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WHEEL SLIP CONTROL BASED ON COMPOSITE NONLINEAR FEEDBACK

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ABSTRACT

To produce faster vehicle acceleration and avoid wheelspin on slippery roads, the wheel slip must be controlled to achieve maximum traction. Recent researches in slip control always had to compromise between speed of time response and overshooting. This research studies the application of Composite Nonlinear Feedback (CNF) controller for vehicle wheel slip control, particularly for in-wheel electric vehicle. A strategy for applying the CNF controller which involves feedback linearization is proposed. The CNF is a combination of a linear feedback law and a nonlinear feedback law without any switching element. The CNF control focuses on improving the transient performance. The proposed control strategy is validated by simulation.

Keywords: wheel slip ratio, composite nonlinear feedback, feedback linearization, partially nonlinear system.

INTRODUCTION

Different strategies have been developed to control the traction forces in electric vehicles (EVs) [1, 2]. Some of these proposals chose to control the wheel slip ratio [3-5], since the wheel slip ratio has a direct relationship with the friction force. In these approaches the control problem is to ensure the wheel slip stabilizes at a value that gives the optimum longitudinal friction force. Composite Nonlinear Feedback (CNF) is an attractive control technique, which has remarkable ability to improve the transient performance [6]. Lin, Pachter and Banda [7] first introduced the algorithm for the design of the composite nonlinear feedback law to improve the tracking performance. Chen, Lee and Venkataramanan. [8, 9] further developed the CNF control technique for a more general class of systems with measurement feedback. Lan [10] extended it to partially linear composite systems. Peng, Chen, Cheng and Lee [11] enhanced CNF with integrator to enable removal of steady-state bias and Cheng, Peng, Chen and Lee [12] continued the development of CNF for tracking general references. CNF has been successfully applied to DC motor speed control [13], robot manipulator tracking control [14], HDD servo system [15], and overhead crane servo system [16] among many others. To the best of our knowledge CNF control technique has not yet been implemented to wheel slip control.

Single wheel vehicle model

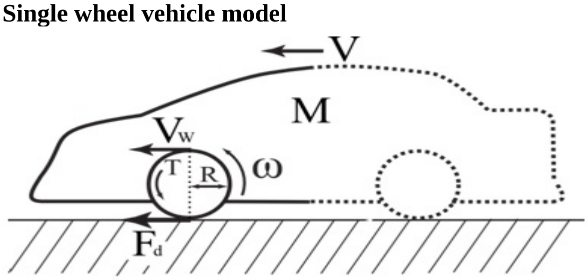

Figure-1. Single wheel vehicle model.

The forces acting on a single wheel vehicle such as shown in Figure-1 can be described in the following equations [3]:

$$J\dot{\omega} = T - RF_d \tag{1}$$

$$M\dot{V} = F_d \tag{2}$$

where J is the wheel moment, $\dot{\omega}$ is wheel angular velocity, T is motor torque, R is wheel radius, M is ½ of the vehicle mass, V is the vehicle velocity, and F_d is traction force defined by:

$$F_d = \mu(\lambda)F_n \tag{3}$$

where $\mu(\lambda)$ is the tire-road friction coefficient, and F_n is the normal force acting on a single wheel. μ is described by Pacejka's Magic Formula [3] in Equation (4), and the wheel slip ratio λ during acceleration is defined as in Equation (5). Slip occurs whenever a torque is applied to the wheel as it is essential for generating friction at the tire-road interface [4].

$$\mu = D\,sin\left[C\,tan^{-1}\left(B\lambda - E\left(B\lambda - tan^{-1} B\lambda\right)\right)\right] \tag{4}$$

$$\lambda = \frac{V_W - V}{V_W} \quad V_W = R\omega \tag{5}$$

Using the ideas from [19], the wheel slip system can be linearized via feedback linearization to obtain an equivalent system of a simpler form. Substituting F_d from Equation (3) and rearranging Equation (1) and Equation (2), we get:

