



PARAMETRIC SENSITIVITY ANALYSIS OF A HEAVY DUTY PASSENGER VEHICLE SUSPENSION SYSTEM

T. Ram Mohan Rao and G. Venkata Rao

Department of Mechanical Engineering, Vasavi College of Engineering, Hyderabad, India

E-Mail: trmrao@yahoo.com

ABSTRACT

Suspension system design plays an important role in improving passenger comfort and road holding capabilities of an automobile. There is a compromise between the road holding and ride comfort. While Ride comfort is directly related to the acceleration sensed by passengers when traveling on a rough road, road holding ability is associated with the contact forces of the tires and road surface. Suspension travel or working space refers to the relative displacement between the sprung- mass and the un- sprung masses of the vehicle. The present mathematical work aims to determine the discomfort, road holding and working space in a passenger bus by solving the relevant probabilistic equations using MATLAB through a quarter car model. The variability's in the parameters of spring stiffness and damping are used to evaluate the standard derivations of the vertical vehicle body accelerations, tire radial force and relative displacement between wheel and vertical body. The rational selection of damping and suspension stroke and an estimation of speed limits can be had from these studies.

Keywords: vehicle, suspension system, discomfort, road holding, working space, quarter car model.

1. INTRODUCTION

The main task of the passenger vehicle suspension designer is reducing both vehicle body acceleration and the dynamic tire loads. This is to insure good ride comfort for the passenger and reduce the damage to the vehicle structure. Also, the improvement of ride quality in a vehicle can reduce the passenger fatigue, thereby resulting in increased safety and comfort and vehicle control for a driver.

It has been observed that movement of a vehicle on random road surfaces is one of the main reasons of generating vibrations in its components [1, 2]. Several optimization techniques were used earlier to optimize parameters of suspension system in which different objective functions were used. Optimal suspension parameters, particularly damping co-efficient were generated and by using the optimized suspension parameters the vehicle ride quality was improved.

The suspension system reduces the transmission of oscillations to the vehicle body from road surface disturbances. The chassis should be well isolated from the road surface with the minimal suspension travel, yet provide good handling performance [3].

Generally a vehicle suspension system may be categorized as either passive, semi active and fully active systems. Passive suspension system includes the conventional leaf springs and shock absorbers used in most passenger heavy vehicles. Passive system does not have any control elements incorporated in them and therefore are inexpensive.

The springs are assumed to have almost linear characteristics while most of the shock absorbers exhibit nonlinear relationship between force and velocity. In a passive system, these elements have fixed characteristics and hence have no mechanism for feed back control.

In a vehicle suspension system there are a variety of performance parameters, which need to be optimized.

There are three important parameters given below which should be carefully considered in designing a vehicle suspension system [4].

Ride comfort is directly related to the acceleration sensed by passengers when traveling on a rough road. Road holding ability is associated with the contact forces of the tires and the road surface. These contact forces are assumed to depend linearly on the tire deflections. Working space refers to the relative displacement between the sprung and un sprung masses. It should be lesser than the rattle space.

In the present paper, a mathematical representation for ride comfort, road holding and working space is made by considering the standard deviations of the three suspension parameters. The three suspension parameters are vertical vehicle body acceleration (\ddot{y}_s), tire radial force (F_1) and displacement between wheel and vehicle body ($y_s - y_u$).

One of the early papers which describe the probabilistic analysis of suspension systems is by Kong – Huiguo [5], where the methodology is described in detail. The method has been over the years, refined and the recent book by Mastinu Gobbi [6] describes the 1S-PSD and 2S-PSD methodologies used in this paper.

1.1 Modeling of the passenger bus as a passive system

The two degrees of freedom quarter model shown in Figure-1 is the most commonly used model in the design studies for passive suspension system. It consists of a spring and a damper connecting the body (sprung mass) to a single wheel (UN sprung mass), which in turn is connected to the ground via the tire spring.

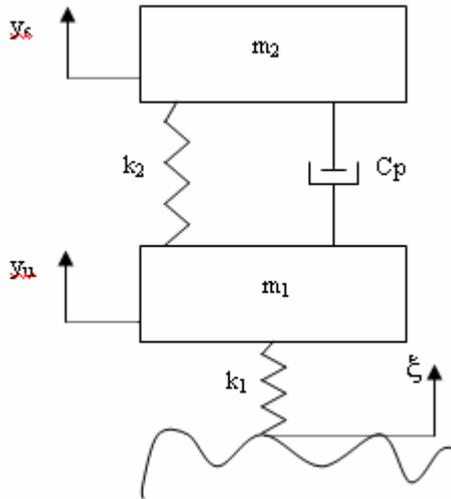


Figure-1. Quarter car model.

According to Newton’s second law, the sum of forces acting on a mass equals to the mass times its acceleration. In this case, the force acting on sprung mass (m_2) is the spring force due to (K_2) and damper force due to (C_p) exerted on the sprung mass. The force acting on the UN sprung mass (m_1) is the spring (k_1, K_2) and damper force (C_p). The linear equations of motions pertaining to the system model are

$$m_2 \ddot{y}_s - C_p (\dot{y}_s - \dot{y}_u) - k_2 (y_s - y_u) + k_1 (y_u - \xi) = 0 \quad (1)$$

$$m_2 \ddot{y}_s + C_p (\dot{y}_s - \dot{y}_u) + k_2 (y_s - y_u) = 0 \quad (2)$$

Where

- m_1 - mass of the wheel plus part of the mass of the suspension arms i.e. un sprung mass
- m_2 - $1/4$ of the body mass i.e., sprung mass
- k_1 - tire radial stiffness
- ξ - Road irregularity (sinusoidal road surface profile)
- C_p - suspension damping
- K_2 - suspension stiffness

$$D(j\omega) = k_1 k_2 + j k_1 C_p \omega - (k_2 m_1 + k_1 m_2 + k_2 m_2) \omega^2 + j C_p (m_1 + m_2) \omega^3 + m_1 m_2 \omega^4 \quad (4)$$

The transfer function between the imposed displacement ξ and y_s reads

$$y_s(j\omega) = \frac{k_1(k_2 + j C_p \omega)}{D(j\omega)} \quad (5)$$

The transfer functions between ξ and \ddot{y}_s is

$$H_1(j\omega) = -\omega^2 Y_s(j\omega) \quad (6)$$

The transfer function between ξ and F_1 is

$$H_2(j\omega) = k_1(1 - y_s(j\omega)) \quad (7)$$

The transfer function between ξ and $(y_s - y_u)$ is

$$H_3(j\omega) = y_s(j\omega) - y_u(j\omega) \quad (8)$$

y_s - Vertical displacement of sprung mass
 y_u - vertical displacement of un sprung mass
 The responses of the vehicle model are respectively, the vertical vehicle body acceleration (\ddot{y}_s), the force applied between road and wheel (F_1), the relative displacement between wheel and vehicle body ($y_s - y_u$).

