>1.2( T )</td>
</tr>
<tr>
<td>PID</td>
<td>0.6/( \alpha )</td>
<td>( T )</td>
</tr>
</tbody>
</table>

Table-2. Values of PID parameter.

<table>
<thead>
<tr>
<th>Controller Type</th>
<th>With 0% overshoot</th>
<th>With 20% overshoot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( K_p )</td>
<td>( T_i )</td>
</tr>
<tr>
<td>P</td>
<td>4.20478605</td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>4.90558373</td>
<td>0.034924</td>
</tr>
<tr>
<td>PID</td>
<td>8.40957211</td>
<td>0.029104</td>
</tr>
</tbody>
</table>

Table-3. Time domain analysis.

<table>
<thead>
<tr>
<th>Controller Type</th>
<th>With 0% overshoot</th>
<th>With 20% overshoot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time Setting</td>
<td>Over- shoot</td>
</tr>
<tr>
<td>P</td>
<td>0.0130</td>
<td>2.9826</td>
</tr>
<tr>
<td>PI</td>
<td>0.0130</td>
<td>2.9984</td>
</tr>
<tr>
<td>PID</td>
<td>0.0130</td>
<td>1.3779</td>
</tr>
</tbody>
</table>
Figure 4. Step response of the closed loop system with various parameter of PID controller.

Estimation of output vector and state vector

\[
\hat{y}_k = y_k - h(\hat{x}_{k|k-1}) \tag{15}
\]

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \hat{y}_k \tag{16}
\]

Estimation of error covariance matrix

\[
P_{k|k} = (1 - K_k H_k) P_{k|k-1} \tag{17}
\]

The most important of the EKF as an estimation system is to determine the best initial values for the three covariance matrices, namely, Q, R, and P, since these initial values are highly contributing on the filter stability, i.e., the estimation values of the rotor speed and the rotor speed, and the convergence time. The difficulty of the EKF system to estimate the rotor speed, especially at a lower speed, makes the error of the estimation of the rotor position becomes too high (Ejlali, 2012), so that the controller of the sensorless drive is no longer working properly.

Ensemble Kalman Filter

The EnKF estimation system is a suboptimal estimator, where the error statistics are predicted by using a Monte Carlo or ensemble integration method to solve the Fokker-Planck equation. Unlike the EKF system, the evaluation of the filter gain \( K_k \) in the EnKF system does not involve the approximation of the nonlinearity functions \( f(x, u) \) and \( h(x) \). Hence, the computational burden of evaluating the Jacobians of \( f(x, u) \) and \( h(x) \) does no longer exist in the EnKF system. The starting point for the EnKF estimation system as a particle filters is done by choosing a set of sample points, which is an ensemble of state estimations that captures the initial probability distribution of the state. These state estimation points are then propagated through the true nonlinear system, so that the probability density function of the actual state can be approximated by the ensemble of the estimation system (EnKF).

The EnKF estimation method consists of three stages. The first step is called the forecast step: where to represent the error statistics of the system, assume that at time \( k \), there are an ensemble of \( q \) forecasted state estimations with random sample errors. We denote these ensemble as \( X^f_k \in \mathbb{R}^{n \times q} \), where

\[
X^f_k \triangleq \begin{pmatrix} x^f_{k,1}, x^f_{k,2}, \ldots, x^f_{k,q} \end{pmatrix}, \tag{18}
\]

with the superscript \( f \) refers to the \( i \)-th forecast ensemble member. with \( x^f_{k,i} \) is calculated through (5). Then, the ensemble mean \( \bar{x}^f_k \in \mathbb{R}^n \) is defined by

\[
\bar{x}^f_k \triangleq \frac{1}{q} \sum_{i=1}^{q} x^f_{k,i} \tag{19}
\]

Since the true state \( x_k \) is not known, we approximate (18) by using the ensemble members. We define the ensemble error matrix \( E^f_k \in \mathbb{R}^{n \times q} \) around the ensemble mean by

\[
E^f_k \triangleq \begin{bmatrix} x^f_{k,1} - \bar{x}^f_k, \ldots, x^f_{k,q} - \bar{x}^f_k \end{bmatrix} \tag{20}
\]

and the ensemble of the output error \( E^a_{y_k} \in \mathbb{R}^{p \times q} \) by

\[
E^a_{y_k} \triangleq \begin{bmatrix} y^f_{k,1} - \bar{y}_k, \ldots, y^f_{k,q} - \bar{y}_k \end{bmatrix} \tag{21}
\]
We then approximate the $P^f_k$ by $\tilde{P}^f_k$, the $P^a_{yy_k}$ by $\tilde{P}^a_{yy_k}$, and the $P^f_{yy_k}$ by $\tilde{P}^f_{yy_k}$, respectively, where

$$
\tilde{P}^a_k = \frac{1}{q-1} E^a_k (E^a_k)^T \quad \tilde{P}^f_k = \frac{1}{q-1} E^f_k (E^f_k)^T
$$

Thus, we interpret the forecast ensemble mean as the best forecast estimations of the state, and the spread of the ensemble members around the mean as the error between the best estimations and the actual state.