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$$\dot{V} = \frac{F_n}{M}\mu(V, y) \tag{6}$$

$$\dot{x} = \frac{-RF_n}{J}\mu(V, y) + \frac{1}{J}u \tag{7}$$

$$y = x \tag{8}$$

Here ω is set as the state, u is the input motor torque, and the controlled output is set as $y = x$. To cancel the nonlinear term $\frac{-RF_n}{J}$ in the second equation, we chose the input as $u = J\left(\frac{RF_n\mu}{J} - x + v\right)$, where v is the new control input to be defined by the CNF design procedure. This result in the following partially linear system:

$$\dot{V} = \frac{F_n}{M}\mu(V, y) \tag{9}$$

$$\dot{x} = -x + v \tag{10}$$

$$y = x \tag{11}$$

Composite nonlinear feedback

$$u = u_L + u_N = F_x + G_r + \rho(r, y)B^T P(x - x_e).$$

The CNF control law is designed following the procedures given by [5].

Composite nonlinear feedback for partially linear composite system

Consider a partially linear composite system characterized by

$$\dot{\xi} = f(\xi, y), (\xi, 0) \tag{12}$$

$$\dot{x} = Ax + Bu.x(0) = x_0 \tag{13}$$

$$y = Cx \tag{14}$$

where $(\xi, x) \in R^m \times R^n$ the state, $u \in R$ the control input, and $y \in R$ the output of the system, f is a smooth (i.e., C^∞) function, A, B and C are appropriate dimensional constant matrices. The following assumptions are made

A1: (A, B) is controllable;
A2: (A, B, C) is invertible and has no invariant zero at $s = 0$; and
A3: There exists a C^1 positive definite function $V_\xi(\xi)$ and class K_∞ functions α_1 and α_2 such that

$$\alpha_1\|\xi\| \le V_\xi(\xi) \le \alpha_2(\|\xi\|), \tag{15}$$

$$\frac{\partial V_\xi(\xi)}{\partial \xi} f(\xi, r) < 0 . \tag{16}$$

for all $\xi \in R^m$.

The CNF control law is constructed by the following procedure [5].

Step-1. Design a linear feedback law

$$u_L = Fx + Gr \tag{17}$$

where r is a step command input and F is chosen such that $A+BF$ is Hurwitz and the output of the following system,

$$\dot{x} = (A + BF)x, y = Cx \tag{18}$$

has $\|y(t)\| \le ke^{-at}$ for some $k > 0$; and the closed loop system $C = (sI - A - BF)^{-1}$ has certain desired properties, e.g., having a small damping ratio. G is scalar given by

$$G = -\left[C(A + BF)^{-1} B\right]^{-1}. \tag{19}$$

Step-2. Given a positive definite matrix $W \in R^{n \times n}$, solve the Lyapunov equation

$$(A + BF)^T P + P(A + BF) = -W \tag{20}$$

for P > 0. Then the nonlinear feedback control law $u_N(t)$ is given by

$$u_N = \rho(r, y)B^T P(x - x_e). \tag{21}$$

where $\rho(r, y)$ is any non-positive function locally Lipschitz in y and

$$x_e = G_e r = -(A + BF)^{-1} BGr . \tag{22}$$

The choice of $\rho(r, y)$ is not unique. Usually $\rho(r, y)$ is chosen as a function of the tracking error r-y, such that $\rho(r, y)$ has the following properties, 1) when the system is far away from the desired set point, $\rho(r, y)$ is small in magnitude and thus the effect of the nonlinear part is very limited compared to the linear part of the CNF; and 2) when the system draws near to the set point, $\rho(r, y)$ becomes larger and larger in magnitude, and the nonlinear part of the control law will become effective [7]. The following choice is one of the suiTable-candidates [8],

$$\rho(r, y) = -\beta e^{-\alpha|y - r|} . \tag{23}$$

where $\beta > 0$ and $\alpha > 0$ are tuning parameters.