The discomfort is evaluated by computing the standard deviation of the vertical vehicle body acceleration ($\sigma_{\ddot{y}_s}$). The higher the standard deviation, the higher is the discomfort. The road holding is evaluated by computing the standard deviations of the tire radial force (σ_{F_1}). The variation of tyre radial force can lead to a loss of contact with the ground and poor handling ability. The working space is evaluated by computing the standard deviations of the relative displacement between wheel and vehicle body ($\sigma_{y_s - y_u}$) i.e., the working space is related to design and packing constraints, as well as to wheel lateral vibrations. Road holding and working space is strictly related to active safety.

Discomfort ($\sigma_{\ddot{y}_s}$), road holding (σ_{F_1}) and working space ($\sigma_{y_s - y_u}$) are the objective functions.

2. TRANSFER FUNCTIONS OF THE PROPOSED MODEL

The ratio of the Laplace transform of the out put variable to the input variable (i.e.) the transfer function, under the assumption that all initial conditions are zero, can be written as follows:

The transfer function between displacement ξ and y_u is given by

$$y_u(j\omega) = \frac{k_1(k_2 + j C_p \omega - m_2 \omega^2)}{D(j\omega)} \quad (3)$$

The displacement ξ (road irregularity) may be represented by a random variable defined by a stationary and ergodic stochastic process with zero mean value. The power spectral density (PSD) of the process may be determined on the basis of experimental measurements which are collected from literature [4].

2.1 Power spectral density of the process

In the present work, two representations for Power spectral densities are considered.

$$PSD_{\xi_1}(\omega) = \frac{A_b v}{\omega^2} \quad (9)$$



$$PSD_{\xi^2}(\omega) = \frac{A_v \omega_c}{\omega_c^2 + \omega^2} \quad \dots\dots\dots (10)$$

Where A_b, A_v are the road roughness parameters.

and $\omega_c = a.v$

The value of the coefficient a (rad/m) depends on the shape of the road irregularity spectrum and speed v (m/sec).

In a log-log scaled plot, Equation (9) takes the shape of a one-sloped power spectral density, which can be indicated as 1S-PSD.

A better correlation with measured spectra can be obtained by resorting to more complex spectra as suggested by different researchers. In a log-log scaled plot of the equation (10), power spectra density takes the shape of a two-slope which can be indicated as 2S-PSD. An idea of power spectral density for different load conditions can be had from Figure-2 and table as formulated by ISO [7].

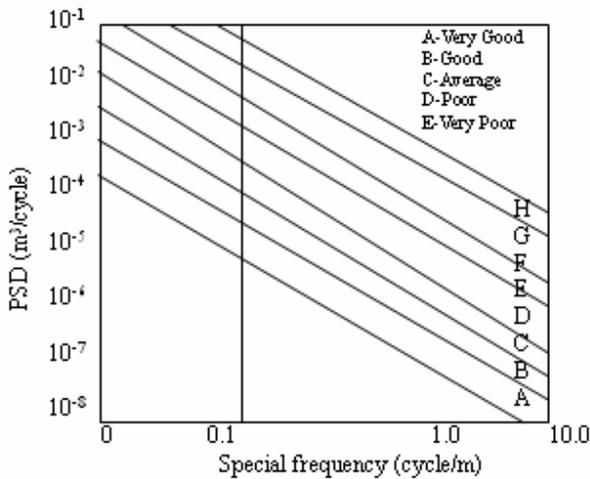


Figure-2. Road roughness classification by ISO.

Table-1. Road roughness values classified by ISO.

Degree of roughness $S(\Omega) \times 10^{-6}$		
Road class	Range	Geometric mean
A (very good)	< 8	4
B (Good)	8 – 32	16
C (average)	32 – 128	64
D (poor)	128 – 512	256
E (very poor)	512 – 2048	1024
F	2048 – 8192	4096
G	8192 – 32768	16384
H	> 32768	

2.2 Derivation of standard deviations in analytical form

The variance of a random variable described by a stationary and ergodic stochastic process is

$$\sigma_l^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} P_{SDI}(\omega) d\omega \quad \dots\dots\dots (11)$$

Analytical solution for σ_l^2 for PSDI can be written as

$$P_{SDI} = \frac{N_{k-1}(j\omega)N_{k-1}(-j\omega)}{D_k(j\omega)D_k(-j\omega)} \quad \dots\dots\dots (12)$$

Where D_k a polynomial of degree K, and N_{k-1} is a polynomial of a degree k-1.

3. FORMULAE REFERRING TO THE 1S-PSD

The analytical formulae giving the discomfort, road holding and working space are obtained by solving analytically the equation (11).