The second step is the analysis step: To obtain the analysis estimations of the state, the EnKF performs an ensemble of parallel data assimilation cycles, where for $i = 1, \ldots, q$

$$
x^2_k = x^a_k + \tilde{R}_k \left( \hat{y}_k - h \left( x^a_k \right) \right)
$$

The perturbed observations $\hat{y}_k^i$ are given by

$$
\hat{y}_k^i = y_k + v_k^i
$$

where $v_k^i$ is a zero-mean random variable with a normal distribution and covariance $R_k$. The sample error of the covariance matrix computed from the $v_k^i$ converges to $R_k$ as $q \to \infty$. We approximate the analysis of the error of the covariance matrices $P^a_k$ by $\tilde{P}^a_k$, where

$$
\tilde{P}^a_k = \frac{1}{q-1} E^a_k E^a_k^T
$$

And $E^a_k$ is defined by with $x^a_k$ replaced by $x_k^2$ and $\tilde{x}_k^i$ replaced by the mean of the analysis estimate ensemble members. We use the classical Kalman filter gain expression and the approximations of the error covariances to determine the filter gain by $\tilde{R}_k$ by

$$
\tilde{R}_k = \tilde{P}^a_{xy_k} (\tilde{P}^f_{yy_k})^{-1}
$$

The last step is the prediction of error statistics in the forecast step:

$$
x_{k+1}^f = f(x_k, u_k) + w^i_k
$$

where the values are $w^i_k$ sampled from a normal distribution with average zero and covariance $Q_k$. The sample error covariance matrix computed from the $w^i_k$ converges to $Q_k$ as $q \to \infty$. Finally, we summarize the analysis and forecast steps.

Analysis Step:

$$
\tilde{R}_k = \tilde{P}^f_{xy_k} (\tilde{P}^f_{yy_k})^{-1}
$$

RESULTS AND DISCUSSIONS

A sensorless speed estimation and control system has been simulated using MATLAB. Simulation parameters of the BLDC motor are given as follows: the stator resistance $R=0.7\Omega$, the equivalent inductance of the stator $L_s=5.21\times10^-3$ $H$, the maximum of each phase winding permanent magnet flux $\lambda_m=0.05238Wb$, the inertial $J=0.022\times10^-3$ $kgm^2$, the viscous friction coefficient $B=0$ $Nms$, the poles of the permanent magnet $p=4$ and simulation step length $T=5\times10^-3$s, $x_0=[0 0 -1 1 0]^T$.

For the EKF system, the $\hat{x}_{k|k-1}$ is calculated from (5) and the Jacobian matrix $F = \frac{\partial}{\partial x}A$ is calculated from (8), while for the EnKF the same equation is used to calculate $\tilde{x}_{k|k}$, with the upper index $f$, the number of the ensemble member, and in this paper is determined to be 8.

In order to reach an insight distinct from the whole system performance of the proposed sensorless algorithm, the motor operation needs to be evaluated under different conditions. The effectiveness of the proposed algorithm and its comparison will be analyzed based on the performance of the estimation systems from a low speed operation added with the various speed increment operations.

In the first experiment, low speed operation performance is conducted. The reference speed is determined to be 200 rpm, and the experimental results (i.e. the performance curves) are presented in Figure 5.As can be seen in from this figure, the EnKF estimation system performed a better predictive ability compare with that of the EKF estimation system. It is also clearly seen that both the rotor position estimation error and the rotor speed estimation error of the EKF estimation is increased as increasing the experimental time, while those error values for EnKF estimation system are close to constantly zero. In such conditions, the estimations error for both of the rotor position and the rotor speed of the EKF
estimation system are growing to be very high, that makes
the controller and the sensorless drive is no longer possible
to effectively control the motor.

These phenomena are in contrast with the
response of the EnKF estimation system, which show
nearly zero estimations error, for both the rotor position
and the rotor speed.

The second experiment is performed to evaluate
the response of the EKF and the EnKF estimation systems
on various speed changes operation. In this experiment, the
reference speed changes from 2000 to 3600 rpm within
t=0.25s; then the reference speed is decreased from 3600
to 1600 rpm within t=0.5s, and from 1600 to 2800 rpm
within t=0.75s, and the experimental results (i.e. the
performance curves) are depicted in Figure 6. As it was
expected from the theory, the performance curves of the
EnKF estimation system works properly with very high
reliability, with the estimated value of the actual speed
could be the same as the speed changes. However, as can
also be clearly seen from this figure, the performance
curves of the EKF estimation system are showing a
fluctuated estimation rotor speed values, providing a high
degree of estimation error rate for both rotor speed and the
rotor position.

Figure-5. Performance curves of low speed.
CONCLUSIONS

This paper focused on the estimation of BLDC motor speed for different speed references with EKF and EnKF. In order to evaluate the estimate performance, simulation experiments are presented in the paper. It is obvious to see that, from the simulation results, the accurate estimation performance can be obtained and the effectiveness of EnKF algorithm can be demonstrated. Moreover, the sensorless BLDC motor can be controlled precisely according to the designed EnKF algorithm.

REFERENCES