Step-3. The CNF control law is given by combining the linear and nonlinear feedback law derived in the previous steps,

$$u = u_L + u_N = Fx + Gr + \rho(r, y)B_T P(x - x_e) \tag{24}$$

Wheel slip control based on cnf for partially linear composite system

Following this procedure for the wheel slip control problem, we obtain a state feedback gain matrix

$$F = -100098.5 \tag{25}$$

Next, we choose $W = 1.0100$. Solving the Lyapunov equation of (20), we obtain

$$P = 5.0002 \times 10^{-5} \tag{26}$$

which is indeed positive-definite. The nonlinear gain function is selected as follows:

$$\rho(e) = -5 \times 10^{8} e^{-10^{3}|y-r|} \tag{27}$$

Finally, the CNF control law for the wheel slip control system is given by

$$\dot{V} = \frac{F_N}{M}\mu(V, y), \tag{28}$$

$$\dot{x} = -x + v \tag{29}$$

and

$$v = -1000985x + 1000995r - 5\times 10^{8} e^{-10^{3}|y-r|} \times 5.0002\times 10^{-5}(x - x_e) \tag{30}$$

The control structure is shown in Figure-2 where λ^* is desired slip ratio and ω^* is reference wheel speed. The reference signal for the CNF controller is calculated by a wheel speed reference generator as in Eq. 31. Vehicle speed is assumed to be directly available thus is not estimated.

$$\omega^* = \frac{V}{R(1-\lambda^*)} \tag{31}$$

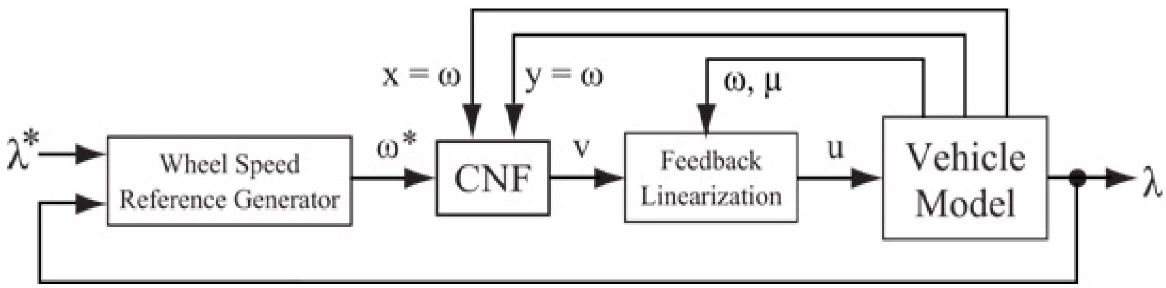

Figure-2. Control structure.

SIMULATION AND RESULTS

The parameters for the simulation are set as $M = 900[kg]$, $J = 1.0[Nms^{-2}]$, and $R = 0.31725$. The reference slip ratio is set at $\lambda^* = 0.168$ which corresponds to a point in the neighbourhood of the peak of the Pacejka's curve with the coefficients B, C, D, and E set to 10, 1.9, 1, and 0.97 respectively.

Table-1 shows that the nonlinear control law in CNF contributed to faster rise and settling time compared with control using linear feedback law only. Figure-3 shows the simulation results using only the linear control law part of CNF and using full CNF control.

Table-1. Comparison between linear and CNF control.

Control	Rise Time [s]	Settling Time [s]
Linear	0.0053	0.0269
CNF	0.0027	0.0133

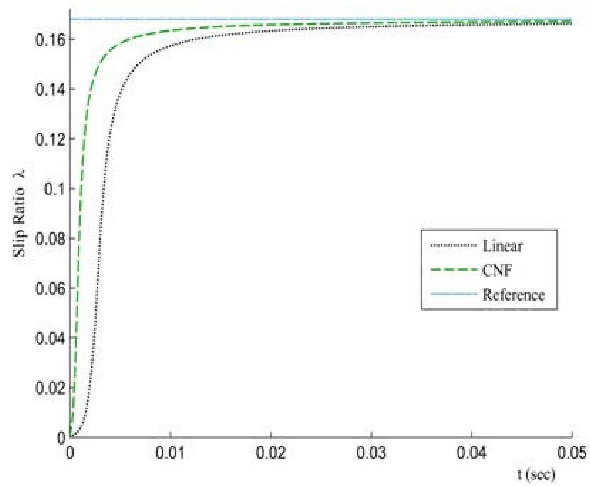

Figure-3. Linear control and CNF control.