Variance of the vehicle body acceleration $\sigma_{\ddot{y}_s}$ (square of $\sigma_{\ddot{y}_s}$),

$$\sigma_{\ddot{y}_s}^2 = 1/2 A_b v \bar{\sigma}_{\ddot{y}_s}^2 \quad \dots\dots\dots (13)$$

$$\bar{\sigma}_{\ddot{y}_s}^2 = \frac{1}{m_2^2} \left(\frac{(m_2 + m_1)k_2^2}{c_p} + k_1 c_p \right) \quad \dots\dots\dots (14)$$

Variance of the force acting between road and wheel F_1 (square of σ_{F_1})

$$\sigma_{F_1}^2 = 1/2 A_b v \bar{\sigma}_{F_1}^2 \quad \dots\dots\dots (15)$$

$$\bar{\sigma}_{F_1}^2 = (m_2 + m_1)^2 (P) \quad \dots\dots\dots (16)$$

$$P = \left(\frac{(m_2 + m_1)k_2^2}{m_2^2 c_p} - \frac{2k_1 k_2 m_1}{m_2 c_p (m_2 + m_1)} + \frac{k_1^2 m_1}{c_p (m_2 + m_1)^2} + \frac{k_1 c_p}{m_2^2} \right)$$

Variance of the relative displacement between wheel and vehicle body $y_s - y_u$ (square of $\sigma_{y_s - y_u}$)

$$\sigma_{y_s - y_u}^2 = 1/2 A_b v \bar{\sigma}_{y_s - y_u}^2 \quad \dots\dots\dots (17)$$

$$\bar{\sigma}_{y_s - y_u}^2 = \frac{m_2 + m_1}{c_p} \quad \dots\dots\dots (18)$$

3.1 Analysis for the passenger bus using 1S-PSD

Table-2 gives reference values for the different variables for the passenger bus under investigation. Also shown are the lower and upper bound values, which are the limits to which the reference values can vary. By solving equations (13) to (18) using MATLAB, along with the reference vehicle parameters, acceleration, road holding and working space are computed.



Table-2. Data of the reference road vehicle for analysis.

Design variable	Reference value (r)	Lower and upper bound
m_1 (kg)	(m_{1r}) 2050	4100-1025
m_2 (kg)	(m_{2r}) 100	200-50
k_1 (N/m)	(k_{1r}) 2000000	4000000-1000000
k_2 N/m)	(k_{2r}) 400000	800000-200000
c_p (Ns/m)	(c_{pr}) 5000	10000-2500

Note: subscript r indicates reference values

Table-3. Data of the road roughness for analysis.

Parameter	Reference value
A_b (m)	1.4×10^{-5}
$a = \omega_c / v$ (rad / m)	0.4
A_v (m ²)	3.5×10^{-5}

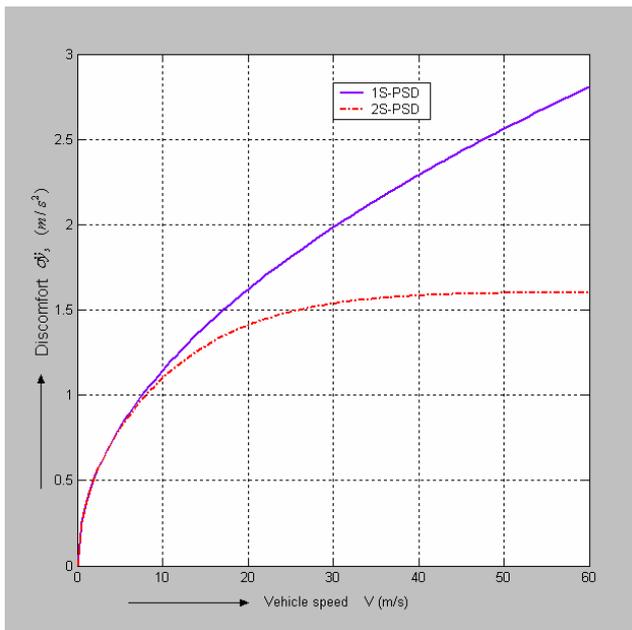


Figure-3. Discomfort as a function of vehicle speed (1S-PSD and 2S-PSD models).

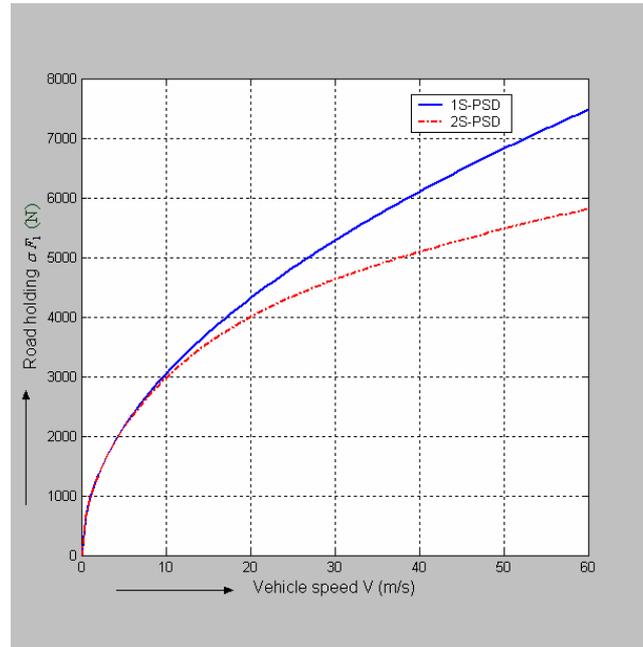


Figure-4. Road holding as a function of vehicle speed ((1S-PSD and 2S-PSD models).

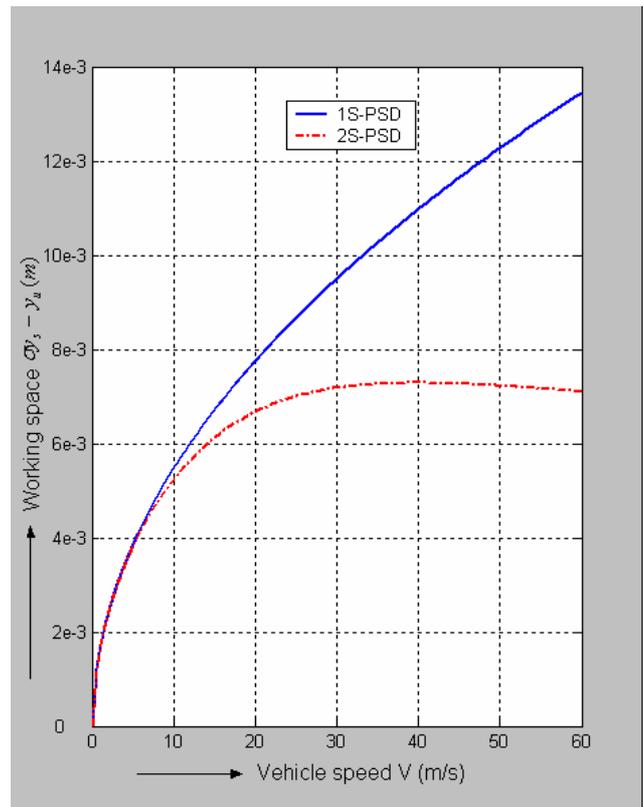


Figure-5. Working space as a function of vehicle speed (1S-PSD and 2S-PSD models).