Augmenting composite nonlinear feedback with integrator

Using the procedures described in [9,10], we follow the usual practice to augment an integrator into the given system. We define an auxiliary state variable

$$\dot{x}_i = K_i(y - r) = K_i Cx - K_i r \tag{32}$$

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which is implement table-as y is assumed to be measurable, and K_i is a positive scalar to be selected to yield an appropriate integration action. The augmented system is then given as follows:

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{B}_r r \tag{33}$$

$$\bar{y} = \bar{C}\bar{x} \tag{34}$$

Where

$$\bar{x} = \begin{pmatrix} x_i \\ x \end{pmatrix}, \ \bar{x}_0 = \begin{pmatrix} 0 \\ x_0 \end{pmatrix} \tag{35}$$

$$\bar{A} = \begin{pmatrix} 0 & K_i C \\ 0 & A \end{pmatrix},$$

$$\bar{B} = \begin{pmatrix} 0 \\ B \end{pmatrix}, \ \bar{B}_r = \begin{pmatrix} -K_i \\ 0 \end{pmatrix}, \ \bar{C} = \begin{pmatrix} 0 & C \end{pmatrix} \tag{36}$$

The enhanced CNF control law is designed as follow [9]:
Design a linear feedback control law

$$u_L = F\bar{x} + Gr \tag{37}$$

where F is chosen such that 1) $\bar{A} + \bar{B}F$ is an asymptotically sTable-matrix, and 2) the closed-loop system $C(A+BF)^{-1}B$ has certain desired properties. Let us partition $F = [F_i \quad F_x]$ in conformity with x_i and x. G is chosen as

$$G = -\left[C(A + BF_x)^{-1B}\right]^{-1} \tag{38}$$

Given a positive definite matrix $W \in R^{(n+1)\times(n+1)}$, solve the following Lyapunov equation:

$$(\bar{A} + \bar{B}F)^T P + P(\bar{A} + \bar{B}F) = -W \tag{39}$$

For $P>0$. The nonlinear feedback portion of the enhanced CNF control law u_N is given by

$$u_N = \rho(e)\bar{B}^T P(\bar{x} - \bar{x}_e) \tag{40}$$

where $\rho(e)$, with $e = y - r$ being the tracking error, is a smooth and non-positive function of $\|e\|$, and tends to a constant as $t \to \infty$. Next $\bar{x}_e$ is defined as

$$\bar{x}_e = G_e r \ \text{and} \ G_e = \begin{bmatrix} 0 \\ -(A + BF_x)^{-1} BG \end{bmatrix} \tag{41}$$

The linear feedback control law and nonlinear feedback parts derived in the previous steps are now combined to form an enhanced CNF control law.

The simulation results of normal CNF control compared with enhanced CNF is shown in Figure-4 and Table-2. Table-3 and Figure-5 show the overall comparison. Figure-5 shows that only linear and CNF control augmented with integrator tracked the reference $\lambda^* = 0.168$ at the end of the simulations. On the other hand, linear and CNF control not augmented with integrator settled at 0.166 and 0.1671 respectively. This suggests that CNF control is not very efficient in removing steady state bias.

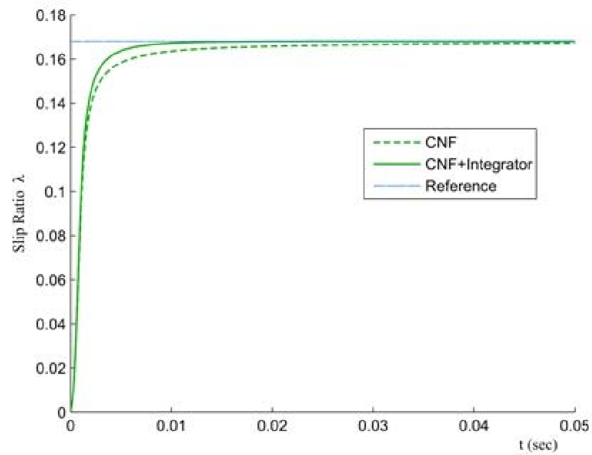

Figure-4. Normal and enhanced CNF control.

$$u = u_L + u_N = \bar{F}\bar{x}\rho(e)\bar{B}^T P(\bar{x} - \bar{x}_e) \tag{42}$$

Wheel slip control case based on enhanced CNF

Following the above procedure for the wheel slip control problem, we obtain a state feedback gain matrix

$$F = [F_i \quad F_x] = [-9999.9 \quad -10098.5] \tag{43}$$

Next, we choose $W=I_2$. Solving the Lyapunov equation of (39), we obtain

$$p = \begin{bmatrix} 0.0100 & 4.9999\times10^{-5} \\ 4.9999\times10^{-5} & 5.0002\times10^{-5} \end{bmatrix} \tag{44}$$

which is indeed positive-definite. K_i is chosen to be 100, and the nonlinear gain function is selected as follows:

$$\rho(e) = -5\times10^{-8} e^{-10^3|y-r|} \tag{45}$$

Finally, the enhanced CNF control law for the wheel slip control system is given by

$$\begin{pmatrix} \dot{x}_i \\ \dot{x} \end{pmatrix} = \begin{bmatrix} 0 & 100 \\ 0 & -1 \end{bmatrix}\begin{pmatrix} x_i \\ x \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v + \begin{bmatrix} -100 \\ 0 \end{bmatrix} r \tag{46}$$

and

$$v = -[9999.9 \quad 10098.5]\begin{pmatrix} \dot{x}_i \\ \dot{x} \end{pmatrix} + 10099.5r - \frac{5\times10^3}{e^{10^3|y-r|}}[4.9999 \quad 5.0002]\begin{bmatrix} x_i \\ x - x_e \end{bmatrix} \tag{47}$$

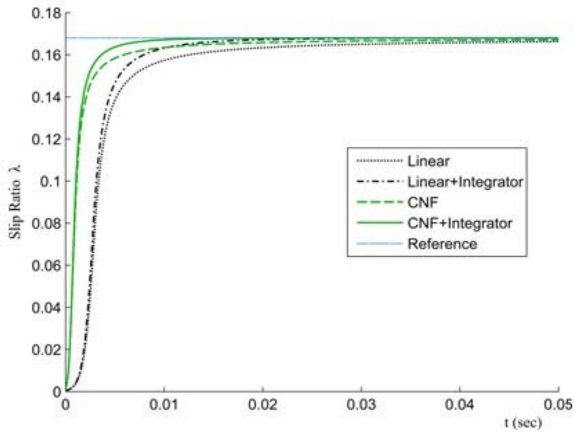

Figure-5. Overall comparison.

Table-2. Comparison between normal control and enhanced CNF with integrator.

Control	Rise Time [s]	Settling Time [s]
CNF	0.0027	0.0133
CNF with Integrator	0.0020	0.0057

Table-3. Overall comparison.

Control	Rise Time [s]	Settling Time [s]
Linear	0.0053	0.0269
Linear with Integrator	0.0039	0.0117
CNF	0.0027	0.0133
CNF with Integrator	0.0020	0.0057

CONCLUSIONS

In this paper, CNF has been successfully applied to wheel slip control. The nonlinear feedback law of the CNF improved the transient performance of the wheel slip control especially in rise time and settling time.

However CNF control alone has not been efficient in removing steady state error. Augmenting the CNF with an integrator improved its capability of removing steady state bias.

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