Figures 3, 4 and 5 shows, respectively the standard deviations for discomfort (σ_{ϕ_d}), road holding (σ_{F_1}) and working space ($\sigma_{y_s - y_u}$) as function of the



vehicle speed (v) considering the reference vehicle while using 1S-PSD model.

Variance of the vehicle body acceleration $\sigma_{\ddot{y}_s}$ (square of $\sigma_{\ddot{y}_s}$)

3.2 Formulae referring to the 2S-PSD

The analytical formulae giving the discomfort, road holding and working space are obtained by solving analytically equation (12)

$$\sigma_{\ddot{y}_s}^2 = c_{rv} \frac{k_1^2 c_p^2 (k_2 + c_p \omega_c + m_2 \omega_c^2) + k_1 k_2^2 (m_1 + m_2) (k_2 + c_p \omega_c) + m_1 m_2 \omega_c^2}{(m_2^2 c_p (D_s))} \quad (19)$$

Where

$$D_s = k_1 k_2 + k_1 c_p \omega_c + k_2 (m_1 + m_2) \omega_c^2 + k_1 m_2 \omega_c^2 + (m_1 + m_2) c_p \omega_c^3 + m_1 m_2 \omega_c^4$$

$$\omega_c = a v$$

$$c_{rv} = 1/2 A_v a v$$

Variance of the force acting between road and wheel F_z (square of σ_{F_z})

$$\sigma_{F_z}^2 = c_{rv} \frac{A_{2s} + B_{2s} + C_{2s}}{k_1^2 m_2^2 c_p^2 (D_s)} \quad (20)$$

Where

$$A_{2s} = k_1^4 (m_1 + m_2) c_p (-2k_2 m_1 m_2 + m_1 c_p^2 + m_2 c_p^2) (k_2 + c_p \omega_c + m_2 \omega_c^2)$$

$$B_{2s} = k_1^3 k_2^2 (m_1 + m_2)^2 c_p (k_2 m_1 + k_2 m_2 + m_1 c_p \omega_c + m_2 c_p \omega_c + m_1 m_2 \omega_c^2)$$

$$C_{2s} = k_1^4 m_1 m_2^2 c_p (k_1 k_2 + k_1 c_p \omega_c + k_1 m_2 \omega_c^2 + k_2 m_2 \omega_c^2 + m_2 c_p \omega_c^3)$$

Variance of the relative displacement between wheel and vehicle body $y_s - y_u$ (square of $\sigma_{y_s - y_u}$)

$$\sigma_{y_s - y_u}^2 = c_{rv} \frac{k_1 (k_2 m_1 + k_2 m_2 + m_1 c_p \omega_c + m_2 c_p \omega_c + m_1 m_2 \omega_c^2)}{c_p (D_s)} \quad (21)$$

The main difference between the formulae referring to the 1S-PSD equations (13 –18) and those referring to 2S-PSD (19 –21) is that in the first set of formulae, the running condition parameters A_b and v are always not mixed with system model parameters (m_1, m_2, k_1, k_2, c_p). The opposite occurs for 2S-PSD formulae in which running conditions parameters ω_c, c_{rv} are mixed with model parameters (m_1, m_2, k_1, K_2, c_p).

This implies that for 1S-PSD excitation, the minima of $\sigma_{\ddot{y}_s}, \sigma_{F_z}, \sigma_{y_s - y_u}$ (as function of the suspension parameters) do not depend on running conditions (A_b, v).

3.3 Analysis of passenger bus using 2S-PSD

The results for the passenger bus using 2S-PSD are given in Figures 3, 4 and 5 along with those for 1S-PSD model. It can be seen that the results vary substantially, especially at high speeds. Thus, the 2S-PSD model provides a more realistic analysis of the suspension parameters.

4. PARAMETER SENSITIVITY ANALYSIS

The dynamic response of the road vehicle system model in Figure-1 is analyzed on the basis of Equations (13-21). By considering now the upper and lower bounds for different parameters. The formulae derived earlier have a general meaning and can be used for simulating the comfort, road holding and working space of every road vehicle that could be modeled as in Figure-1. For every new design, this procedure needs to be repeated.

An examination of equations (13), (15) and (17) for IS-PSD model shows that the non-dimensional standard deviations do not depend on vehicle speed. The opposite occurs for the non-dimensional standard deviations derived from equations (19-21) given by 2S-PSD model. For this reasons these non-dimensional standard deviations are analyzed at two different vehicle speeds, low speed (30m/s) and high speed (60 m/s).

The results of the various analyses are plotted as a function of parameter ratios like k_1/k_{1r} (i.e., present stiffness of spring to the reference vehicle spring stiffness). Similarly other ratios for which the response is plotted are $k_2/k_{2r}, m_1/m_{1r}, m_2/m_{2r}$ and c_p/c_{pr} .

The parameters are varied within wide ranges. The data are presented in non-dimensional form, i.e., the



standard deviation of interest σ_j is divided by the corresponding one (σ_{jr}) computed by considering the parameters at their reference values reported in Table 2, i.e.,

$$\sigma_{\sigma_{jr}} = \sigma_{\sigma_j} (m_{1r}, m_{2r}, k_{1r}, k_{2r}, c_{pr})$$

$$\sigma_{F1r} = \sigma_{F1} (m_{1r}, m_{2r}, k_{1r}, k_{2r}, c_{pr})$$

The results of the parameter sensitivity analysis are shown in Figures 6 to 20.

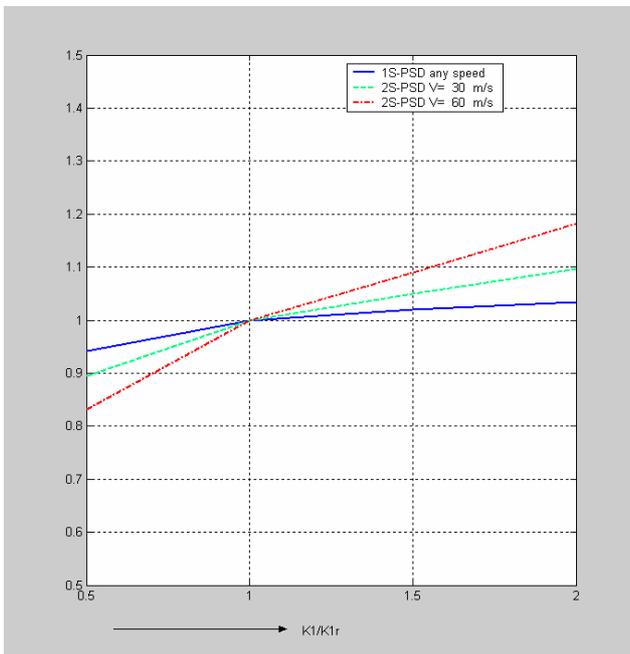


Figure-6. $\sigma_{\sigma_j} / \sigma_{\sigma_{jr}}$ vs. dimensionless ratio k_1/k_{1r} .

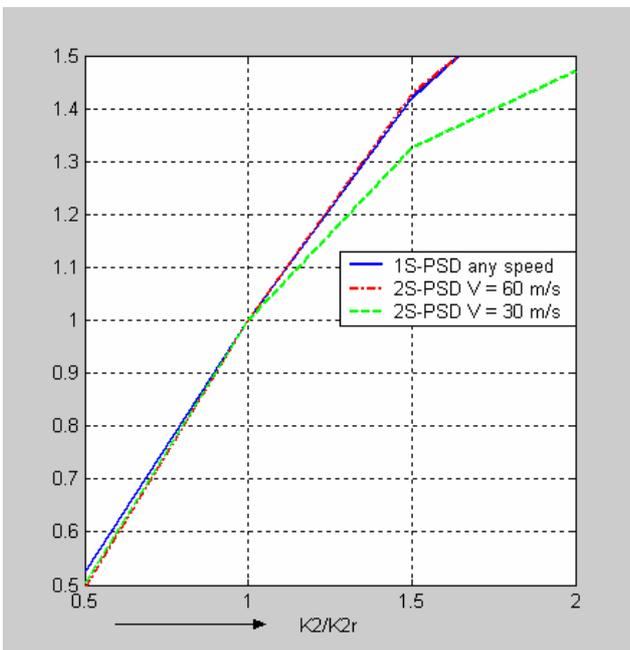


Figure-7. $\sigma_{\sigma_j} / \sigma_{\sigma_{jr}}$ vs. dimensionless ratio k_2/k_{2r} .

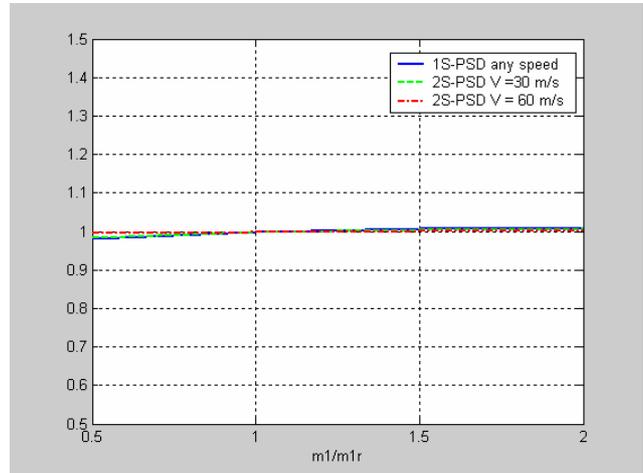


Figure-8. $\sigma_{\sigma_j} / \sigma_{\sigma_{jr}}$ vs. dimensionless ratio m_1/m_{1r} .

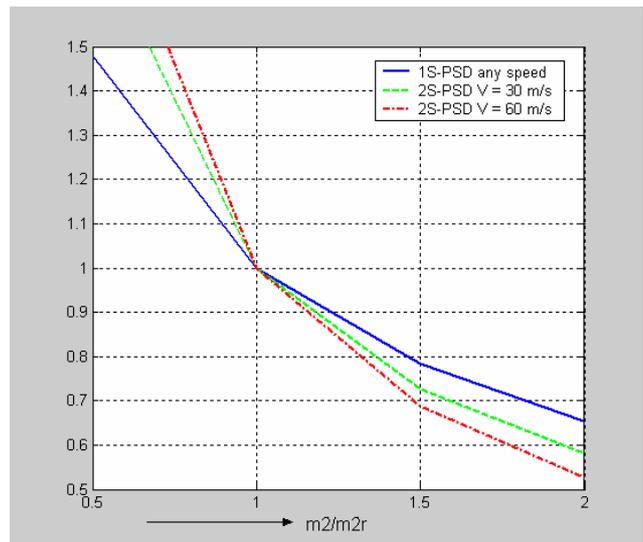


Figure-9. $\sigma_{\sigma_j} / \sigma_{\sigma_{jr}}$ vs. dimensionless ratio m_2/m_{2r} .

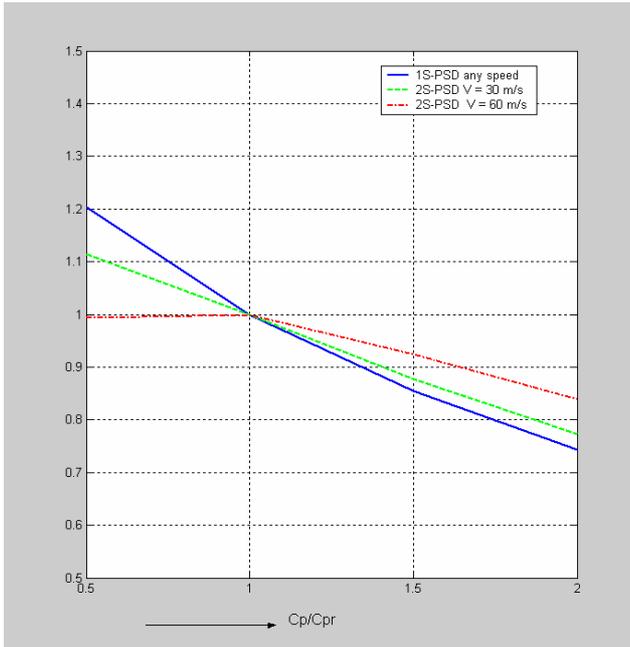


Figure-10. $\sigma_w / \sigma_{w,r}$ vs. dimensionless ratio $c_p/c_{p,r}$.

Figures 6 to 10 give the results of standard deviation of the discomfort i.e., $\sigma_w / \sigma_{w,r}$ as function of model parameters. Each diagram has been obtained by varying one single parameter, the other ones being constant and equal to those of the reference vehicle.

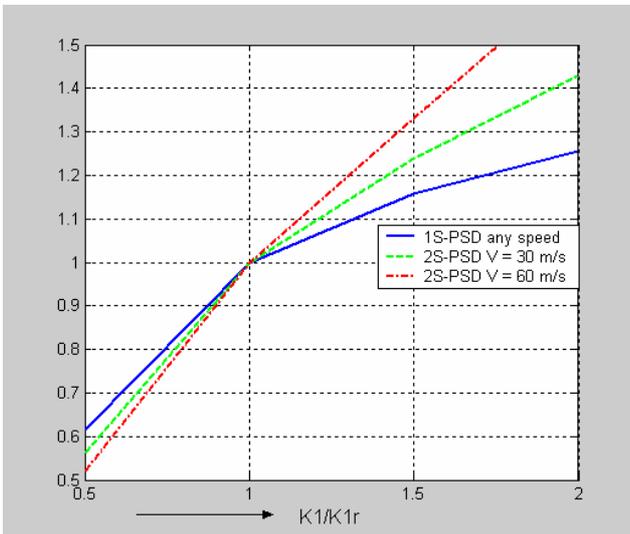


Figure-11. $\sigma_{F1} / \sigma_{F1,r}$ vs. dimensionless ratio $k_1/k_{1,r}$.

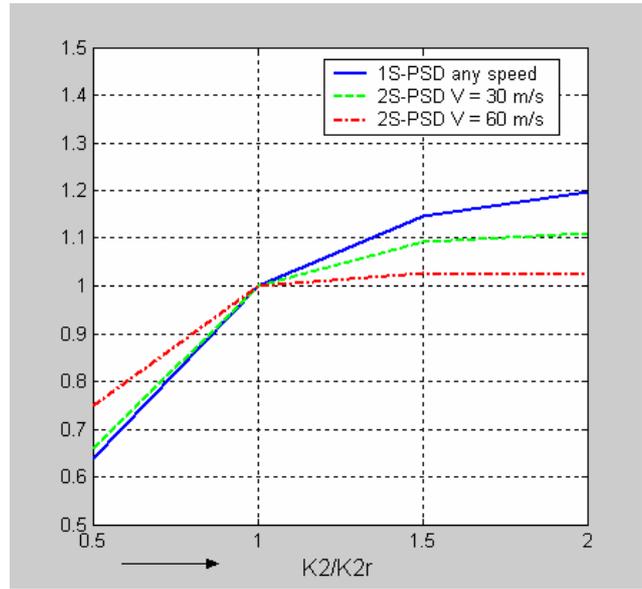


Figure-12. $\sigma_{F1} / \sigma_{F1,r}$ vs. dimensionless ratio $k_2/k_{2,r}$.

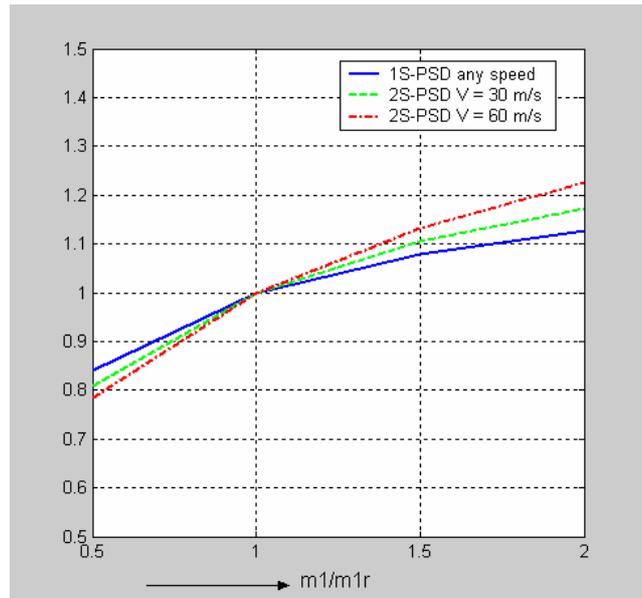


Figure-13. $\sigma_{F1} / \sigma_{F1,r}$ vs. dimensionless ratio $m_1/m_{1,r}$.

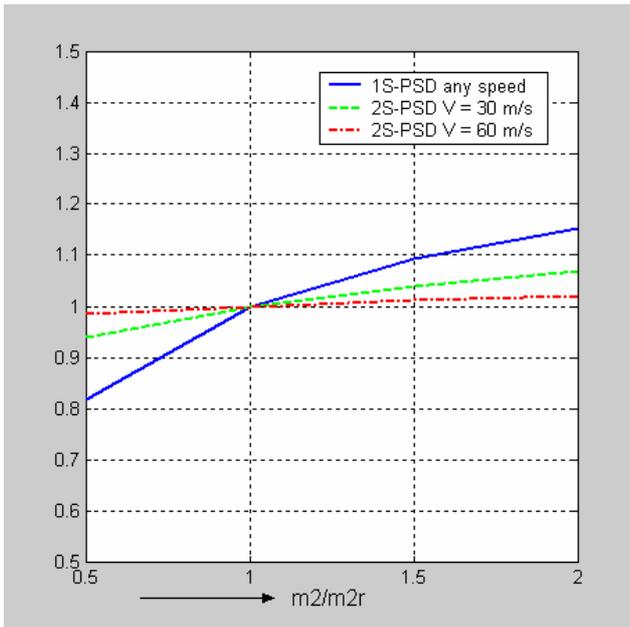


Figure-14. $\sigma_{F1} / \sigma_{F1r}$ vs. dimensionless ratio m_2/m_{2r} .

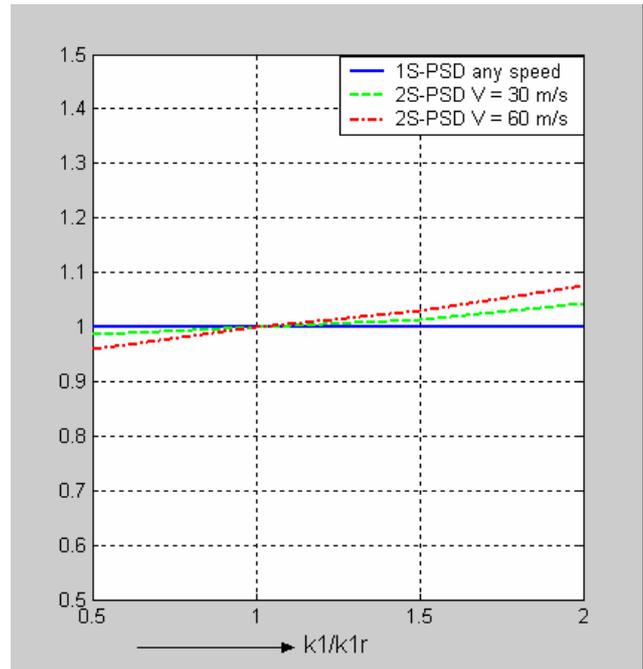


Figure-16. $\sigma_{y_s-y_u} / \sigma_{y_s-y_{ur}}$ vs. dimensionless ratio k_1/k_{1r} .

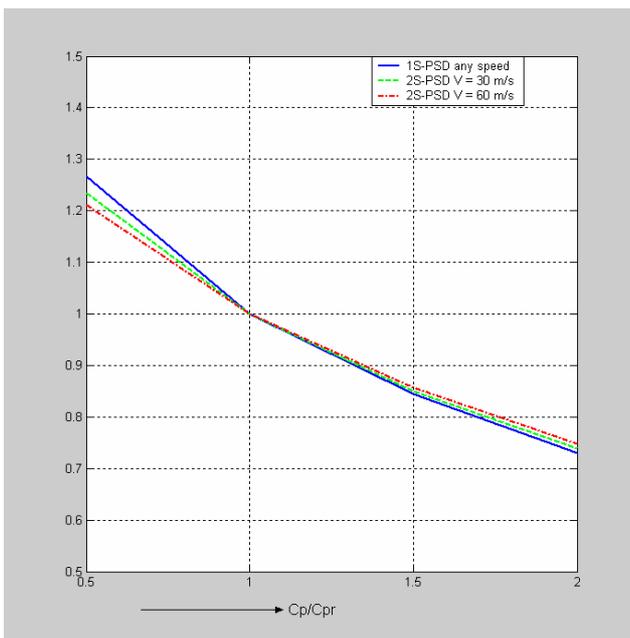


Figure-15. $\sigma_{F1} / \sigma_{F1r}$ vs. dimensionless ratio c_p/c_{pr} .

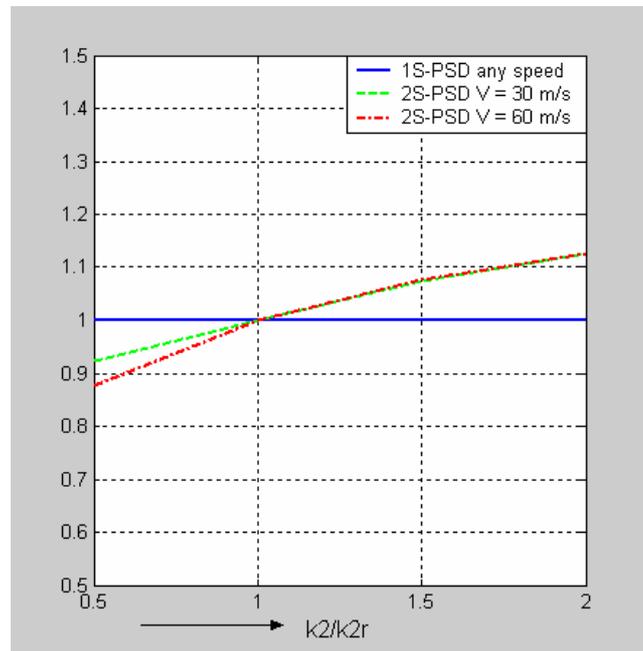


Figure-17. $\sigma_{y_s-y_u} / \sigma_{y_s-y_{ur}}$ vs. dimensionless ratio k_2/k_{2r} .

Figures 11 to 15 give the results of standard deviation of the road holding i.e., $\sigma_{F1} / \sigma_{F1r}$ as a function of model parameters. Each diagram has been obtained by varying one single parameter, the other ones being constant and equal to those of the reference vehicle.

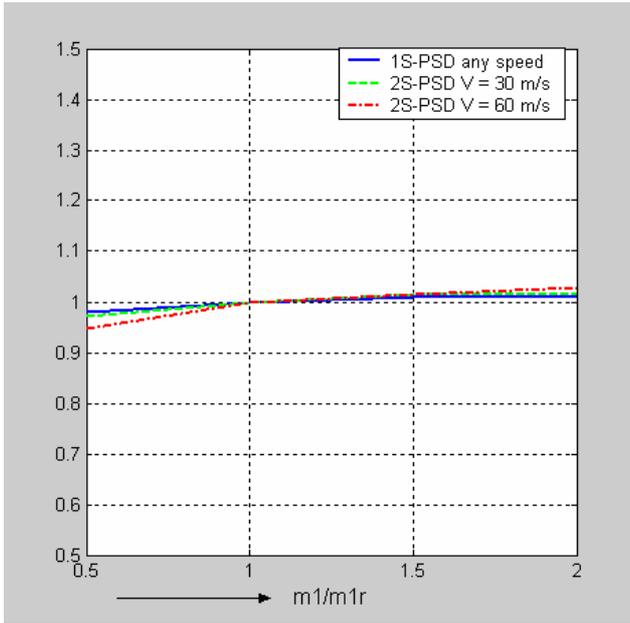


Figure-18. $\sigma_{y_s-y_u} / \sigma_{y_s-y_{ur}}$ vs. dimensionless ratio m_1/m_{1r} .

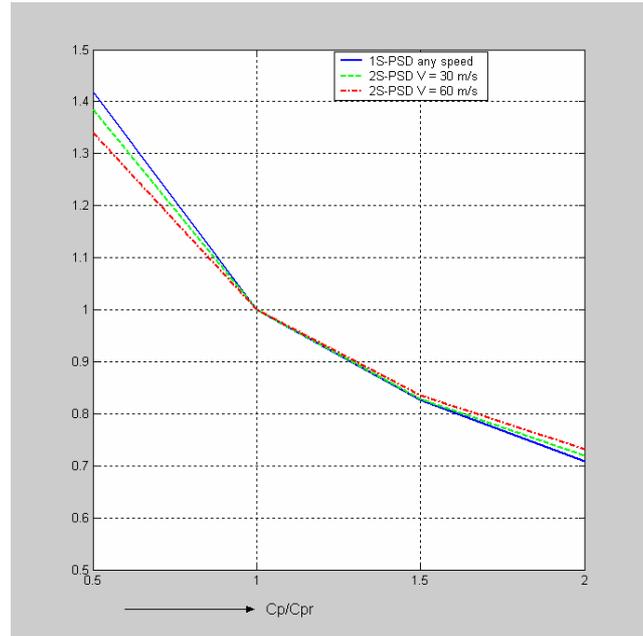


Figure-20. $\sigma_{y_s-y_u} / \sigma_{y_s-y_{ur}}$ vs. dimensionless ratio C_p/C_{pr} .

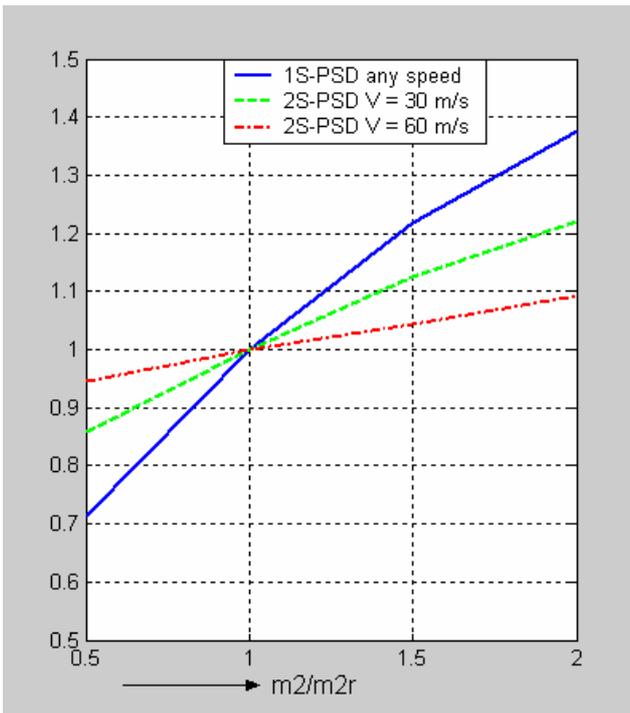


Figure-19. $\sigma_{y_s-y_u} / \sigma_{y_s-y_{ur}}$ vs. dimensionless ratio m_2/m_{2r} .

Figures 16 to 20 give the results of standard deviation of the working space i.e., $\sigma_{y_s-y_u} / \sigma_{y_s-y_{ur}}$ as function of model parameters. Each diagram has been obtained by varying one single parameter, the other ones being constant and equal to those of the reference vehicle.

5. CONCLUSIONS

The practical significance of the above described analysis can be appreciated if one considers the deterioration / variation of stiffness and damping due to wear and tear of components and presence of dust and mud collected.

It is observed from Figures 6 to 10 that:

- The tyre radial stiffness k_1 influences significantly (the influence is stronger at high speed considering the 2S-PSD)
- σ_{F1} Increases with the suspension stiffness k_2
- σ_{F1} Does not depend significantly on the wheel mass m_1
- σ_{F1} Depends strongly on the vehicle body mass m_2
- the suspension damping c_p has influence on the standard deviation

From the Figures 11 to 15, it is observed that:

- σ_{F1} Depends linearly on the tyre stiffness k_1
- σ_{F1} Increases with the suspension stiffness k_2 (almost the opposite occurs at high speed considering the 2S-PSD)
- σ_{F1} Increases with the wheel mass m_1



- σ_{F1} Does not depend significantly on the vehicle body mass m_2
- The suspension damping c_p has significant influence on the standard deviation σ_{F1}

It is also observed that the working space (y_s-y_u) is such that from the Figures, 16-20.

- $\sigma_{y_s-y_u}$ Is not influenced by k_1 and k_2 for the 1S-PSD excitation
- The influence of m_2 on $\sigma_{y_s-y_u}$ is less important at high speed for the 2S-PSD excitation
- $\sigma_{y_s-y_u}$ is strongly influenced by the suspension damping

As remarked by many earlier authors, the quarter car model has been a good aid for the preliminary design of vehicle suspension system. The quarter car when analyzed coupled with the parameter variabilities can yield results which are substantially better for performance improvement.

REFERNECES

- [1] Dokaimish M.A. and El-Madany M.H. 1980. Random response of tractor-semi trailer system Vehicle system dynamics. 9: 87-112.
- [2] Paddan G.S. and Grilfin M.J. 2002. Evaluation of whole- body vibration in vehicles. Journal of sound and vibrations. 253(11): 195-213.
- [3] Rill. G. 2005. Vehicle dynamics. Lecture notes, University of applied science, Regensburg.
- [4] Rovat D.H. and Hubbard .M. 1981. Optimal vehicle suspension minimizing Rms Rattle space spring-mass acceleration and jerk. Transaction of the ASME. 103: 228-236.
- [5] Kong Huiguo. Statistical analysis of vehicle vibration and dynamic load, and selection of suspension design parameters. UM-MEAM-82-15.
- [6] Mastinu G., Gobbi M, Miano C. 2006. Optimal design of complex mechanical system- with applications to vehicle engineering. Springer.
- [7] Esmailzadeh E and Taghirad H.D. 1996. Active vehicle suspensions with optimal state feedback control. Journal of Mechanical Science.